Week I

Overview

- Admin Topics;
- Graph Theory;
- Graph Representation;
- Depth-First Search (DFS).

Graph Theory

- directed vs undirected graphs;
- walk, path and edge progression;
- reachability.

DFS

Algorithm 1: Depth First Search (DFS)

Input: undirected graph G, vertex $s \in V(G)$

Output: tree $(R, T) \subseteq G$, R reachable from s

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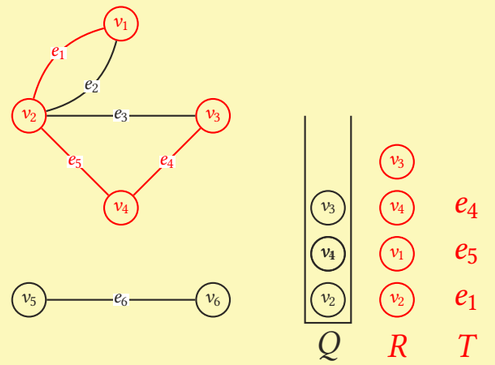
1 set R := {s}, Q := {s} and T = ∅;
2 if Q = ∅ then return R, T;
3 else v := last vertex added to Q;
4 choose w ∈ V(G) \ R with {v, w} ∈ E(G);
5 if there is no such w then
6 |   set Q := Q \ {v} and go to 2
7 set R := R ∪ {w}, Q := Q ∪ {w},
    T := T ∪ {{v, w}}, go to 2;
```



Graph Representation

incidence matrix	adjacency matrix	adjacency list
$A \in \{0,1\}^{\lvert V\rvert \times \lvert E\rvert}$, $a_{v,e} = \begin{cases} 1, & \text{if } v \in e \\ 0, & \text{if } v \notin e \end{cases}$	$A \in \mathbb{Z}^{\lvert V\rvert \times \lvert V\rvert}$, $a_{v,w} = \lvert\{e = \{v,w\} \in E\}\rvert$	$L = [\ell(v) \colon v \in V]$, $\ell(v) = [e \colon e = \{u,v\} \in E]$
$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	$\ell(v_1) = [e_1, e_2]$ $\ell(v_2) = [e_1, e_2, e_3, e_5]$ $\ell(v_3) = [e_3, e_4]$ $\ell(v_4) = [e_4, e_5]$ $\ell(v_5) = [e_6]$ $\ell(v_6) = [e_6]$
$O(\lvert V\rvert\lvert E\rvert)$	$O(\lvert V\rvert^2)$	$O(\lvert E\rvert \log \lvert V\rvert)$

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