

In class -exercises 8.-10.1.2024

The In class -exercises are to be done in the exercise session and the assistant will give advice on how to do them if necessary. The correct solutions to the problems will be discussed together. To obtain points for these exercises, you only need to be present.

Here are some “review” problems from calculus and analytic geometry. And some parametric curve material that is discussed in the first lecture. To give you the best advantage, the exercises are meant to be done by hand without the help of any software.

1. Calculate the following derivatives.

(a) $\frac{d}{dx}(x^2 - 3)^5$

(b) $\frac{d}{dx}\left(\frac{x-4}{x+2}\right)$

(c) $\frac{d}{dt}(t^3 \cdot e^{-2t})$

(d) $\frac{d}{du}(u \cdot \sqrt{1+u^2} + \ln(u + \sqrt{1+u^2}))$

2. Calculate the following integrals.

(a) $\int_0^4 t\sqrt{t^2+9}dt.$

(b) $\int_{-\pi}^{\frac{\pi}{4}} \sin(2x)dx$

3. Sketch the region bounded by the two parabolas $x = y^2$ and $y = x^2/8$, and calculate its area.

4. Curve and its parametric presentation

(a) Sketch the curve $x^2 + y^2 = 4$. What is this curve called?

(b) It is said, that the plane curve in (a) has a parametrization $\mathbf{r}(t) = (2 \cos t, 2 \sin t) \in \mathbb{R}^2$, where $0 \leq t \leq 2\pi$ (t is called the parameter). Show that this is true.

(c) Can you find an another parametrization for the curve in (a)?

(d) Find a parametrization for the curve $y = x^2 - 3$ between points $(0, -3)$ and $(2, 1)$.

(e) Sketch the curve $x(t) = 3 \cos(t), y(t) = 5 \sin(t)$ for $0 \leq t \leq 4\pi$. What is this curve called? How $\mathbf{r}(t) = (x(t), y(t))$ moves along the curve when t goes from 0 to 4π ?