Week IV

Overview

- Matchings;
- Maximal vs Maximum;
- Matching and Flow.

Definitions

- $M \subset E$ is called *matching* if all $e \in M$ are pairwise disjoint, i.e., if the endpoints are all different
- $M \subset E$ is a *maximum matching* in G if M is a matching with highest cardinality, i.e.,

$$|M'| \leq |M| \quad \text{for all matchings } M'$$

Definition (Maximal Matching)

***No** other edge can be added to this match;*

Definition (Maximum Matching)

A match with the largest possible number of edges;

Definition (Maximum vs Maximal)

A maximum matching is the maximal matching with the maximum number of edges.

Maximum Matching

Algorithm 1: Maximum Matching

Input: undirected graph $G = (V, E)$
Output: maximum matching M

```
set M = ∅
while there exists M-augmenting path in G do
    choose M-augmenting path P
    set M = (M \ E(P)) ∪ (E(P) \ M)
return M
```

Maximum Matching - Bipartite

Algorithm 2: Maximum Matching Bipartite Graphs

Input: undirected bipartite graph $G = (V, E)$
Output: maximum matching M

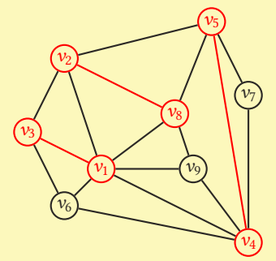
```
set M = ∅
construct G'
while there exists s-t-path in G' do
    choose s-t-path P
    set M = (M \ E(P)) ∪ (E(P) \ M)
    update G'
return M
```

Maximum Matching ILP

$$\max \quad \sum_{e \in E} x_e \tag{1a}$$

$$\text{s.t.} \quad \sum_{e \in \delta(v)} x_e \leq 0 \qquad v \in V \tag{1b}$$

$$x_e \in \{0, 1\} \qquad e \in E \tag{1c}$$



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