Combinatorial Optimization

Flows

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§ Week III §

Problem 1: Spanning Trees

Answer each of these true/false questions about minimum spanning trees:

1. A MST contains cycles;
2. If we remove an edge from a MST, the resulting subgraph is still a MST;
3. If we add an edge from a MST, the resulting graph is still a MST;
4. If there are V nodes in a given graph, a MST of that resulting graph contains $|V| - 1$ edges.

Problem 2: Conference Organizer

Suppose you are organizing a conference where researchers present articles they have written. Researchers who want to present an article send a paper to the conference organizers. The conference organizers have access to a committee of reviewers who are each willing to read up articles each.

Each paper submission gets reviewed by up to reviewers. Moreover, each submission has a particular topic and each reviewer has a specialization for a set of topics, so papers on a given topic only get reviewed by those reviewers who are experts on that topic.

The conference organizers need to decide which reviewers will review each article (or equivalently, which articles will be reviewed by which reviewers). Explain how they could use a flow network to solve this problem.

Problem 3: Social Network

Suppose you have a bunch of computers networked together (haphazardly) using wires. You want to send a message to every other computer as fast as possible. Unfortunately, some wires are being monitored by some shadowy organization that wants to intercept your messages.

After doing some reconnaissance, you were able to assign each wire a "risk factor" indicating the likelihood that the wire is being monitored. For example, if a wire has a risk factor of zero, it is extremely unlikely to be monitored; if a wire has a risk factor of 10, it is more likely to be monitored.
The smallest possible risk factor is 0; there is no largest possible risk factor.

Implement an algorithm that selects wires to send your message such that:

1. every computer receives the message and;
2. you minimize the total risk factor. The total risk factor is defined as the sum of the risks of all of the wires you use.

Problem 4: Disjoint Roads

A number k of trucking companies, $c_1, \ldots, c_k$, want to use a common road system, which is modeled as a directed graph, for delivering goods from source locations to a common target location. Each trucking company c_i has its own source location, modeled as a node s_i in the graph, and the common target location is another vertex t. (All these $k + 1$ vertices are distinct.)

The trucking companies want to share the road system for delivering their goods, but they want to avoid getting in each other's way while driving. Thus, they want to find k edge-disjoint paths in the graph, one connecting each source s_i to the target t. We assume that there is no problem if trucks of different companies pass through a common vertex.

Design an algorithm for the companies to use to determine k such paths, if possible, and otherwise return "impossible".

Problem 5: Ford vs Fulkerlson

Using the Ford-Fulkerson method, compute a maximal flow in the following network:

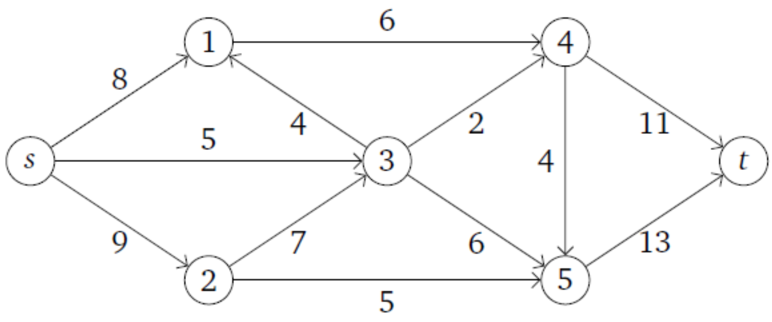

Figure 1: Example of flow network