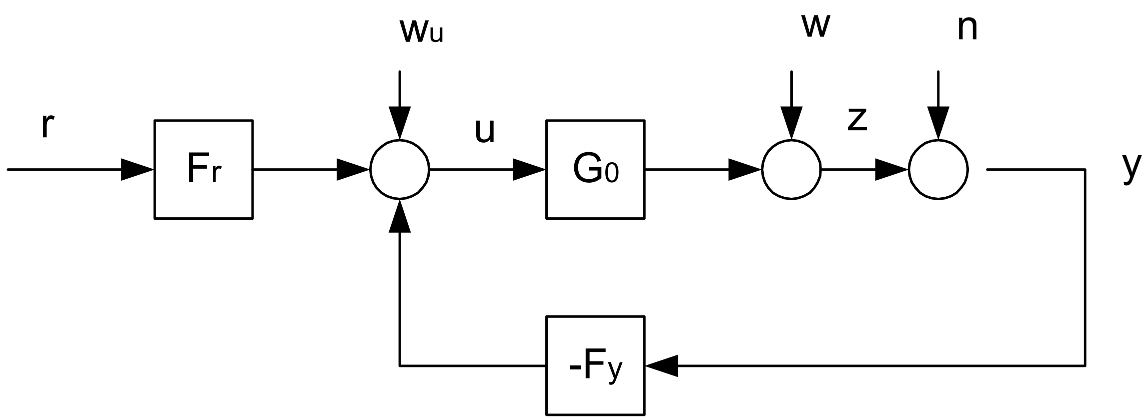
Chapter 7

- LQG control
- Robustness of LQG control
- Loop transfer recovery (LTR)
- Formal loop shaping
- Some extra material for your own reading (not required)
- Final steps in the course
- Course end

Chapter 7: *LQG*-control



$$z = Gu + w$$

$$u = F_r r - F_y y$$

$$e = z - r$$

$$y = z + n$$

Note:

LQG theory (Linear-Quadratic-Gaussian) means optimal (LQ) control with Gaussian noise disturbances present. The stochastic theory of continuous time systems is difficult.

It is possible to present the theory in a ”simplified” form, but that is omitted here. The important thing is to know that noise intensities (variance does not exist for continuous time stochastic signals) are mostly used as tuning parameters only. The optimal state estimator, Kalman filter, is used to estimate the states, which are fed back according to the LQ theory.

LQG = Kalman filter + LQ

Spectral description of disturbances

To a *m*-dimensional *u*(*t*) a hermitian *m x m*-matrix $\Phi_u(\omega)$

is attached, (spectrum, spectral density)

If *G* is a linear and stable system, then

$$y(t) = G(p)u(t)$$

$$\Phi_y(\omega) = G(i\omega)\Phi_u(\omega)G^*(i\omega)$$

The spectral density is defined

$$\Phi_u(\omega) = U(i\omega)U^*(i\omega)$$

in which

$$U(i\omega) = \int_{-\infty}^{\infty} u(t)e^{-i\omega t}dt$$

Fourier-transformation of the signal

$$R_u = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_u(\omega)d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}U(i\omega)U^*(i\omega)d\omega = \int_{-\infty}^{\infty}u(t)u^T(t)dt$$

Covariance vs. signal size (Parseval's theorem) *m x m*-dimensional matrix

$$R_u = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_u(\omega)d\omega$$

measures the size of signal *u* by weighting the frequency components.

Cross-correlation spectral density :

$$z = \begin{bmatrix} y \\ u \end{bmatrix} \qquad \Phi_z(\omega) = \begin{bmatrix} \Phi_{yy}(\omega) & \Phi_{yu}(\omega) \\ \Phi_{uy}(\omega) & \Phi_{uu}(\omega) \end{bmatrix}$$

hermitian $\quad \Phi_{yu}(\omega) = \Phi^{*}_{uy}(\omega)$

Two signals do not correlate, if their cross-correlation spectral density is identically zero.

The signal is called **white noise**, intensity *R*, if its spectral density is constant at the the frequency range

$$\Phi_e(\omega) \equiv R$$

R is known as the **intensity** of the signal.

The history of a white noise signal does not give any information of the future values of the signal.

General state-space realization of the process is

$$\dot{x} = Ax + Bu + Nv_1$$
$$z = Mx$$
$$y = Cx + v_2$$

and the criterion

$$\min\left(\|e\|_{Q_1}^2 + \|u\|_{Q_2}^2\right) = \min\int\left[e^T(t)Q_1e(t) + u^T(t)Q_2u(t)\right]dt$$

$$e = z - r$$

Consider the *regulator problem* ($r = 0$)

$$\dot{x} = Ax + Bu + Nv_1$$
$$z = Mx$$
$$y = Cx + v_2$$

white noise $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ with intensity $\begin{bmatrix} R_1 & R_{12} \\ R_{12}^T & R_2 \end{bmatrix}$

Determine

$u(t) = -F_y(p)y(t)$ such that

$V = \|z\|_{Q_1}^2 + \|u\|_{Q_2}^2$ is minimized

Solution (without proof)**:** Let (A,B) be stabilizable and (A,C) detectable.

The optimal control law is

$$\boxed{\begin{aligned} u(t) &= -L\hat{x}(t) \\ \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + K\left[y(t) - C\hat{x}(t)\right] \end{aligned}}$$

in which the Kalman gain K is obtained by the Riccati equation

$$AP + PA^T + NR_1N^T - \left(PC^T + NR_{12}\right)R_2^{-1}\left(PC^T + NR_{12}\right)^T = 0$$

$$K = \left(PC^T + NR_{12}\right)R_2^{-1}$$

and the state feedback coefficient L

$$L = Q_2^{-1} B^T S$$

where S is the solution to the stationary Riccati equation (LQ)

$$A^T S + SA + M^T Q_1 M - SBQ_2^{-1} B^T S = 0$$

(symmetric and positive semidefinite solution)

(Note that only infinite optimization horizons are considered here, so that the stationary Riccati equations can be used.)

The solution has the *separation property* : the optimal state observer and optimal state feedback can be designed independent of each other. The whole solution is then the ”combination” of these.

If the states are measurable , $y = x$, the Kalman-filter is not needed and the state feedback is formed directly from the state.

The theory guarantees that the resulting closed-loop system is stable.

Terms: LQ (linear quadratic)
LQG (linear quadratic gaussian)
ARE (algebraic Riccati equation)
(Separation principle)

Matlab:

kalman, estim
lqgreg
lqr, dlqr
lqe, dlqe
lqrd
sigma, dsigma

But how about the robustness of the *LQ* (*LQG*) – controller ?

$$e = (I - G_c)r - Sw + Tn$$

$$u = G_{ru} r + G_{wu}(w + n)$$

Not necessarily very good!

The weight matrices and noise intensities can be thought to be *tuning parameters* in control design.

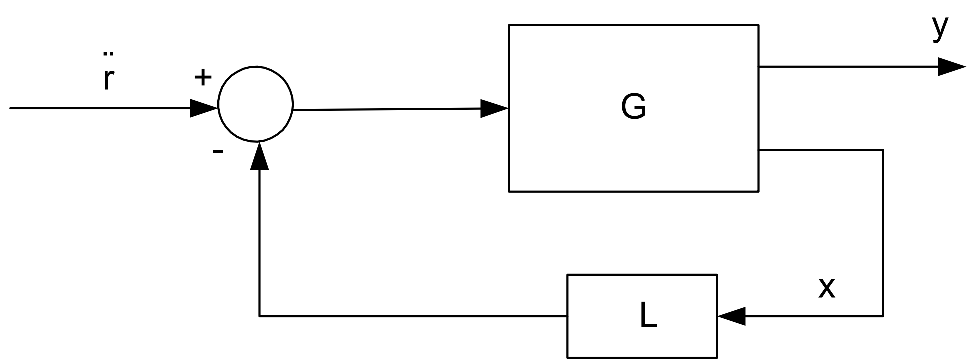
After design the frequency domain analysis and simulations must be carried out to verify the performance of the controller. Next, consider robustness a bit closer.

Robustness of LQ/LQG-controllers

LQ:

$$u(t) = -Lx(t)$$

$$\dot{x}(t) = Ax(t) + Bu(t)$$



The loop transfer function (gain in control signal)

$$G_k(s) = L\left(sI - A\right)^{-1} B$$

But in that transfer function

$$H(s) = L(sI - A)^{-1} B$$

L is determined from the equations

$$L = Q_2^{-1} B^T S$$
$$A^T S + SA + Q_1 - SBQ_2^{-1} B^T S = 0$$

and now it holds (apply the lemma 5.2 in the textbook)

$$\left[I + H(-i\omega)\right]^T Q_2 \left[I + H(i\omega)\right] \geq Q_2$$

which in *SISO*-case is

$$\left|1 + H(i\omega)\right| \geq 1$$

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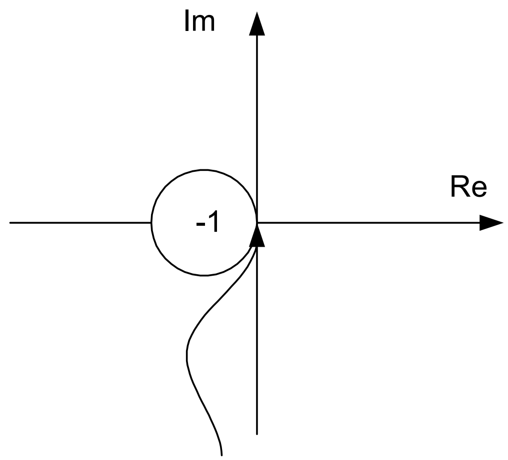


That means that the Nyquist curve will never go inside the circle shown in the figure:

-phase margin at least 60 degrees
-gain margin infinite
-the magnitude of the sensitivity function is less than one
-the magnitude of the complementary sensitivity function is smaller than two.

LQG: $$u(t) = -L\hat{x}(t) + \tilde{r}(t)$$

$$p\hat{x}(t) = A\hat{x}(t) + Bu(t) + K\left[y(t) - C\hat{x}(t)\right]$$

It follows

$$u(t) = -F_y(p)y(t) + F_r(p)\tilde{r}(t)$$

$$F_y(p) = L\left(pI - A + BL + KC\right)^{-1} K$$

$$F_r(p) = I - L\left(pI - A + BL + KC\right)^{-1} B$$

and the loop gains are

$$GF_y = C\left(sI - A\right)^{-1} BL\left(sI - A + BL + KC\right)^{-1} K$$

$$F_yG = L\left(sI - A + BL + KC\right)^{-1} KC\left(sI - A\right)^{-1} B$$

looked from the output or input side of the process . (For *SISO*-systems the functions are the same.)

But now the good robustness properties do not necessarily hold, even though K and L have been chosen according to the *LQG*-formulas (the phase margin can even be arbitrarily small).

An idea to fix that problem: let L be chosen as above. Can K be chosen such that

$$F_y G = L\left(sI - A + BL + KC\right)^{-1} KC\left(sI - A\right)^{-1} B \approx L(sI - A)^{-1} B$$

which would make it possible to enjoy the ”ideal” loop transfer function again.

Result: Yes, it is possible by choosing

$$K = \rho B$$

where ρ is large enough. That holds generally and also in MIMO case; the number of inputs and outputs must be the same.

This technique is called the **loop transfer recovery** (*LTR*).

The idea is to calculate *L* as the solution to the optimal control problem and then change *K* as described above. (To increase ρ until the desired sensitivity functions are obtained.) But there is no guarantee that the filter remains stable.

Use another procedure to aim at $K = \rho B$

$$\dot{x} = Ax + Bu + Nv_1$$

$$z = Mx$$

$$y = Cx + v_2$$

$$K = PC^T R_2^{-1}$$

$$PA^T + AP - PC^T R_2^{-1} CP + NR_1 N^T = 0$$

(Kalman)

Choose

$R_1 = \alpha R_2, \quad N = B$ α being ”large”. Then

$$PA^T + AP - KR_2 K^T + \alpha BR_2 B^T = 0$$

where the last two terms dominate, as α grows. Hence

$$K \approx \sqrt{\alpha} B$$

holds, and also the Kalman-filter remains Ok.

Now the tuning parameters were N, R_1 and R_2.

Note. The presented method was *input-LTR*, because the loop transfer function was $F_y G$ (gain of the input signal)

There exists also an *output-LTR* method based on GF_y

In *SISO*-case the two methods are the same.

Formal "Loop shaping"

- Classical idea: use a compensator to get a desired loop gain
- New approach: minimization of H_∞, H_2 –norms
- Use of weights, sensitivity functions

System $\quad y = Gu + w$

Control law $\quad u = -F_y y$

Sensitivity functions

$$S = \left(I + GF_y\right)^{-1}$$

$$T = I - S = \left(I + GF_y\right)^{-1} GF_y = GF_y \left(I + GF_y\right)^{-1}$$

$$G_{wu} = -\left(I + F_y G\right)^{-1} F_y = -F_y \left(I + GF_y\right)^{-1} = -F_y S$$

and weight matrices

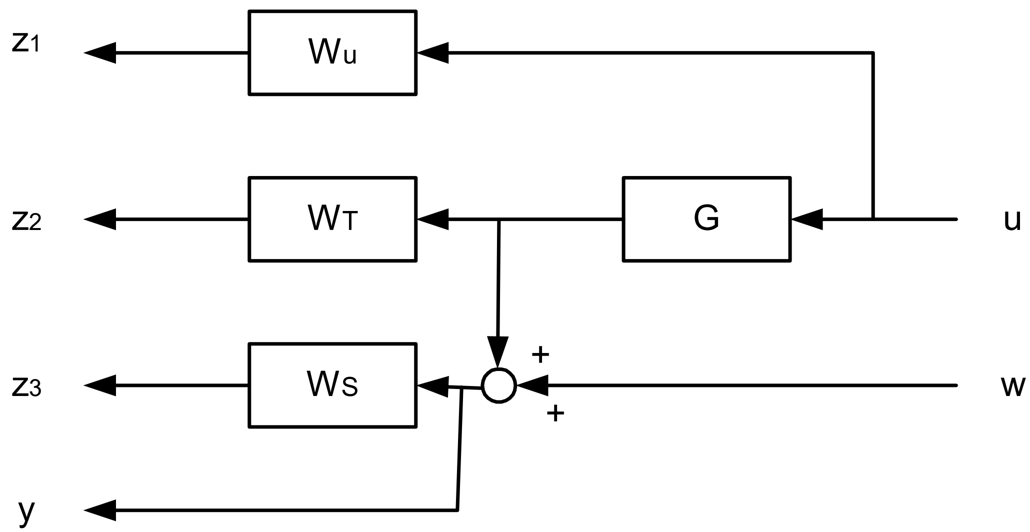
$$W_S(i\omega)S(i\omega)$$

$$W_T(i\omega)T(i\omega)$$

$$W_u(i\omega)G_{wu}(i\omega)$$

which are used to ”shape” the sensitivity functions. But the dimension of the system and the compensator grow.

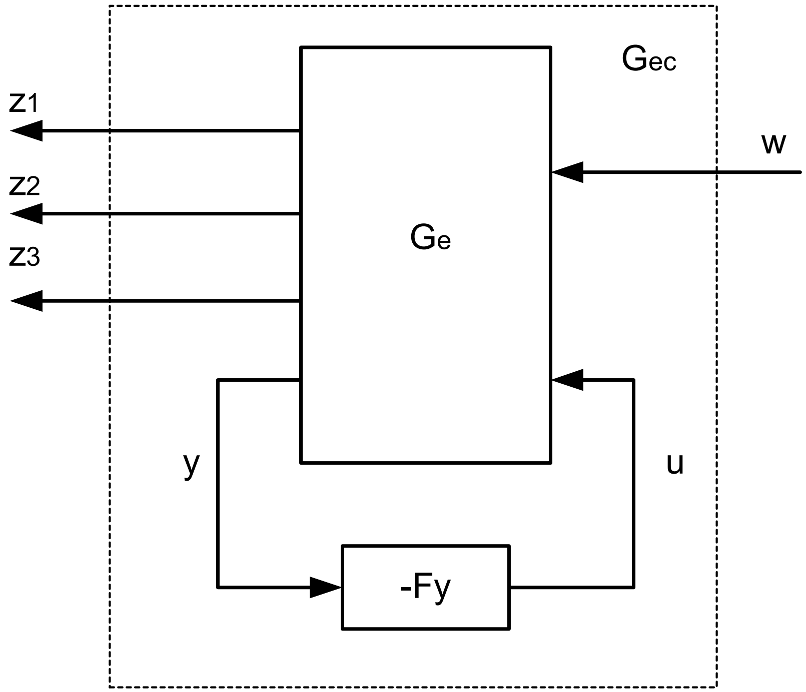
Generalized control configuration



$$z_1 = W_u u$$
$$z_2 = W_T G u$$
$$z_3 = W_S (Gu + w)$$
$$y = Gu + w$$

"weights" are used to form an augmented system

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$



Generalized control configuration. When the control loop is closed, the transfer function from *w* to *z* is obtained.

$$z = \begin{bmatrix} W_u G_{wu} \\ -W_T T \\ W_S S \end{bmatrix} w = G_{ec} w$$

and the motivation of using the weights becomes obvious. Similarly, it is clear why the norms between *w* and *z* are minimized for performance.

Control design can be carried out based on the generalized process model *Ge* e.g. by using the Matlab commands *mixsyn, hinfsyn* and *h2syn* (Robust Control Toolbox).

But look at the issue from theoretical viewpoint. Form a realization of the open loop system *Ge* with inputs *u,w* and outputs *z,y*

$$\dot{x} = Ax + Bu + Nw$$

$$z = Mx + Du$$

$$y = Cx + w$$

where certain assumptions have been made (*z* does not depend directly from *w*, *y* does not depend directly from *u*). Moreover, assume that $D^T \begin{bmatrix} M & D \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}$

The assumptions are sometimes restrictive (they are needed for the mathematic solution to be appropriate), but often they can be relaxed, if needed . (In this case the solution gets more difficult and nasty to derive.)

To see how this can be done, let $D^T D$ be invertible. Change variables from *u* to

$$\tilde{u} = \left(D^T D\right)^{1/2} u + \left(D^T D\right)^{-1/2} D^T M x$$

which gives

$$z = \tilde{M}x + \tilde{D}\tilde{u}, \quad \tilde{M} = \left(I - D\left(D^T D\right)^{-1} D^T\right)M, \quad \tilde{D} = D\left(D^T D\right)^{-1/2}$$

Now $\tilde{D}^T\left[\tilde{M} \;\; \tilde{D}\right] = [0 \;\; I]$ which is easy to verify.

Example. DC motor

$$G(s) = \frac{20}{s(s+1)}$$

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 20 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

Use simple weights $W_u = 1, W_T = 1, W_S = 1/s$

An augmented system model is obtained by taking the new state (see the figure) $z_3 = x_3, \quad x_3 = \frac{1}{p}(Gu + w)$

giving

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w$$

$$z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x + w$$

which also fulfils $D^T \begin{bmatrix} M & D \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}$

Control design is done from this model, the dimension of which has grown because of weights. In more complex cases it is more difficult to form the extended state representation. But see Matlab command *augw*.

In Matlab, the command *mixsyn* turns out to be helpful here.

```
[K,CL,GAM,INFO]=mixsyn(G,W1,W2,W3)

 mixsyn H-infinity mixed-sensitivity synthesis method for robust
 control design. Controller K stabilizes plant G and minimizes
 the H-infinity cost function

              ||   W1*S ||
              || W2*K*S ||
              ||   W3*T ||
 where
     S := inv(I+G*K)        % sensitivity
     T := I-S = G*K/(I+G*K)  % complementary sensitivity
     W1, W2 and  W3 are stable LTI 'weights'

 Inputs:
    G         LTI plant
    W1,W2,W3  LTI weights (either SISO or compatibly dimensioned MIMO)
           To omit weight, use empty matrix (e.g., W2=[] omits W2)
```

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Outputs:

K H-infinity Controller

CL CL=[W1*S; W2*K*S; W3*T]; weighted closed-loop system

GAM GAM=hinfnorm(CL), closed-loop H-infinity norm

INFO Information STRUCT, see HINFSYN documentation for details

Example:

```
G=ss(-1,2,3,4);   % plant to be controlled
   w0=10;     % desired closed-loop bandwidth
   A=1/1000;  % desired disturbance attenuation inside bandwidth
   M=2 ;      % desired bound on hinfnorm(S) & hinfnorm(T)
   s=tf('s'); % Laplace transform variable 's'
   W1=(s/M+w0)/(s+w0*A); % Sensitivity weight
   W2=[];                % Empty control weight
   W3=(s+w0/M)/(A*s+w0); % Complementary sensitivity weight
   [K,CL,GAM,INFO]=mixsyn(G,W1,W2,W3);
```

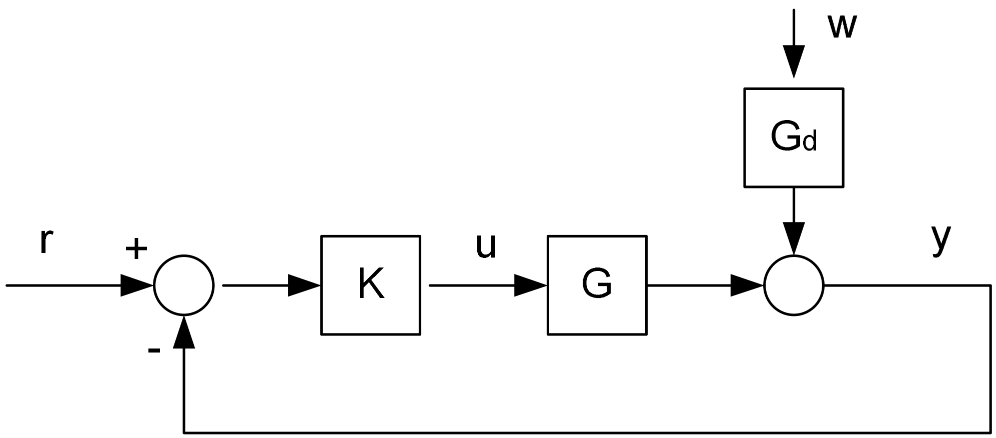
Plot results of successful design:

```
   L=G*K;  % loop transfer function
   S=inv(1+L); % Sensitivity
   T=1-S;      % complementary sensitivity
```

Mixsyn does the H infinity problem formulation automatically and solves the problem. If you use the command *hinfsyn*, you have to form the augmented plant yourself and pose the problem accordingly.

This is *Mixed Sensitivity Design*, an advanced form of *Loop Shaping Control*.

Example of control design



$$G(s) = \frac{200}{(10s+1)(0.05s+1)^2}$$

$$G_d(s) = \frac{100}{10s+1}$$

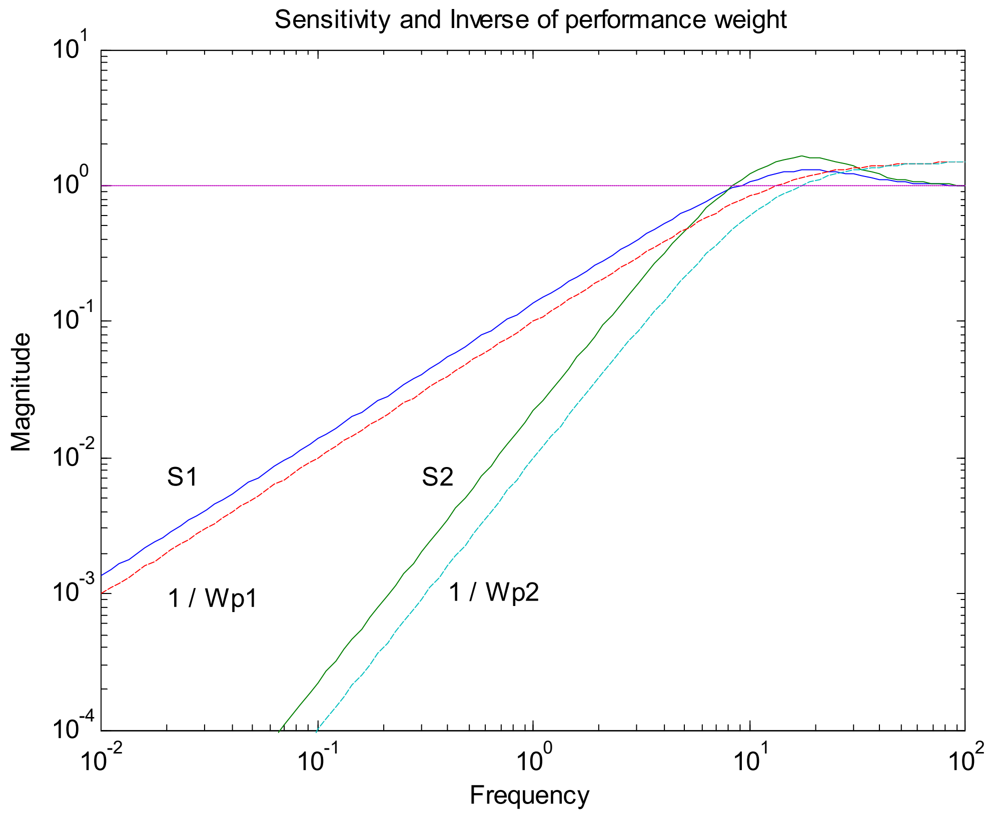
Command tracking + disturbance rejection problem

Both demands are difficult to meet simultaneously (trade-off in control design)

Let us try loop shaping by H_∞ control.

Example of control design...

```
% Mixed sensitivity design
%
% Uses the Robust Control Toolbox
%
s=tf('s');
G=200/(10*s+1)/(0.05*s+1)^2;
Gd=100/(10*s+1);
M=1.5; wb=10; A=1e-4;
Ws=tf([1/M wb], [1 wb*A]); Wu=1;
[Fy,CL,gopt]=mixsyn(G,Ws,Wu,[]);
```

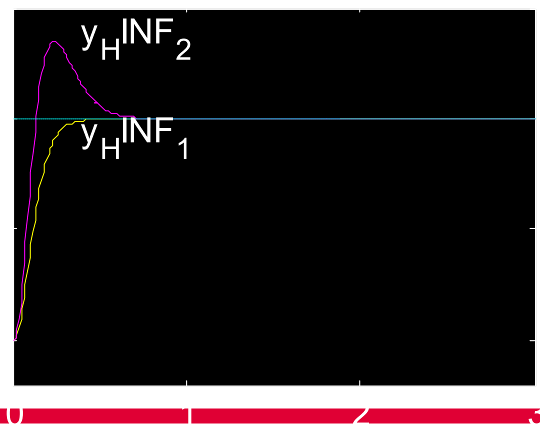


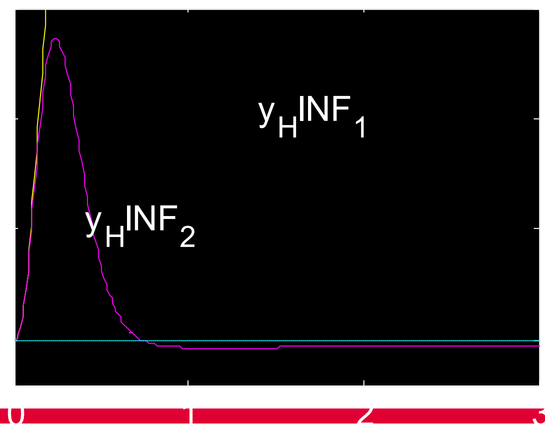
Note: $W_P = W_S$

$$W_{P1} = \frac{s/M + \omega_B^*}{s + \omega_B^* A}; \quad M = 1.5,\ \omega_B^* = 10,\ A = 10^{-4}$$

$$W_{P2} = \frac{(s/M^{1/2} + \omega_B^*)^2}{(s + \omega_B^* A^{1/2})^2}; \quad M = 1.5,\ \omega_B^* = 10,\ A = 10^{-4}$$

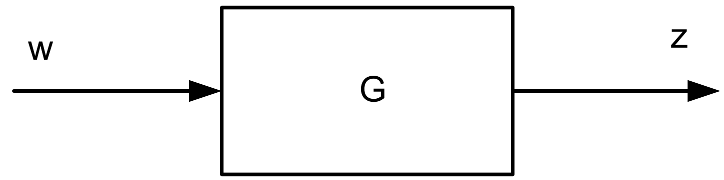
Because the load response is very poor in design 1, higher gains for the controller at low frequencies are needed (integral action).





To that end, use W_{P2} ,and the result is clearly better.

Some norm theory (**not** required…..to the end of slides):



Measure the output *z* by using the 2-norm

$$\left\| z(t) \right\|_2 = \sqrt{\sum_i \int_{-\infty}^{\infty} \left| z_i(\tau) \right|^2 d\tau}$$

Note. In what follows the norm is denoted by ”two bars” to make a distinction to the absolute value of a scalar value. (The textbook uses, for some obscure reason, two bars only in the case of a system norm).

The system 2-norm (euclidian norm) is

$$\|G(s)\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} \operatorname{tr}\left(G(i\omega)^* G(i\omega)\right) d\omega}$$

where

$$\operatorname{tr}\left|G(i\omega)^* G(i\omega)\right| = \sum_{i,j}\left|G_{ij}(i\omega)\right|^2 = \|G(i\omega)\|_F^2$$

is called the Frobenius norm. The system must be ”strictly proper”, *D*=0, in order the 2-norm to be finite. By using the Parseval theorem

$$\|G(s)\|_2 = \|g(t)\|_2 = \sqrt{\int_0^{\infty} \operatorname{tr}\left(g(\tau)^T g(\tau)\right) d\tau} = \sqrt{\int_0^{\infty}\sum_{i,j}\left|g_{ij}\right|^2 d\tau} = \sqrt{\sum_{i,j}\int_0^{\infty}\left|g_{ij}\right|^2 d\tau}$$

and it is seen that the 2-norm can be interpreted as a size measure of the output, when impulses are fed at the input. That has a connection to the stochastic interpretation, because impulse inputs can be interpreted to be white noise.

H2 –norm is then:

$$\left\|G(s)\right\|_2 = \max \left\|z(t)\right\|_2$$

when input *w* is composed of unit impulses.

Let the system be ”proper” (not necessarily ”strictly”, *D* can be non-zero). Define the *H*∞ - norm

$$\left\|G(s)\right\|_\infty = \max_{\omega} \bar{\sigma}\left(G(i\omega)\right)$$

the maximum of the largest singular value of the frequency function

It can be shown that

$$\left\| G(s) \right\|_\infty = \max_{\omega(t) \neq 0} \frac{\left\| z(t) \right\|_2}{\left\| w(t) \right\|_2}$$

is the largest gain to non-zero input signals.

Differences between H2 – and H∞- norms:

$$\left\| G(s) \right\|_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{i,j} \left| G_{ij}(i\omega) \right|^2 d\omega} = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{i} \sigma_i^2 (G(i\omega)) d\omega}$$

(because it can be shown that the Frobenius norm can be written by means of singular values; not proved here)

It is seen that the minimization of these norms means:

H∞ : minimize the maximum of the largest singular value

-H2: minimize all singular values of all frequencies

But what are the consequences of all this? We considered the closed-loop system

$$z = \begin{bmatrix} -W_u G_{wu} \\ -W_T T \\ W_S S \end{bmatrix} w = G_{ec} w$$

and now

$$\|G_{ec}(s)\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{tr}\left(G_{ec}(i\omega)^* G_{ec}(i\omega)\right) d\omega}$$

so that

$$\|G_{ec}(s)\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\|W_S(i\omega)S(i\omega)\|_2^2 + \|W_T(i\omega)T(i\omega)\|_2^2 + \|W_u(i\omega)G_{wu}(i\omega)\|_2^2\right] d\omega$$

These should be ”pushed down” on the whole frequency range. But as that could be interpreted as the minimization of the impulse response, set the criterion

$$V(F_y) = \|z\|_2^2 = \|Mx + Du\|_2^2 = \left[(Mx+Du)^T(Mx+Du\right]^2$$
$$= \left[x^T M^T Mx + x^T M^T Du + u^T D^T Mx + u^T D^T Du\right]^2$$
$$= \|Mx\|_2^2 + \|u\|_2^2$$

where the assumption has been used

$$D^T\begin{bmatrix} M & D \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix} \Rightarrow D^T M = 0,\, D^T D = I$$

The familiar *LQ* (*LQG)* –criterion was obtained. So, H2 –minimization corresponds to *LQ(G)* –control. The difference is the more general formulation (generalized model, input and output variables,weights), when compared to the conventional *LQ*-theory. But note that it is easy to formulate this kind of a control problem, which does not have a solution; H2-norm is then not finite.

The solution is a state feedback from reconstructed states (if the states cannot be measured, the Kalman filter must be used).

But what about $H\infty$-control: The norm to be minimized is

$$\left\| G_{ec} \right\|_{\infty} = \max_{\omega} \ \bar{\sigma}\left(G_{ec}(i\omega) \right)$$

the largest singular value of the closed-loop system.

But that cannot be made analytically! Instead, try to find a controller, which fulfils

$$\left\| \mathrm{G}_{ec} \right\|_{\infty} \leq \gamma$$

find iteratively the smallest γ, for which a corresponding controller exists.

Result: Consider the open loop

$$\dot{x} = Ax + Bu + Nw$$
$$z = Mx + Du$$
$$y = Cx + w$$

where $D^T \begin{bmatrix} M & D \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}$

If the Riccati equation

$$A^T S + SA + M^T M + S\left(\gamma^{-2} NN^T - BB^T\right) S = 0$$

has a positive semidefinite solution, then for the system controlled by

$$u = -B^T S\hat{x}$$

it holds that

$$\|z\|_2 \le \gamma \|w\|_2$$

for all inputs w. But the 2-norm of the signals induces the system ∞-norm. Then the ∞-norm of the system is smaller than γ.

The design procedure:

1. Determine the generalized plant *G*.
2. Design weights W_U, W_S, W_T .
3. Pick γ.
4. If the controller exists, make γ smaller, otherwise make it larger; iterate until the smallest γ has been found (so-called γ-iteration).
5. Investigate the properties of the closed loop; if not good enough, goto item 2.

Note. 1. Because of the weights the controller usually has a high dimension. Use model reduction techniques to reduce the dimension without changing much the controller properties.

Note. 2. γ-iteration and the design is done automatically by the command *hinfsyn* in the Robust Control-toolbox of Matlab. (Corresponding to *h2syn*).

The iteration need not be programmed by the designer.

Final steps

- Today's lecture no 12 (29. 11) is the last lecture. The 12th exercise on Thursday is the last exercise. The last homework no 6 has been published.

- Second Intermediate exam (IE2) on Thursday 7.12, 14:00-16:00, room T3 (T-house).
- First full exam (Kurssitentti) on Tuesday 12.12, 13:00-16:00, hall AS1.

- No registrations are needed for these exams. You can participate in both if you wish. Intermediate exams cannot be repeated. For the full exams later (next: 8th of January 2024) you have to register.
- For the grading of the course, see lecture slides, Chapter 1. (If you participate in both intermediate and full exam, then the better of (IE1+IE2), (Full exam), counts.) Note that when you have been given a grade of the course, it can never go lower even if you participate in later exams (trying to improve the grade).

Contents

- Classical control theory: SISO-systems, linear or linearized system models
- Extension to multivariable (MIMO) systems
- Performance and limitations of control
- Uncertainty and robustness,
- IMC-control,
- LQ and LQG control
- Optimal control
- Introduction to Model Predictive Control

The end

Good luck for the future!