KETJUSÄÄNTÖ

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$$

Pinta: $z = f(x, y)$

$$\left.\begin{aligned} x &= u(t) \\ y &= v(t) \end{aligned}\right\} \Rightarrow z = f(u(t), v(t)) = g(t)$$

Voidaan kysyä, mikä on $g'(t)$?

Erotusosamäärä:

$$\lim_{h \to 0} \frac{g(t+h) - g(t)}{h} =$$

$$\lim_{h \to 0} \frac{f(u(t+h), v(t+h)) - f(u(t), v(t))}{h}$$

$$= \lim_{h \to 0} \frac{f(u(t+h), v(t+h)) - f(u(t), v(t+h))}{h}$$

$$+ \lim_{h \to 0} \frac{f(u(t), v(t+h)) - f(u(t), v(t))}{h}$$

Saadaan siis:

$$g'(t) = f_1(u(t), v(t))\, u'(t) + f_2(u(t), v(t))\, v'(t)$$

Eli $z = f(x, y)$

(a) Oletetaan: x, y t:n suhteen jatkuvasti derivoituvia

Pätee:

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

(b) x, y s:n ja t:n suhteen jatkuvasti derivoituvia

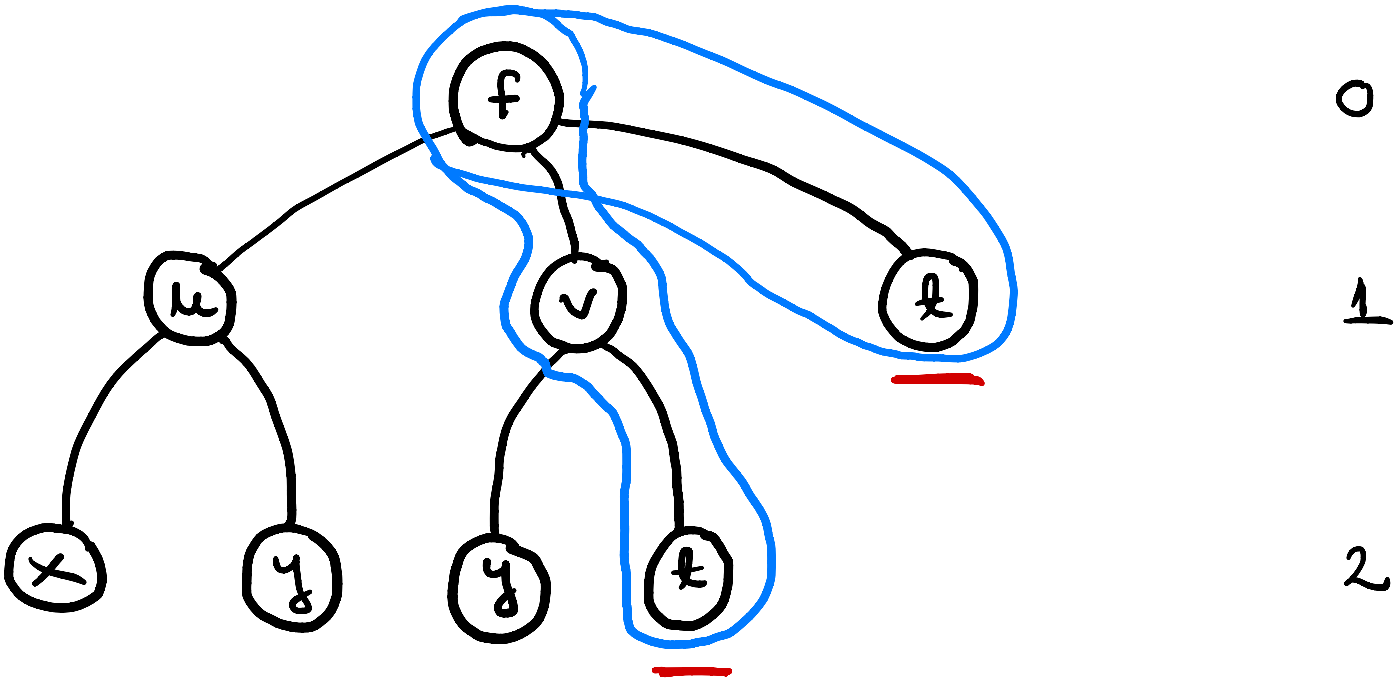
$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

Yleisempi tapaus:

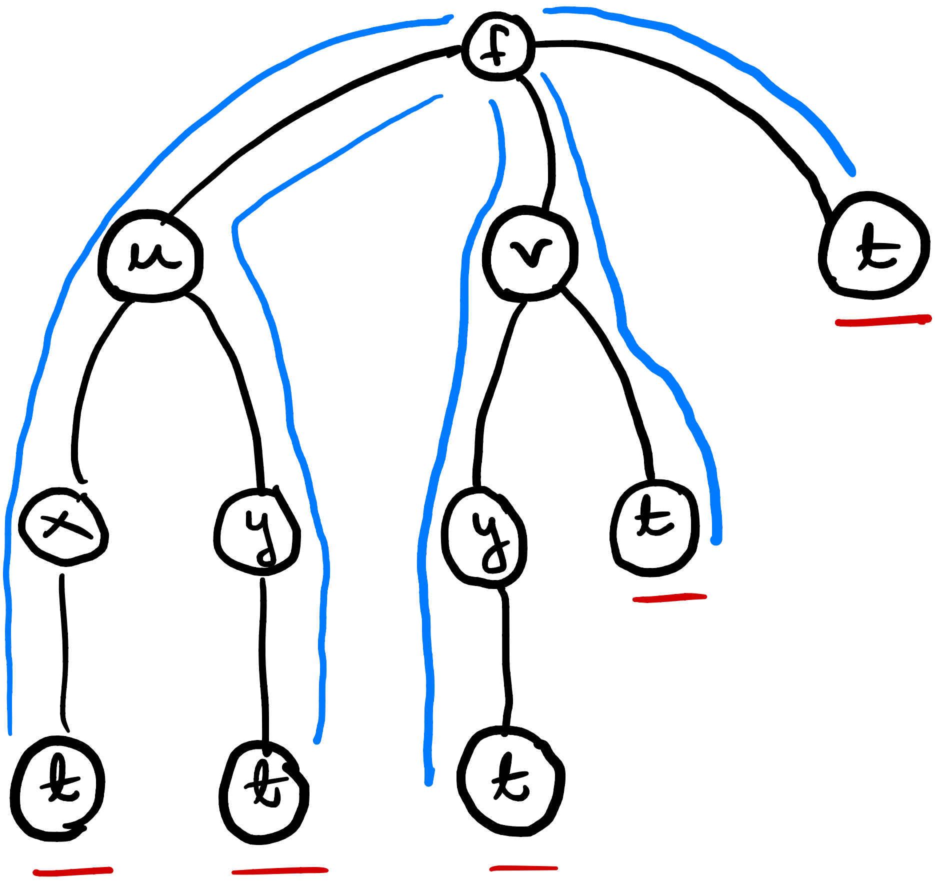
(1) $z = f(u, v, t)$,

$u = u(x, y)$
$v = v(y, t)$



$$\frac{\partial z}{\partial t} = \frac{\partial f}{\partial v}\frac{\partial v}{\partial t} + \frac{\partial f}{\partial t}$$

(2) $z = f\big(u(x(t), y(t)),\ v(y(t), t),\ t\big)$



$$\frac{dz}{dt} = \frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}\right)$$

$$+ \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial y}\frac{dy}{dt} + \frac{\partial v}{\partial t}\right)$$

$$+ \frac{\partial f}{\partial t}$$

$\longrightarrow$ viisi termiä

Esimerkki $\frac{\partial}{\partial x} f(x^2 y, x+2y)$

$$= f_1(x^2 y, x+2y) \frac{\partial}{\partial x}(x^2 y) + f_2(x^2 y, x+2y) \frac{\partial}{\partial x}(x+2y)$$

$$= 2xy\, f_1(\cdot, \cdot) + f_2(\cdot, \cdot)$$

LINEAARISET APPROKSIMAATIOT

Tangenttitaso: piste (a,b)

$$f(x,y) \simeq L(x,y) = f(a,b) + f_1(a,b)(x-a) + f_2(a,b)(y-b)$$

Esimerkki $f(x,y) = \sqrt{2x^2 + e^{2y}}$

Linearisoidaan pisteessä $(2,0)$
tarkastellaan " $(2.2, -0.2)$

$f(a,b) = 3$

$$f_1(x,y) = \frac{2x}{\sqrt{2x^2+e^{2y}}} \quad ; \quad f_1(2,0) = \frac{4}{3}$$

$$f_2(x,y) = \frac{e^{2y}}{\sqrt{2x^2+e^{2y}}} \quad ; \quad f_2(2,0) = \frac{1}{3}$$

$$L(x,y) = 3 + \frac{4}{3}(x-2) + \frac{1}{3}(y-0)$$

$$f(2.2,-0.2) \simeq L(2.2,-0.2) = 3.2$$

("Tarkka" 3.2172)

⟹ Osittaisderivaattojen olemassaolo ei takaa funktion $f(x,y)$ jatkuvuutta (!)

DIFFERENTIOITUVUUS

Määritelmä

$f(x,y)$ on differentioituva pisteessä (a,b), jos

$$\lim_{(h,k)\to(0,0)} \frac{f(a+h,b+k) - f(a,b) - f_1(a,b)h - f_2(a,b)k}{\sqrt{h^2+k^2}} = 0$$

Lause Jos osittaisderivaatat ovat jatkuvia (a,b):n ympäristössä, on f differentioituva.

<u>Esimerkki</u> $f(x,y) = x^3 + xy^2$

Differentioituvuus:

$$(x+h)^3 + (x+h)(y+k)^2 - (x^3 + xy^2)$$

$$- (3x^2 + y^2)h - 2xy\,k$$

$$= 3x\underline{h^2} + \underline{h^3} + 2y\underline{hk} + \underline{hk^2} + x\underline{k^2}$$

h & k - termit lähestyvät nollaa nopeammin kuin $\sqrt{h^2+k^2}$, kun $(h,k) \to (0,0)$