Advanced Microeconomics 2
Helsinki GSE
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You have two hours to complete the examination. Answer all questions.

1. Answer the following short questions.

   (a) Three agents share and consume a single divisible good. The total available amount of the good is $\overline{x}$. A feasible allocation is a vector (x_1, x_2, x_3) where x_i denotes the consumption of $i \in \{1, 2, 3\}$. What are the Pareto efficient allocations if all i have strictly increasing utility functions u_i that depend solely on their own consumption? How can you find Pareto-optima if each i cares about the consumption utility of all agents so that their utility function is $U_i(x_1, x_2, x_3) = \sum_j \lambda_{ij} u_j(x_j)$, where $\lambda_{ij} > 0$ for all i and all $j \in \{1, 2, 3\}$, and each u_j is a strictly increasing function?

   (b) Explain how the Gale-Shapley algorithm (also know as the deferred acceptance algorithm) works in a matching model. Show that for all preference profiles, the outcome of the algorithm is pairwise stable.

2. Answer the following problems for an exchange economy.

   (a) Two agents $i \in \{1, 2\}$ have preferences over two goods $\{x, y\}$ represented by utility functions:

   $$u_1(x_1, y_1) = x_1 + 2y_1,$$

   $$u_2(x_2, y_2) = \min\{2x_2, y_2\},$$

   where (x_i, y_i) denotes the consumption vector of agent i. Let $(\bar{x}, \bar{y})$ denote the total amounts of the two goods in the economy. Find the Pareto-efficient allocations $((x_1, y_1), (x_2, y_2))$ for this society.

   (b) Suppose the agents have initial endowments $\omega_1 = \omega_2 = (1, 1)$. Compute a competitive equilibrium for this economy.

(c) Suppose that a third agent with utility function:

$$u_3(x_3, y_3) = 3x_3 + 3y_3,$$

and an initial endowment $\omega_3 = (5, 5)$ is added to the economy. Compute a competitive equilibrium for this economy.

3. Three consumers choose optimal consumption under uncertainty. There are three possible states s of the world: $s \in \{1, 2, 3\}$. The objective probabilities of the states are given by (π_1, π_2, π_3), where $\pi_1 \geq \pi_2 \geq \pi_3 > 0$ and $\sum_{s=1}^{3} \pi_s = 1$ There is a single consumption good for each state and the (Bernoulli) utility for each consumer $i \in \{1, 2, 3\}$ from consumption x_{is} in state $s \in \{1, 2, 3\}$ is given by:

$$u_i(x_{is}) = \ln(x_{is}).$$

The consumers $i \in \{1, 2, 3\}$ have von Neumann - Morgenstern expected utility preferences for consumption vectors (x_{i1}, x_{i2}, x_{i3}).

(a) Suppose that the consumers have initial endowments across the three states given by:

$$\omega_1 = (3, 1, 1), \quad \omega_2 = (1, 3, 1), \quad \omega_3 = (1, 1, 3).$$

Let (p_1, p_2, p_3) be the prices for consumption in the three states. Formulate the agents' optimization problems for contingent consumptions.

(b) Solve for the competitive equilibrium prices and the equilibrium allocation. Which agent has the highest expected utility in equilibrium?

(c) Suppose that the economy has only two assets: a safe asset paying one unit of the consumption good in all states and a risky asset that pays one unit of the consumption good only in state 1. Is the asset market complete and does the first welfare theorem apply to this setting?

(d) Assume that $p_1 = p_2 = p_3$ and formulate the utility maximization problems of the agents where the first asset sells at price 1 and the second at price q. At equilibrium prices, the aggregate demand for each asset is zero. Determine the sign of the equilibrium trade for each type of agent.