Combinatorial Optimization

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Introduction to Combinatorics

§ Week I §

Problem 1: All representations

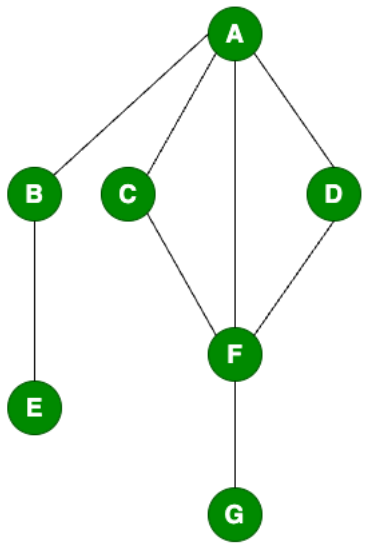

Figure 1: Example of an undirected graph.

Represent the graph above using **incidence matrix** and **list adjacency**.

Solution:

Incidence matrix:

$$\begin{array}{cc} & \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Adjacency list:

$$A : \{B, C, D, F\}$$
$$B : \{A, E\}$$
$$C : \{A, F\}$$
$$D : \{A, F\}$$
$$E : \{B\}$$
$$F : \{A, C, D, G\}$$
$$G : \{F\}$$

Problem 2: DFS by Hand

Considering the graph from the previous problem, write up all the steps required to complete the **DFS** algorithm.

Solution:

We have a selection of nodes to follow to build into another data structure S:

- Starting from node A, we have four options: B, C, D and F. Then, add A in S ($S\{A\}$);
- Picking B and put into S ($S = \{A, B\}$;
- From B, the next option to go is E which we would add to S ($S = \{A, B, E\}$;
- E is a dead-end, so it is returning to B, which does not have any other alternative to go, so it goes back to A;
- From A, it goes to C ($S = \{A, B, E, C\}$) then goes to F ($S = \{A, B, E, C, F\}$;
- From F, it goes to G, which is a dead-end, so it returns to F ($S = \{A, B, E, C, F, G\}$;
- From F, the only available route is D resulting in $S = \{A, B, E, C, F, G, D\}$

Note that there is no single solution, and different DFS are possible.

Problem 3: Escape Room

Using any graph representation of your choice, express the following **maze as a graph**:

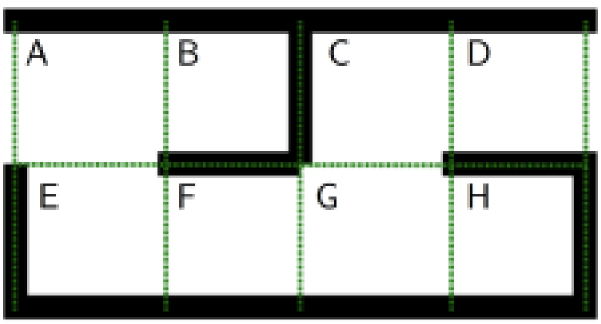

Figure 2: Search for the keys

Solution:

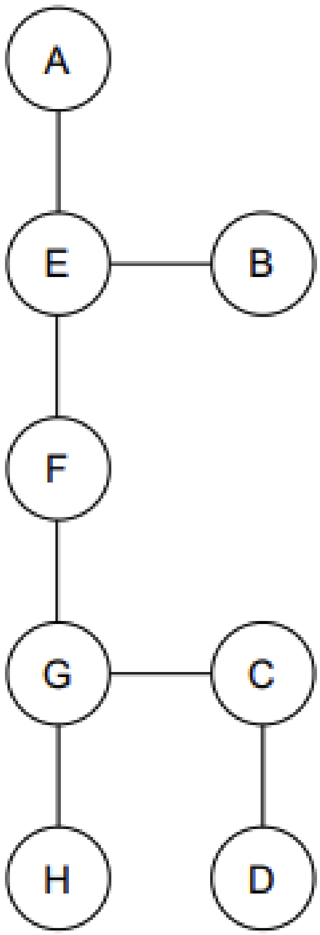

Figure 3: Graph from maze

Using **visual representation**:

Using **incidence matrix**:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} \begin{pmatrix} A & B & C & D & E & F & G & H \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 and \end{pmatrix}$$

Using **adjacency list**:

$$\begin{aligned}
&A : \{B, E\}\\
&B : \{A\}\\
&C : \{D, G\}\\
&D : \{C\}\\
&E : \{A, F\}\\
&F : \{E, G\}\\
&G : \{C, H\}\\
&H : \{G\}
\end{aligned}$$

Problem 4: Escaping

Using DFS, find at least **two different** routes to reach **room H**.

Solution:

First, using DFS:

1. Starting for A, we can go to B ($S = \{A, B\}$);
2. B is a dead-end; then we return to A and take E instead ($S = \{A, B, E\}$);
3. From E, it goes to G, where it has two options: first, let us take C ($S = \{A, B, E, F, G, C\}$);
4. From C, it goes to D and then reaches a dead-end ($S = \{A, B, E, F, G, C, D\}$);
5. Returns to G, the only option available is H ($S = \{A, B, E, F, G, C, D, H\}$).

The two ways to reach H could be from A ($A- > E- > F- > G- > H$ and from D ($D- > C- > G- > H$).

Problem 5: DFS vs Reachability

In a sufficient large graph, a **DFS algorithm** showed that such a graph is disconnected. How would you **justified** that?

Solution: If the size of DFS is the same as the size of the original graph G, all nodes in the DFS are connected in the original graph.

Problem 6: Building blocks organically

The hydrocarbons known as alkanes have chemical formula C_pH_{2p+2}, alkanes where C and H represent atoms of carbon and hydrogen, respectively. Graphs can represent alkane molecules. Draw a figure of a methane C_1H_4 molecule. How many “different” C_3H_8 molecules are there?

Solution:

If you know the chemical properties of alkanes, many different molecules are available. However, those graphs would be connected and have the same DFS.

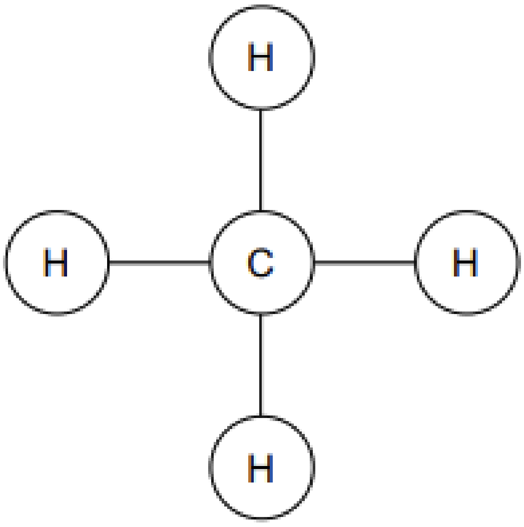

Figure 4: Methane

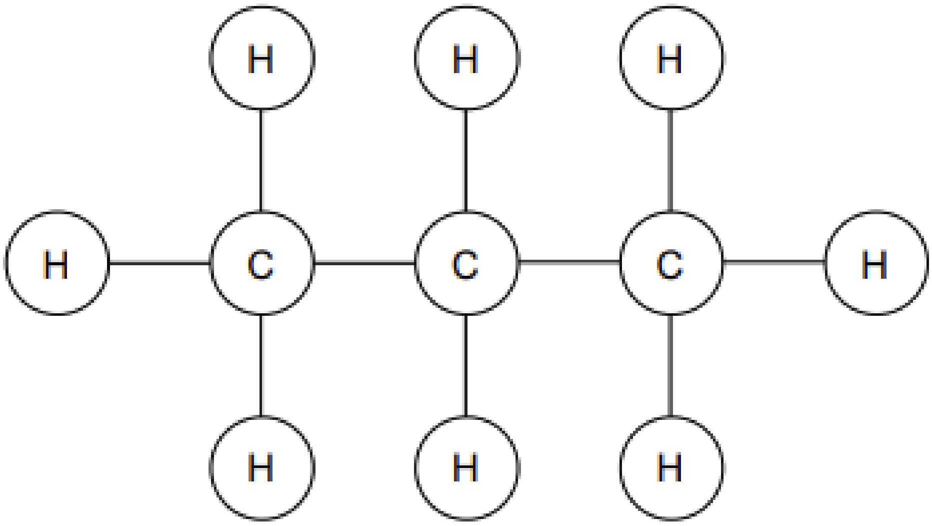

Figure 5: Propane