EXAMPLE 11–1 Radiation Emission from a Black Ball

Consider a 20-cm-diameter spherical ball at 800 K suspended in air as shown in Figure 11–12. Assuming the ball closely approximates a blackbody, determine (*a*) the total blackbody emissive power, (*b*) the total amount of radiation emitted by the ball in 5 min, and (*c*) the spectral blackbody emissive power at a wavelength of 3 μm.

SOLUTION An isothermal sphere is suspended in air. The total blackbody emissive power, the total radiation emitted in 5 minutes, and the spectral blackbody emissive power at 3 mm are to be determined.

Assumptions The ball behaves as a blackbody.

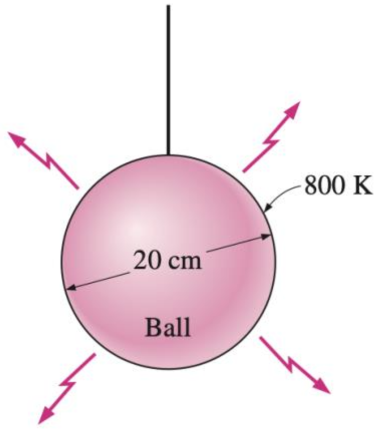

FIGURE 11–12
The spherical ball considered in Example 11–1.

Analysis (*a*) The total blackbody emissive power is determined from the Stefan–Boltzmann law to be

$$E_b = \sigma T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(800\ \text{K})^4 = \mathbf{23.2 \times 10^3\ W/m^2 = 23.2\ kW/m^2}$$

That is, the ball emits 23.2 kJ of energy in the form of electromagnetic radiation per second per m^2 of the surface area of the ball.

(*b*) The total amount of radiation energy emitted from the entire ball in 5 min is determined by multiplying the blackbody emissive power obtained above by the total surface area of the ball and the given time interval:

$$A_s = \pi D^2 = \pi(0.2\ \text{m})^2 = 0.1257\ \text{m}^2$$

$$\Delta t = (5\ \text{min})\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 300\ \text{s}$$

$$Q_{\text{rad}} = E_b A_s\, \Delta t = (23.2\ \text{kW/m}^2)(0.1257\ \text{m}^2)(300\ \text{s})\left(\frac{1\ \text{kJ}}{1000\ \text{W} \cdot \text{s}}\right)$$

$$= \mathbf{876\ kJ}$$

That is, the ball loses 876 kJ of its internal energy in the form of electromagnetic waves to the surroundings in 5 min, which is enough energy to raise the temperature of 1 kg of water by 50°C. Note that the surface temperature of the ball cannot remain constant at 800 K unless there is an equal amount of energy flow to the surface from the surroundings or from the interior regions of the ball through some mechanisms such as chemical or nuclear reactions.

(*c*) The spectral blackbody emissive power at a wavelength of 3 μm is determined from Planck's distribution law to be

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\dfrac{C_2}{\lambda T}\right) - 1\right]} = \frac{3.743 \times 10^8\ \text{W} \cdot \mu\text{m}^4/\text{m}^2}{(3\ \mu\text{m})^5\left[\exp\left(\dfrac{1.4387 \times 10^4\ \mu\text{m} \cdot \text{K}}{(3\ \mu\text{m})(800\ \text{K})}\right) - 1\right]}$$

$$= \mathbf{3848\ W/m^2 \cdot \mu m}$$

EXAMPLE 11–2 **Emission of Radiation from a Lightbulb**

The temperature of the filament of an incandescent lightbulb is 2500 K. Assuming the filament to be a blackbody, determine the fraction of the radiant energy emitted by the filament that falls in the visible range. Also, determine the wavelength at which the emission of radiation from the filament peaks.

SOLUTION The temperature of the filament of an incandescent lightbulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

Assumptions The filament behaves as a blackbody.

Analysis The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.4\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that $T = 2500$ K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 11–2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(2500\ \text{K}) = 1000\ \mu\text{m}\cdot\text{K} \longrightarrow f_{\lambda_1} = 0.000321$$

$$\lambda_2 T = (0.76\ \mu\text{m})(2500\ \text{K}) = 1900\ \mu\text{m}\cdot\text{K} \longrightarrow f_{\lambda_2} = 0.053035$$

That is, 0.03 percent of the radiation is emitted at wavelengths less than 0.4 μm and 5.3 percent at wavelengths less than 0.76 μm. Then the fraction of radiation emitted between these two wavelengths is (Fig. 11–15)

$$f_{\lambda_1-\lambda_2} = f_{\lambda_2} - f_{\lambda_1} = 0.053035 - 0.000321 = \mathbf{0.0527135}$$

Therefore, only about 5 percent of the radiation emitted by the filament of the lightbulb falls in the visible range. The remaining 95 percent of the radiation appears in the infrared region in the form of radiant heat or "invisible light," as it used to be called. This is certainly not a very efficient way of converting electrical energy to light and explains why fluorescent tubes are a wiser choice for lighting.

The wavelength at which the emission of radiation from the filament peaks is easily determined from Wien's displacement law to be

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \rightarrow \lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{2500\ \text{K}} = \mathbf{1.16\ \mu m}$$

Discussion Note that the radiation emitted from the filament peaks in the infrared region.

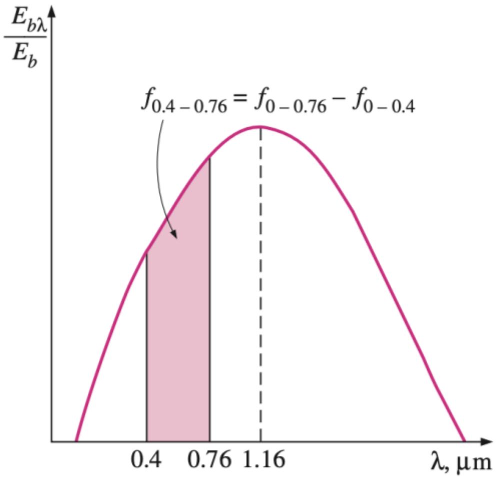

FIGURE 11–15
Graphical representation of the fraction of radiation emitted in the visible range in Example 11–2.

EXAMPLE 11–3 **Radiation Incident on a Small Surface**

A small surface of area $A_1 = 3$ cm^2 emits radiation as a blackbody at $T_1 = 600$ K. Part of the radiation emitted by A_1 strikes another small surface of area $A_2 = 5$ cm^2 oriented as shown in Figure 11–23. Determine the solid angle subtended by A_2 when viewed from A_1, and the rate at which radiation emitted by A_1 that strikes A_2.

SOLUTION A surface is subjected to radiation emitted by another surface. The solid angle subtended and the rate at which emitted radiation is received are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from Eq. 11-12 to be

$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos \theta_2}{r^2} = \frac{(5 \text{ cm}^2) \cos 40°}{(75 \text{ cm})^2} = \mathbf{6.81 \times 10^{-4} \text{ sr}}$$

since the normal of A_2 makes 40° with the direction of viewing. Note that solid angle subtended by A_2 would be maximum if A_2 were positioned normal to the direction of viewing. Also, the point of viewing on A_1 is taken to be a point in the middle, but it can be any point since A_1 is assumed to be very small.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle ω_{2-1}. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(600 \text{ K})^4}{\pi} = 2339 \text{ W/m}^2 \cdot \text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle ω_{2-1} is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle ω_{2-1}. That is,

$$
\begin{aligned}
\dot{Q}_{1-2} &= I_1(A_1 \cos \theta_1)\omega_{2-1} \\
&= (2339 \text{ W/m}^2 \cdot \text{sr})(3 \times 10^{-4} \cos 55^\circ \text{ m}^2)(6.81 \times 10^{-4} \text{ sr}) \\
&= \mathbf{2.74 \times 10^{-4} \text{ W}}
\end{aligned}
$$

Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 2.74×10^{-4} W.

Discussion The total rate of radiation emission from surface A_1 is $\dot{Q}_e = A_1 \sigma T_1^4 =$ 2.204 W. Therefore, the fraction of emitted radiation that strikes A_2 is $2.74 \times 10^{-4}/2.204 = 0.00012$ (or 0.012 percent). Noting that the solid angle associated with a hemisphere is 2π, the fraction of the solid angle subtended by A_2 is $6.81 \times 10^{-4}/(2\pi) = 0.000108$ (or 0.0108 percent), which is 0.9 times the fraction of emitted radiation. Therefore, the fraction of the solid angle a surface occupies does not represent the fraction of radiation energy the surface will receive even when the intensity of emitted radiation is constant. This is because radiation energy emitted by a surface in a given direction is proportional to the *projected area* of the surface in that direction, and reduces from a maximum at $\theta = 0^\circ$ (the direction normal to surface) to zero at $\theta = 90^\circ$ (the direction parallel to surface).