Aalto university
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Exercise sheet 12
Complex Analysis, MS-C1300.
Hand in exercise 1 and 2 in separate files for grading. Deadline Wednesday 29.11 at 23:59. The exercises should be uploaded to the correct folder on MyCourses as pdf-files with name and student number in the file name. **Submission via MyCourses is the only accepted way.** Done during class Thursday 30.11 or Friday 1.12.

(1) Obtain the Laurent series expansions of

$$f(z) = \frac{1}{2z - z^2}$$

and

$$g(z) = \frac{2 - 2z}{(2z - z^2)^2}$$

in:
(a) $\Delta^*(0, 2) = \{z \in \mathbb{C}; 0 < |z| < 2\}$ (2p)
(b) $\Delta^*(2, 2) = \{z \in \mathbb{C}; 0 < |z - 2| < 2\}$ (2p)
(c) $D = \{z \in \mathbb{C}; |z| > 2\}$ (2p)

(2) Let $\gamma(t) = e^{it}$, for $0 \leq t \leq 2\pi$. Calculate

$$\int_\gamma f(z)\, dz$$

when:
(a) $f(z) = z^3 e^{-1/(z^4)}$ (3p)
(b) $f(z) = (z^2 + z)\sin(1/z)$ (3p)

(3) Calculate

$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + \pi^2)^2}\, dx.$$

(*Hint:* Evaluate

$$\lim_{R\to\infty} \int_{\gamma(R)} \frac{e^{iz}}{(z^2 + \pi^2)^2}\, dz,$$

where $\gamma(R) = [-R, R] + \tilde{\gamma_R}$, and $\tilde{\gamma}_R(t) = Re^{it}$, $0 \leq t \leq \pi$.)

(4) Calculate

$$\int_{|z-e|=2} \frac{1}{(z - 1)\operatorname{Log}(z)}\, dz$$

where $|z - e| = 2$ is positively oriented.