ELEC-E8107 - Stochastic models, estimation and control

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Exercises Session 5: Solution

Exercise 1

A nonlinear system dynamic model of a robot moving on the plane is given by the following equation.

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \theta_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta t cos(\theta_k) \\ 0 & 1 & 0 & \Delta t sin(\theta_k) \\ 0 & 0 & 1 & \frac{\Delta t}{L} tan(\phi) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ \theta_k \\ v_k \end{bmatrix} + q_k \tag{1}$$

Where v is the speed of the vehicle, θ is the heading and q_k is the process noise vector with covariance matrix Q_k. This covariance matrix can be assumed to be a diagonal matrix. The parameter ϕ is the steering angle and is considered a known input to the system. The constant parameter L is the distance between the front and back wheels of the robot. Here we assume $L = 15cm$.

Only the positions x and y of the robot are measured. The measurement noise is assumed Gaussian with zero mean and has 0.5 meters standard deviation. The measurement noise of the x-axis and y-axis are assumed independent.

1. Write the measurement equation for the system.
2. Implement the bootstrap particle filter to estimate the state of the system.

Solution Exercise 1

The bootstrap particle filter is one of the simplest (basic) forms of the particle filter method. It uses the dynamic model as the importance distribution. See the provided Matlab script for the implementation.

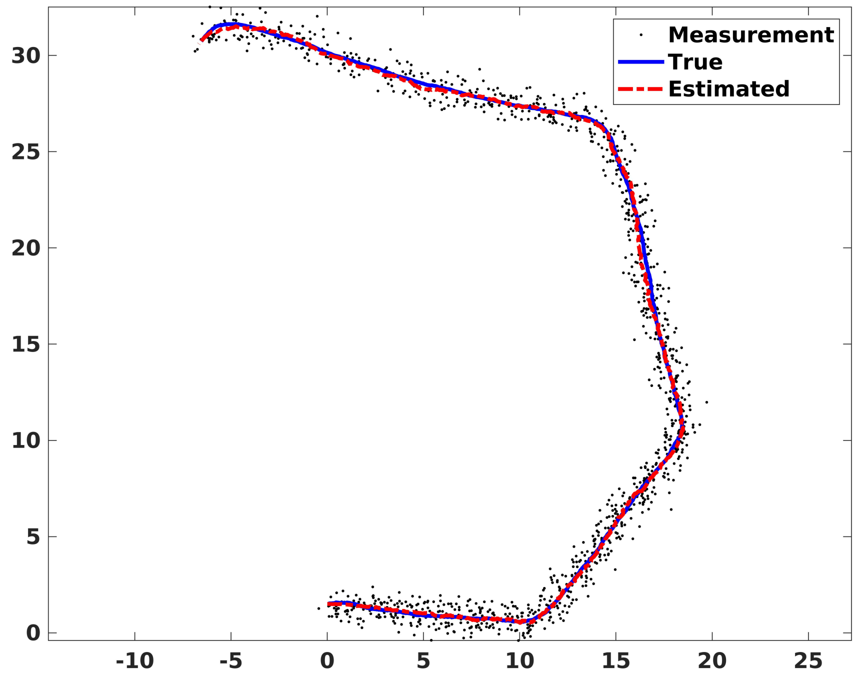

Figure 1: Comparison of PF estimate with the true trajectory and the measurement

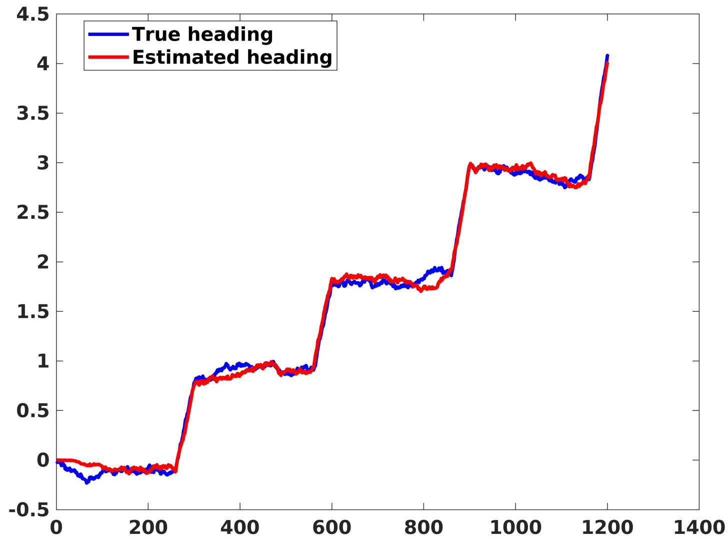

Figure 2: Comparison of PF estimate with the true heading

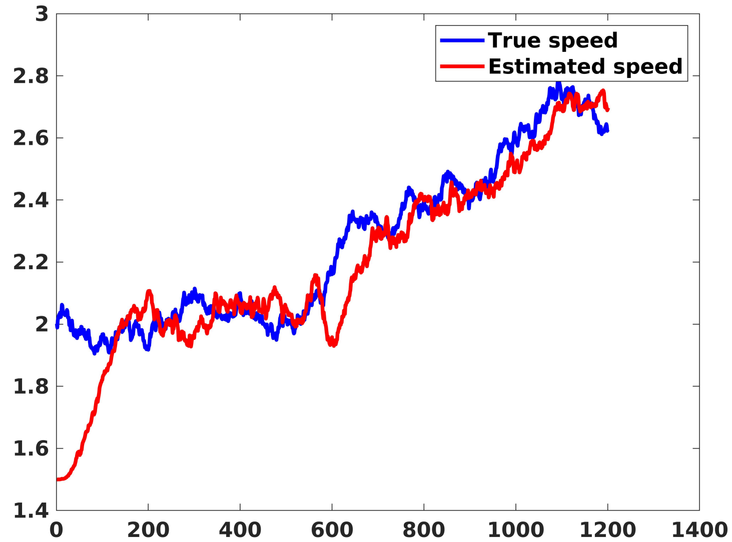

Figure 3: Comparison of PF estimate with the true speed

Exercise 2

The objective is to estimate the pose and movement of a ship moving in the inland waterways. The GPS position and the National Marine Electronics Association (NMEA) standard velocity, true speed, and ground heading (VTG) measurements are available.

The task is to set up an Extended Kalman Filter (EKF) to estimate the instantaneous position of the ship.

Solution Exercise 2

The model equations comprises of equations (2-5), which describe a vessel moving straight ahead with a constant speed. Together we denote them as $f(x, y, s, \psi, k)$. In our case, the control input u_k is not known so there is no deterministic control term in the equation. Thus, all the control inputs to the model can be considered as noises from the modeling point of view.

$$x_{k+1} = x_k + \Delta t s_k \cos(\psi_k) + \eta_k \tag{2}$$
$$y_{k+1} = y_k + \Delta t s_k \sin(\psi_k) + \zeta_k \tag{3}$$
$$s_{k+1} = s_k + \nu_k \tag{4}$$
$$\psi_{k+1} = \psi_k + \kappa_k \tag{5}$$

In the measurement model, the 2D position coordinates in latitude and longitude format together with the heading angle are available as measurements. Note that, one degree in latitude direction corresponds to 111.12 km. The distance corresponding to one degree in longitudinal direction depends on the location of the vessel in latitude coordinates. However, for small traveled distances it can be assumed that the distance corresponding to one degree in the longitudinal direction can be approximated with the cosine of latitude angle in degrees multiplied by 111.12 km. Thus, we write the measurement model as:

$$\mathrm{lat}_k = \frac{x_k}{111120} + \mathrm{lat}_0 + \alpha_k \tag{6}$$
$$\mathrm{lng}_k = \frac{y_k}{111120 \cos(\mathrm{lat}_k)} + \mathrm{lng}_0 + \beta_k \tag{7}$$
$$\mathrm{hdg}_k = \psi_k + \gamma_k \tag{8}$$

The system and measurement models are represented as

$$\begin{aligned} x(k+1) &= f\Big(x(k)\Big) + q(k) \\ y(k) &= h\Big(x(k)\Big) + r(k) \end{aligned}$$

The correction (or, update) step is written as

$$\begin{aligned} \hat{x}(k|k) &= \hat{x}(k|k-1) + G(k)\Big(y(k) - h\Big((\hat{x}(k|k-1)\Big)\Big) \\ G(k) &= P(k|k-1)H^T\Big(HP(k|k-1)H^T + R\Big)^{-1} \\ P(k|k) &= P(k|k-1) - G(k)HP(k|k-1)^T \end{aligned}$$

with the prediction step given as

$$\begin{aligned} \hat{x}(k+1|k) &= f\Big(\hat{x}(k|k)\Big) \\ P(k+1|k) &= FP(k|k)F^T + Q. \end{aligned}$$

Notice that the symbols are otherwise the same as in the KF, but matrices F and H are the Jacobian matrices computed at $\hat{x}(k|k-1)$ in the correction/update step. So, an extra computational workload is present in EKF as these Jacobians have to be computed at each update step. Whereas in the case of KF, the matrices F and H were assumed time-invariant. Moreover, if only one of the systems or measurement models is non-linear, the Jacobian matrix need only be calculated for that model. In our example, the state vector contains $\{x, y, s, \psi\}$ as state variables.
The Jacobians are computed as;

$$\begin{aligned} F &= \begin{bmatrix} \frac{\partial x_{k+1}}{\partial x_k} & \frac{\partial x_{k+1}}{\partial y_k} & \frac{\partial x_{k+1}}{\partial s_k} & \frac{\partial x_{k+1}}{\partial \psi_k} \\ \frac{\partial y_{k+1}}{\partial x_k} & \frac{\partial y_{k+1}}{\partial y_k} & \frac{\partial y_{k+1}}{\partial s_k} & \frac{\partial y_{k+1}}{\partial \psi_k} \\ \frac{\partial s_{k+1}}{\partial x_k} & \frac{\partial s_{k+1}}{\partial y_k} & \frac{\partial s_{k+1}}{\partial s_k} & \frac{\partial s_{k+1}}{\partial \psi_k} \\ \frac{\partial \psi_{k+1}}{\partial x_k} & \frac{\partial \psi_{k+1}}{\partial y_k} & \frac{\partial \psi_{k+1}}{\partial s_k} & \frac{\partial \psi_{k+1}}{\partial \psi_k} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & \Delta t \cos(\psi_k) & -\Delta t s_k \sin(\psi_k) \\ 0 & 1 & \Delta t \sin(\psi_k) & \Delta t s_k \cos(\psi_k) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

with

$$H = \begin{bmatrix} \frac{\partial \text{lat}_k}{\partial x_k} & \frac{\partial \text{lat}_k}{\partial y_k} & \frac{\partial \text{lat}_k}{\partial s_k} & \frac{\partial \text{lat}_k}{\partial \psi_k} \\ \frac{\partial \text{lng}_k}{\partial x_k} & \frac{\partial \text{lng}_k}{\partial y_k} & \frac{\partial \text{lng}_k}{\partial s_k} & \frac{\partial \text{lng}_k}{\partial \psi_k} \\ \frac{\partial \text{hdg}_k}{\partial x_k} & \frac{\partial \text{hdg}_k}{\partial y_k} & \frac{\partial \text{hdg}_k}{\partial s_k} & \frac{\partial \text{hdg}_k}{\partial \psi_k} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{111120} & 0 & 0 & 0 \\ 0 & \frac{1}{111120\cos(\text{lat}_0)} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In principle, the covariance matrices Q and R can be estimated from the instrument calibrations, and/or from the modeling experiments. Ultimately, the covariance matrices are the designer's control parameters, which are mainly determined through the trial-and-error method.