Aalto university
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Exercise sheet 7
Complex Analysis, MS-C1300.
Hand in exercise 1 and 2 in separate files for grading. Deadline Monday 13.11 at 23:59. The exercises should be uploaded to the correct folder on MyCourses as pdf-files with name and student number in the file name. **Submission via MyCourses is the only accepted way.** Done during class Tuesday 14.11 or Wednesday 15.11.

(1) By means of the local Cauchy Theorem (Cauchy's Theorem in a disk), assuming that the circles are positively oriented, calculate:
(a)
$$\int_{|z|=1} \sqrt{9-z^2}\, dz$$
(3p)
(b)
$$\int_{|z|=1} \frac{1}{z^2+2z}\, dz$$
(3p)

(2) Let $\gamma(t) = 2\cos t + i \sin t$ for $0 \le t \le 2\pi$. Evaluate:
(a)
$$\int_\gamma \frac{1}{z}\, dz$$
(3p)
(b)
$$\int_\gamma \frac{1}{z^2+2iz}\, dz$$
(3p)
(*Hint*: Winding numbers and partial fractions help here.)

(3) Calculate
$$\int_{|z+i|=3/2} \frac{1}{z^4+z^2}\, dz$$
where the trajectory $|z+i| = 3/2$ is positively oriented.

(4) Let γ and β be closed, piecewise smooth paths in $\mathbb{C}$ with the same initial point. Show that $n(-\gamma, z) = -n(\gamma, z)$ for every

$z \in \mathbb{C} \setminus |\gamma|$ and that $n(\gamma + \beta, z) = n(\gamma, z) + n(\beta, z)$ for every $z \in \mathbb{C} \setminus (|\gamma| \cup |\beta|)$.