

# COMBINATORIAL OPTIMIZATION

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## Graph Problems I

### § Week II §

#### Problem 1: Shortest Distance

Compute the shortest-paths tree from  $s$  using Dijkstra's algorithm. Sketch each iteration.

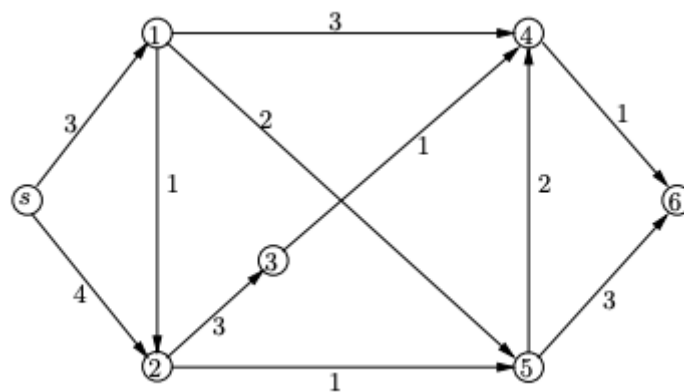


Figure 1: Example of a flow network

#### Problem 2: Prim's vs Kruskal's

Suppose we have an undirected graph with weights that can be either positive or negative. Do Prim's and Kruskal's algorithm produce a MST for such a graph?

#### Problem 3: Prim's variation

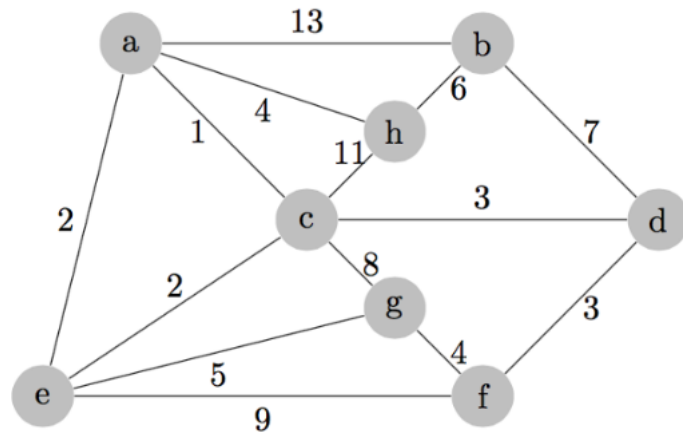
In some problems we want that certain pairs of vertices are directly connected with each other. Modify the Prim's algorithm in order to solve the minimum spanning tree problem with this additional constraint.

#### Problem 4: Reliability

Let  $G = (V, A)$  be a digraph with edge-weights  $0 \geq p_a \geq 1$  for each edge  $a \in A$ . Interpret  $p_a$  as the probability that the arc  $a$  does fail. For an elementary path from a node  $s$  to a node  $t$  the reliability of that path can be understood as the product of all  $p_a$  of edges on that path; namely, it is the probability, that the path does not fail. Show, how a shortest path algorithm can help computing a path from  $s$  to  $t$  with maximal reliability.

## Problem 5: Spanning Tree

For the graph below:



1. What is the cost of a minimum spanning tree?
2. How many minimum spanning trees does it have?