Week V

Overview

- Class P;
- Class NP;
- P vs NP;
- Decision Problem;
- Travelling Salesman Problem (TSP).

Definitions

- **Decision problem** is a yes-or-no problem;
- **TSP** is a graph problem where all nodes are visited only once as part of cycle.

Modelling - Decision variables

$$x_{ij} = \begin{cases} 1, & \text{if path goes from city } i \text{ to city } j \\ 0, & \text{otherwise} \end{cases}$$

Modelling - Objective Function

$$\min \sum_{i}^{n} \sum_{j, j \neq i}^{n} x_{ij} f_{ij}$$

MTZ - Miller-Tucker-Zemlim

$$\text{Minimize} \quad \sum_{i}^{n} \sum_{j, j \neq i}^{n} x_{ij} f_{ij}$$

Subject to:

$$\sum_{i=1, i \neq j}^{n} x_{ij} = 1 \qquad \forall j \in \{1, \cdots, n\}$$

$$\sum_{j=1, j \neq i}^{n} x_{ij} = 1 \qquad \forall i \in \{1, \cdots, n\}$$

$$u_i - u_j + 1 \leq (n-1)(1 - x_{ij}) \qquad 2 \leq i \neq j \leq n$$

$$2 \leq u_i \leq n \qquad 2 \leq i \leq n$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in \{1, \cdots, n\}$$

DFJ - Dantzig-Fulkerson-Johnson

$$\text{Minimize} \quad \sum_{i}^{n} \sum_{j, j \neq i}^{n} x_{ij} f_{ij}$$

Subject to:

$$\sum_{i=1, i \neq j}^{n} x_{ij} = 1 \qquad \forall j \in \{1, \cdots, n\}$$

$$\sum_{j=1, j \neq i}^{n} x_{ij} = 1 \qquad \forall i \in \{1, \cdots, n\}$$

$$\sum_{i \in Q} \sum_{j \neq i, j \in Q} x_{ij} \leq |Q| - 1 \qquad \forall Q \subset \{1, \cdots, n\}, |Q| \geq 2$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in \{1, \cdots, n\}$$

Example - TSP

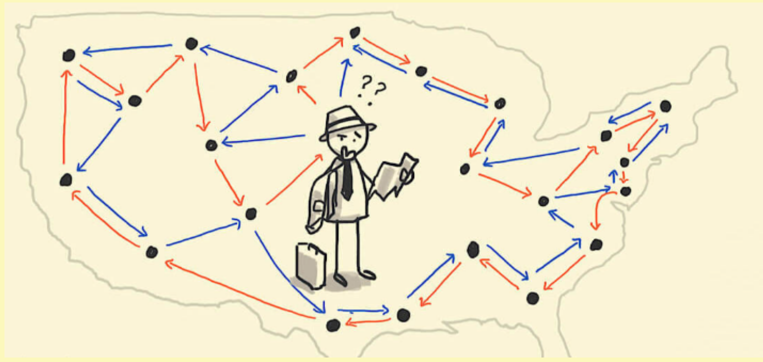
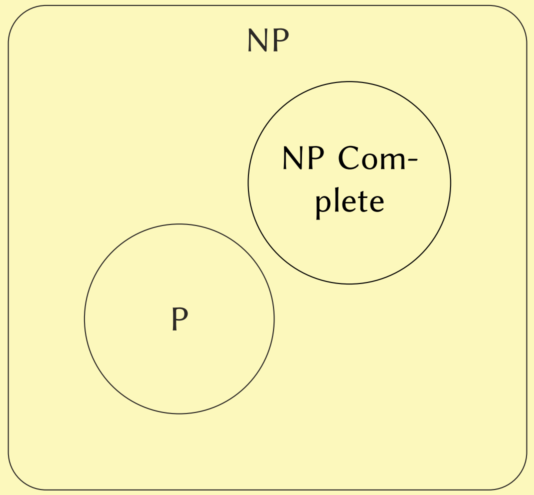


Figure: A lone traveller about to make important decisions

Class P vs Class NP



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