Combinatorial Optimization

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Polyhedral Theory

Dynamic Programming

§ Week IX §

Problem 1: Vending Machines

We have a vending machine with an unlimited supply of coins of denominations $D = \{d_1, d_2, \cdots, d_n\}$ and we want to make change for value V, i.e., we want to get the value V using only the denominations in D. Note that for some instances of the problem, the solution might not exist, e.g., denominations 5 and 10 can make change for 35 but not for 12.

Try to formulate and solve the decision variant of the coin-exchange problem using dynamic programming. You are given the value V – the task is to decide whether the value V can be composed by the denominations from D only.

Problem 2: Shipyard

A shipping company has a cargo ship docked in the harbour onto which it wants to load several shipping containers. There are n containers to choose from. The i-th container weigths $w_P i \geq 0$ tons, and if it is shipped, it will yield a $p_i \geq 0$ dollars profit.

1. Present a recursive DP formulation for the following problem: Given $p[1..n]$ and $w[1..n]$, compute the maximum profit that can be achieved by selecting any subset of the containers, subject to the weight restriction.

2. The shipping company adds the constraint that each ship has a maximum capacity of C containers. Modify your solution to the previous part to handle this constraint.

3. The shipping company has discovered that it has a second ship, identical to the first. Modify your solution to the second part to determine the maximum achievable profit by selecting containers and assigning each to one of these ships.

Problem 3: Pharmacies

A pharmacist has W pills and n empty bottles. Let b_i denote the number of pills that can fit in a bottle i. Let vi denote the cost of purchasing bottle i. Given W, b_i's

- Suppose that you only pay for the fraction of the bottle that is used. For example, if the i-th bottle is half filled with pills, you pay only $\frac{v_i}{2}$. Present an algorithm for this version of the problem.

- Suppose that this assumption does not hold. That is, you must buy the entire bottle, even if only a portion of it is used. Present an algorithm for this version of the problem. (Hint: Use DP. It suffices to give the recursive formulation.)

Problem 4: Ford vs Dijkstra

Use the Bellman-Ford algorithm to calculate the shortest path in this following graph:

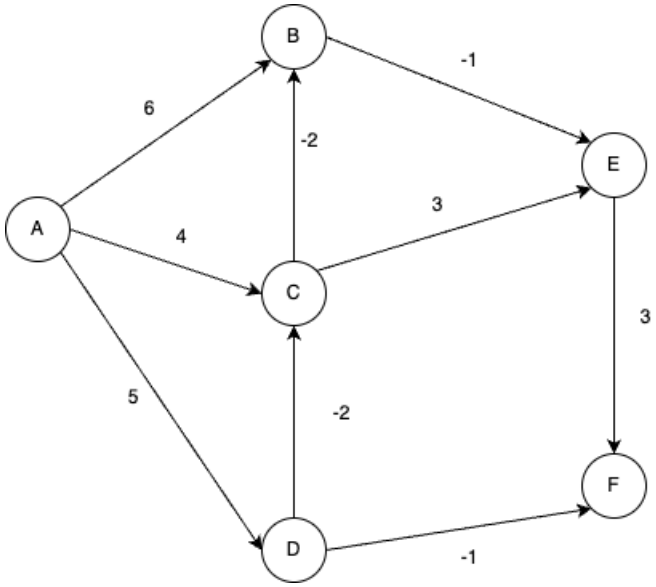

Figure 1: DAG with negative flows