Advanced Microeconomics 2
Fall 2023
Final Examination: Suggested solutions

1. Q1

   (a) Feasible allocations are $(x_1, x_2, x_3) \geq 0$ such that $\sum_{i=1}^3 x_i \leq \overline{x}$. If at (x_1, x_2, x_3), we have $\sum_{i=1}^3 x_i < \overline{x}$, the allocation is not Pareto-efficient since e.g. $(\overline{x} - x_2 - x_3, x_2, x_3)$ is feasible and Pareto-dominates the original. If $\sum_{i=1}^3 x_i = \overline{x}$, then since all u_i are strictly increasing, $u_i(x_i') > u_i(x_i)$ implies that $x_j' < x_j$ for some $j \neq i$ if $(x_1', x_2'.x_3')$ is feasible. Hence $(x_1, x_2, x_3) \geq 0$ such that $\sum_{i=1}^3 x_i = \overline{x}$ are all Pareto-efficient.

   For the second case, If the u_j are concave, then $U_i = \sum_j \lambda_{ij} u_j$ is also concave and you can find Pareto-optimal by solving for all $\mu_1, \mu_2, \mu_3 \geq 0$

$$\max_{(x_1,x_2,x_3)} \sum_i \mu_i \sum_j \lambda_{ij} u_j$$

$$\text{subject to } \sum_i x_i \leq \overline{x}.$$

   If not, then solve for all i, all $j, k \neq i$ and all u', u''

$$\max_{(x_1,x_2,x_3)} u_i$$

$$\text{subject to } U_j(x_1, x_2, x_3) \geq u', \quad U_k(x_1, x_2, x_3) \geq u''.$$

   (The concave case was enough for full points).

   (b) Gale-Shapley and stability, see lecture notes p.29-30.

2. Q2

   (a) A feasible allocation $(x_1, y_1), (x_2, y_2)$ is Pareto efficient if and only if $x_1 + x_2 = \overline{x}, y_1 + y_2 = \overline{y}$ and $2x_2 = y_2$. The first claim follows from agent 1 having strictly increasing utility function. To see the second, note that if $y_2 < 2x_2$, then you can increase agent 1's utility without hurting agent 2 by giving shifting a small amount ϵ of good x from 2 to 1. If $2x_2 = y_2$, then moving to any alternative allocation that gives agent 1 at least as high utility must have strictly more of one of the goods. For a feasible allocation this means that 2 has less of that good and hence a lower utility.

   (b) By first welfare theorem, c.e. allocations are Pareto-efficient. By part a), interior allocations have $2x_2 = y_2$ and the possible corner allocation has $x_2 = 1, y_2 = 2$. The corner allocation is worse for agent 1 than initial endowment and hence cannot be a c.e. allocation. At interior points, the c.e. price is proportional to

$(1, 2)$. Solving simultaneously the equations (budget set and optimal demand for agent 2):

$$x_2 + 2y_2 = 3, 2x_2 = y_2$$

gives $x_2 = \frac{3}{5}, y_2 = \frac{6}{5}$, and therefore $x_1 = \frac{7}{5}, y_1 = \frac{4}{5}$.

(c) If $p_1 \neq p_2$, the demand by agent 3 for one of the goods exceeds 10 and the markets cannot clear. Hence $p_1 = p_2$ at c.e. Optimal demand at these prices for agent 1 is $(0, 2)$ and for agent 2 it is $(\frac{2}{3}, \frac{4}{3})$. Thus the market clearing demand for agent 3 is $(\frac{19}{3}, \frac{11}{3})$.

3. Q3

(a) The maximization problem for agent i is given by:

$$\max_{(x_{i1}, x_{i2}, x_{i3})} \sum_{s=1}^{3} \pi_s \ln(x_{is})$$

$$\text{subject to } \sum_{s=1}^{3} p_s x_{is} \leq \sum_{s=1}^{3} p_s \omega_{is}.$$

(b) Since there is no aggregate uncertainty and since all agents have strictly concave utility functions, we know that all demands are constant across states. First order conditions for maximization imply that $p_s = \pi_s$ for all s. Hence we have:

$$x_{1s} = (3\pi_1 + \pi_2 + \pi_3), x_{2s} = (\pi_1 + 3\pi_2 + \pi_3), x_{3s} = (\pi_1 + \pi_2 + 3\pi_3).$$

(c) Since there are only two assets, the market is not complete and the first welfare theorem does not apply. Agents 2 and 3 cannot get full insurance with this asset structure.

(d) Denote the demand by agent i for assets 1 and 2 by z_{i1}, z_{i2} respectively. Then the problem is written as:

$$\max_{x_{i1}, x_{i2}, x_{i3}, z_{i1}, z_{i2}} \sum_{s=1}^{3} \pi_s \ln(x_{is})$$

$$\text{subject to } z_{i1} = -q z_{i2},$$

$$x_{i1} = \omega_{i1} + z_{i1} + z_{i2}, \quad x_{i2} = \omega_{i2} + z_{i1}, \quad x_{i3} = \quad \omega_{i3} + z_{i1}$$

By substituting the variables from constraints into the objective function, we get an unconstrained problem in z_{i2} only. From the FOC, the demand for asset 2 by agents 2 and 3 is higher than the demand by agent 1 for that asset. Hence 1 must be a net supplier of the asset and a net demander for asset 1. Agents 2 and 3 demand asset 2 and supply asset 1.