Combinatorial Optimization

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Matching

§ Week IV §

Problem 1: Maximal Matching

Find a maximal and a maximum match in the following graph:

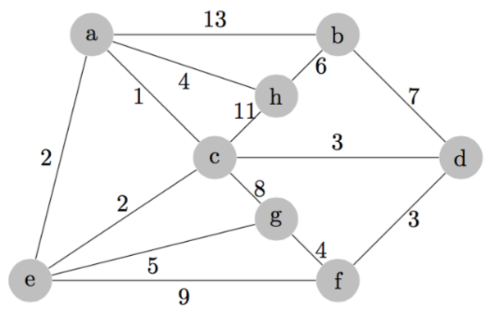

Figure 1: Undirected weighted graph

Problem 2: Ford vs Edmond

Find the maximum flow in the following graph using Ford-Furkelson;

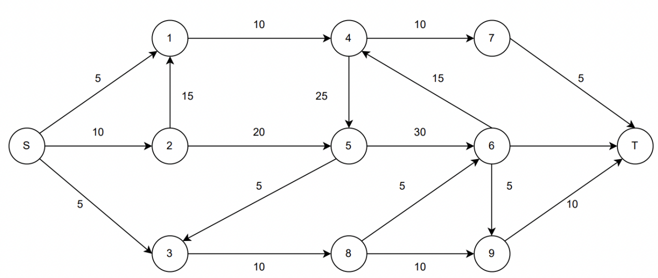

Figure 2: Undirected weighted graph

Problem 3: Flows and matchings

How can Ford-Fulkerson can be used for maximum matchings?

Problem 4: Gale-Shapley

Gale-Shapley is an algorithm developed by two American scientists: David Gale and Lloyd Shapley, to find solutions to stable matching problems. In their formulation, the polynomial algorithm (linear in the input size) works as a truthful mechanism from the point of view of the proposing participants, for whom the solution will always be optimal. In addition, the Gale-Shapley algorithm produces stable matchings on complete bipartite graphs. Consider now complete graphs. Are all matchings stable?

Problem 5: Tic-Tac-Toe

A positional game consists of a set X of positions and a family $W_1, W_2, \cdots, W_m \subset X$ of winning sets (Tic-Tac-Toe has 9 positions corresponding to the 9 boxes, and 8 winning sets corresponding to the three rows, three columns, and two diagonals). Two players alternately choose positions; a player wins when they collect a winning set.

Suppose that each winning set has size at least a and each position appears in at most b winning sets (in Tic-Tac Toe $a = 3$ and $b = 4$). Prove that Player 2 can force a draw if $a \geq 2b$.

Hint: Form a bipartite graph G with bipartition (X, Y) where $Y = \{W_1, W_2, \cdots, W_m\} \cup \{W'_1, W'_2, \cdots, W'_m\}$ with edges xW_j and xW_0 whenever $x \in W_j$. How can Player 2 use a matching in G?