ELEC-E8116 Model-based control systems /exercises 11

Problem 1. Consider the general system representation

$$\dot{x} = Ax + Bu + Nw$$
$$z = Mx + Du$$
$$y = Cx + w$$

where it is assumed that

$$D^T \begin{bmatrix} M & D \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}$$

Show that this assumption can be relaxed by taking

$$\tilde{u} = \left(D^T D\right)^{1/2} u + \left(D^T D\right)^{-1/2} D^T Mx \quad \text{and}$$

$$z = \tilde{M}x + \tilde{D}\tilde{u}, \quad \tilde{M} = \left(I - D\left(D^T D\right)^{-1} D^T\right)M, \quad \tilde{D} = D\left(D^T D\right)^{-1/2}$$

Problem 2. The *Frobenius norm* of a matrix A is defined as

$$\|A\|_F = \sqrt{\sum_{i,j} |a_{ij}|^2} = \sqrt{\text{tr } (A^* A)}$$

Show that $\|A\|_F = \sqrt{\sum_i \sigma_i^2(A)}$, where $\sigma_i(A)$ are the singular values of A.