Example 1 heating Ammonia at Constant pressure

A vertical piston–cylinder assembly containing 0.1 lb of ammonia, initially a saturated vapor, is placed on a hot plate. Due to the weight of the piston and the surrounding atmospheric pressure, the pressure of the ammonia is 20 lbf/in.^{2} Heating occurs slowly, and the ammonia expands at constant pressure until the final temperature is 77°F. Show the initial and final states on T–v and p–v diagrams, and determine

a. the volume occupied by the ammonia at each end state, in ft^3.

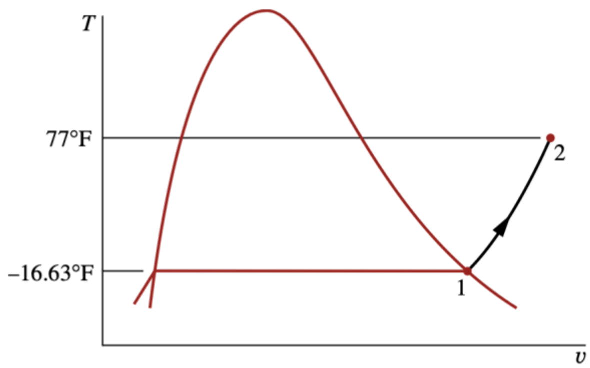
b. the work for the process, in Btu.

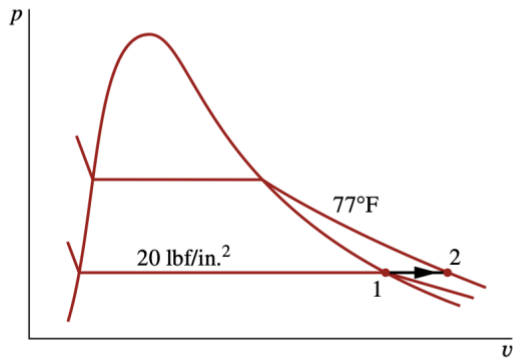
Solution

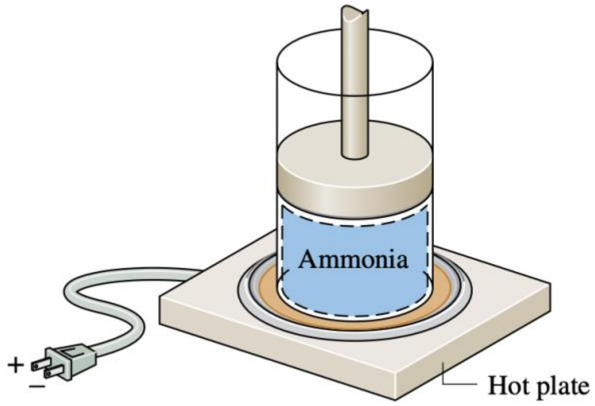
Known Ammonia is heated at constant pressure in a vertical piston–cylinder assembly from the saturated vapor state to a known final temperature.

Find Show the initial and final states on T–v and p–v diagrams, and determine the volume at each end state and the work for the process.

Schematic and Given Data:







Engineering Model

1. The ammonia is a closed system.
2. States 1 and 2 are equilibrium states.
3. The process occurs at constant pressure.
4. The piston is the only work mode.

Analysis The initial state is a saturated vapor condition at 20 lbf/in.^{2} Since the process occurs at constant pressure, the final state is in the superheated vapor region and is fixed by $p_2 = 20$ lbf/in.^{2} and $T_2 = 77$°F. The initial and final states are shown on the T–v and p–v diagrams above.

a. The volumes occupied by the ammonia at states 1 and 2 are obtained using the given mass and the respective specific volumes. From Table A-15E at $p_1 = 20$ lbf/in.^{2}, and corresponding to *Sat.* in the temperature column, we get $v_1 = v_g = 13.497$ ft^3/lb. Thus,

$$V_1 = mv_1 = (0.1 \text{ lb})(13.497 \text{ ft}^3/\text{lb}) = 1.35 \text{ ft}^3$$

Interpolating in Table A-15E at $p_2 = 20$ lbf/in.^{2} and $T_2 = 77$°F, we get $v_2 = 16.7$ ft^3/lb. Thus,

$$V_2 = mv_2 = (0.1 \text{ lb})(16.7 \text{ ft}^3/\text{lb}) = 1.67 \text{ ft}^3$$

b. In this case, the work can be evaluated using Eq. 2.17. Since the pressure is constant

$$W = \int_{V_1}^{V_2} p\, dV = p(V_2 - V_1)$$

Inserting values

❶ $$W = (20 \text{ lbf/in.}^2)(1.67 - 1.35)\text{ft}^3 \left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}}\right| = 1.18 \text{ Btu}$$

---

❶ Note the use of conversion factors in this calculation.

Interpolating in Table A-15E at $p_2 = 20$ lbf/in.^{2} and $T_2 = 77°F$, we get $v_2 = 16.7$ ft^3/lb. Thus,

$$V_2 = mv_2 = (0.1 \text{ lb})(16.7 \text{ ft}^3/\text{lb}) = 1.67 \text{ ft}^3$$

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Inserting values

❶ $$W = (20 \text{ lbf/in.}^2)(1.67 - 1.35)\text{ft}^3 \left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}}\right|$$

$$= 1.18 \text{ Btu}$$

---

❶ Note the use of conversion factors in this calculation.

Example 2 heating Water at Constant Volume

A closed, rigid container of volume 0.5 m^3 is placed on a hot plate. Initially, the container holds a two-phase mixture of saturated liquid water and saturated water vapor at $p_1 = 1$ bar with a quality of 0.5. After heating, the pressure in the container is $p_2 = 1.5$ bar. Indicate the initial and final states on a T–v diagram, and determine

a. the temperature, in °C, at states 1 and 2.

b. the mass of vapor present at states 1 and 2, in kg.

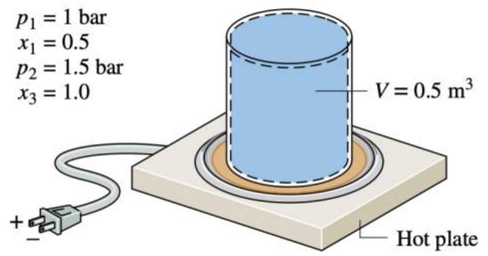
c. If heating continues, determine the pressure, in bar, when the container holds only saturated vapor.

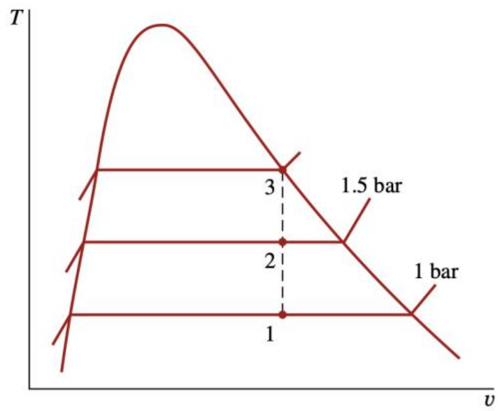
Solution

Known A two-phase liquid–vapor mixture of water in a closed, rigid container is heated on a hot plate. The initial pressure and quality and the final pressure are known.

Find Indicate the initial and final states on a T–v diagram and determine at each state the temperature and the mass of water vapor present. Also, if heating continues, determine the pressure when the container holds only saturated vapor.

Schematic and Given Data:





Engineering Model

1. The water in the container is a closed system.
2. States 1, 2, and 3 are equilibrium states.
3. The volume of the container remains constant.

Analysis Two independent properties are required to fix states 1 and 2. At the initial state, the pressure and quality are known. As these are independent, the state is fixed. State 1 is shown on the T–υ diagram in the two-phase region. The specific volume at state 1 is found using the given quality and Eq. 3.2. That is,

$$\upsilon_1 = \upsilon_{f1} + x_1(\upsilon_{g1} - \upsilon_{f1})$$

From Table A-3 at $p_1 = 1$ bar, $\upsilon_{f1} = 1.0432 \times 10^{-3}\ m^3/kg$ and $\upsilon_{g1} = 1.694\ m^3/kg$. Thus,

$$\upsilon_1 = 1.0432 \times 10^{-3} + 0.5(1.694 - 1.0432 \times 10^{-3}) = 0.8475\ m^3/kg$$

At state 2, the pressure is known. The other property required to fix the state is the specific volume υ_2. Volume and mass are each constant, so $\upsilon_2 = \upsilon_1 = 0.8475\ m^3/kg$. For $p_2 = 1.5$ bar, Table A-3 gives $\upsilon_{f2} = 1.0582 \times 10^{-3}\ m^3/kg$ and $\upsilon_{g2} = 1.59\ m^3/kg$. Since

1 $$\upsilon_f < \upsilon_2 < \upsilon_{g2}$$

2 state 2 must be in the two-phase region as well. State 2 is also shown on the T–υ diagram above.

a. Since states 1 and 2 are in the two-phase liquid–vapor region, the temperatures correspond to the saturation temperatures for the given pressures. Table A-3 gives

$$T_1 = 99.63°\text{C} \qquad \text{and} \qquad T_2 = 111.4°\text{C}$$

b. To find the mass of water vapor present, we first use the volume and the specific volume to find the *total* mass, m. That is,

$$m = \frac{V}{v} = \frac{0.5 \text{ m}^3}{0.8475 \text{ m}^3/\text{kg}} = 0.59 \text{ kg}$$

Then, with Eq. 3.1 and the given value of quality, the mass of vapor at state 1 is

$$m_{g1} = x_1 m = 0.5(0.59 \text{ kg}) = 0.295 \text{ kg}$$

The mass of vapor at state 2 is found similarly using the quality x_2. To determine x_2, solve Eq. 3.2 for quality and insert specific volume data from Table A-3 at a pressure of 1.5 bar, along with the known value of v, as follows

$$x_2 = \frac{v - v_{f2}}{v_{g2} - v_{f2}} = \frac{0.8475 - 1.0528 \times 10^{-3}}{1.159 - 1.0528 \times 10^{-3}} = 0.731$$

Then, with Eq. 3.1

$$m_{g2} = 0.731(0.59 \text{ kg}) = 0.431 \text{ kg}$$

c. If heating continued, state 3 would be on the saturated vapor line, as shown on the T–v diagram of Fig. E3.2. Thus, the pressure would be the corresponding saturation pressure. Interpolating in Table A-3 at $v_g = 0.8475 \text{ m}^3/\text{kg}$, we get $p_3 = 2.11$ bar.

❶ The procedure for fixing state 2 is the same as illustrated in the discussion of Fig. 3.8.

❷ Since the process occurs at constant specific volume, the states lie along a vertical line.

SKILLS DEVELOPED

Ability to…

- define a closed system and identify interactions on its boundary.
- sketch a T–v diagram and locate states on it.
- retrieve property data for water at liquid–vapor states, using quality.

Quick Quiz

If heating continues at constant specific volume from state 3 to a state where pressure is 3 bar, determine the temperature at that state, in °C. Ans. 282°C

Example 3 Stirring Water at Constant Volume

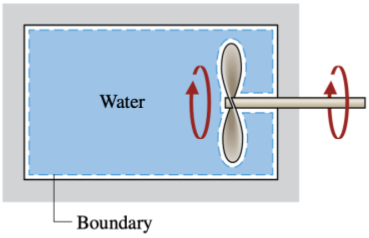
A well-insulated rigid tank having a volume of 10 ft^3 contains saturated water vapor at 212°F. The water is rapidly stirred until the pressure is 20 $lbf/in.^2$ Determine the temperature at the final state, in °F, and the work during the process, in Btu.

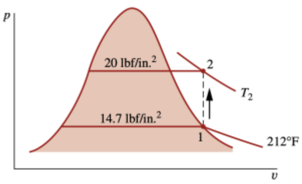
Solution

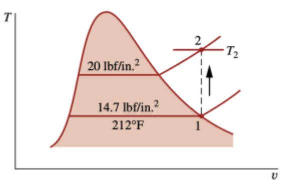
Known By rapid stirring, water vapor in a well-insulated rigid tank is brought from the saturated vapor state at 212°F to a pressure of 20 $lbf/in.^2$

Find Determine the temperature at the final state and the work.

Schematic and Given Data:







Engineering Model

1. The water is a closed system.
2. The initial and final states are at equilibrium. There is no net change in kinetic or potential energy.
3. There is no heat transfer with the surroundings.
4. The tank volume remains constant.

Analysis To determine the final equilibrium state, the values of two independent intensive properties are required. One of these is pressure, $p_2 = 20\ lbf/in.^2$, and the other is the specific volume: $v_2 = v_1$. The initial and final specific volumes are equal because the total mass and total volume are unchanged in the process. The initial and final states are located on the accompanying T–v and p–v diagrams.

From Table A-2E, $v_1 = v_g(212°F) = 26.80\ ft^3/lb$, $u_1 = u_g(212°F) = 1077.6$ Btu/lb. By using $v_2 = v_1$ and interpolating in Table A-4E at $p_2 = 20\ lbf/in.^2$

$$T_2 = 445°F, \qquad u_2 = 1161.6\ \text{Btu/lb}$$

Next, with assumptions 2 and 3 an energy balance for the system reduces to

$$\Delta U + \cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} = \cancelto{0}{Q} - W$$

On rearrangement

$$W = -(U_2 - U_1) = -m(u_2 - u_1)$$

To evaluate W requires the system mass. This can be determined from the volume and specific volume

$$m = \frac{V}{v_1} = \left(\frac{10\ ft^3}{26.8\ ft^3/lb}\right) = 0.373\ lb$$

Finally, by inserting values into the expression for W

$$W = -(0.373 \text{ lb})(1161.6 - 1077.6) \text{ Btu/lb} = -31.3 \text{ Btu}$$

where the minus sign signifies that the energy transfer by work is to the system.

---

1. Although the initial and final states are equilibrium states, the intervening states are not at equilibrium. To emphasize this, the process has been indicated on the T–v and p–v diagrams by a dashed line. Solid lines on property diagrams are reserved for processes that pass through equilibrium states only (quasiequilibrium processes). The analysis illustrates the importance of carefully sketched property diagrams as an adjunct to problem solving.

SKILLS DEVELOPED

Ability to…

- define a closed system and identify interactions on its boundary.
- apply the energy balance with steam table data.
- sketch T–v and p–v diagrams and locate states on them.

Quick Quiz

If insulation were removed from the tank and the water cooled at constant volume from $T_2 = 445°F$ to $T_3 = 300°F$ while no stirring occurs, determine the heat transfer, in Btu. Ans. −19.5 Btu

EXAmPLE 3.4 Analyzing Two processes in Series

Water contained in a piston–cylinder assembly undergoes two processes in series from an initial state where the pressure is 10 bar and the temperature is 400°C.

Process 1–2 The water is cooled as it is compressed at a constant pressure of 10 bar to the saturated vapor state.

Process 2–3 The water is cooled at constant volume to 150°C.

a. Sketch both processes on T–v and p–v diagrams.

b. For the overall process determine the work, in kJ/kg.

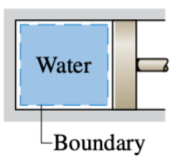
c. For the overall process determine the heat transfer, in kJ/kg.

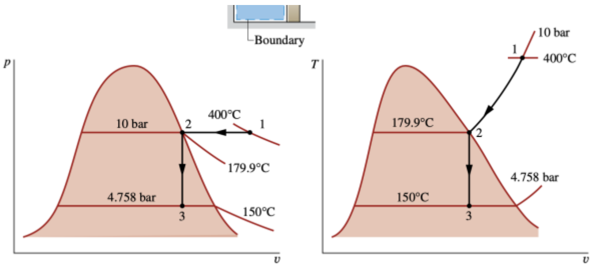
Solution

Known Water contained in a piston–cylinder assembly undergoes two processes: It is cooled and compressed, while keeping the pressure constant, and then cooled at constant volume.

Find Sketch both processes on T–v and p–v diagrams. Determine the net work and the net heat transfer for the overall process per unit of mass contained within the piston–cylinder assembly.

Schematic and Given Data:





Engineering Model

1. The water is a closed system.
2. The piston is the only work mode.
3. There are no changes in kinetic or potential energy.

Analysis

a. The accompanying T–v and p–v diagrams show the two processes. Since the temperature at state 1, $T_1 = 400°C$, is greater than the saturation temperature corresponding to $p_1 = 10$ bar: 179.9°C, state 1 is located in the superheat region.

b. Since the piston is the only work mechanism

$$W = \int_1^3 p\,dV = \int_1^2 p\,dV + \int_2^3 p_2 \cancelto{0}{dV}$$

The second integral vanishes because the volume is constant in Process 2–3. Dividing by the mass and noting that the pressure is constant for Process 1–2

$$\frac{W}{m} = p(v_2 - v_1)$$

The specific volume at state 1 is found from Table A-4 using $p_1 = 10$ bar and $T_1 = 400°C$: $v_1 = 0.3066\ m^3/kg$. Also, $u_1 = 2957.3$ kJ/kg. The specific volume at state 2 is the saturated vapor value at 10 bar: $v_2 = 0.1944\ m^3/kg$, from Table A-3. Hence,

$$\frac{W}{m} = (10\text{ bar})(0.1944 - 0.3066)\left(\frac{m^3}{kg}\right)\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= -112.2\text{ kJ/kg}$$

The minus sign indicates that work is done *on* the water vapor by the piston.

c. An energy balance for the *overall* process reduces to

$$m(u_3 - u_1) = Q - W$$

By rearranging

$$\frac{Q}{m} = (u_3 - u_1) + \frac{W}{m}$$

To evaluate the heat transfer requires u_3, the specific internal energy at state 3. Since T_3 is given and $v_3 = v_2$, two independent intensive properties are known that together fix state 3. To find u_3, first solve for the quality

$$x_3 = \frac{v_3 - v_{f3}}{v_{g3} - v_{f3}} = \frac{0.1944 - 1.0905 \times 10^{-3}}{0.3928 - 1.0905 \times 10^{-3}} = 0.494$$

where v_{f3} and v_{g3} are from Table A-2 at 150°C. Then

$$u_3 = u_{f3} + x_3(u_{g3} - u_{f3}) = 631.68 + 0.494(2559.5 - 631.68)$$
$$= 1584.0 \text{ kJ/kg}$$

where u_{f3} and u_{g3} are from Table A-2 at 150°C.

Substituting values into the energy balance

$$\frac{Q}{m} = 1584.0 - 2957.3 + (-112.2) = -1485.5 \text{ kJ/kg}$$

The minus sign shows that energy is transferred *out* by heat transfer.

SKILLS DEVELOPED

Ability to…

- define a closed system and identify interactions on its boundary.
- evaluate work using Eq. 2.17.
- apply the energy balance with steam table data.
- sketch T–v and p–v diagrams and locate states on them.

Quick Quiz

If the two specified processes were followed by Process 3-4, during which the water expands at a constant temperature of 150°C to saturated vapor, determine the work, in kJ/kg, for the *overall* process from 1 to 4. Ans. $W/m = -17.8$ kJ/kg

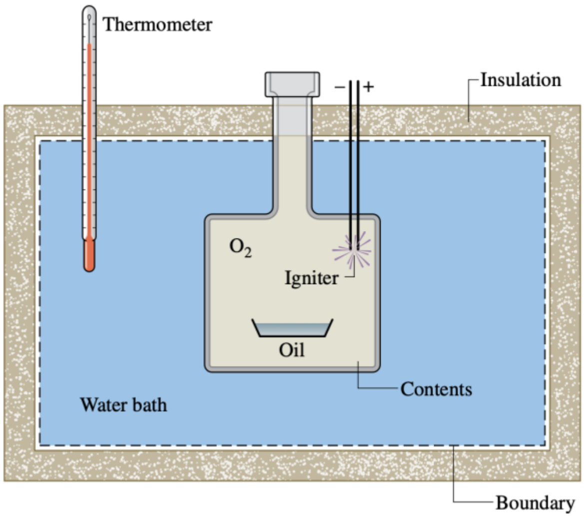
Example 4 Measuring the Calorie Value of Cooking Oil

One-tenth milliliter of cooking oil is placed in the chamber of a constant-volume calorimeter filled with sufficient oxygen for the oil to be completely burned. The chamber is immersed in a water bath. The mass of the water bath is 2.15 kg. For the purpose of this analysis, the metal parts of the apparatus are modeled as equivalent to an additional 0.5 kg of water. The calorimeter is well-insulated, and initially the temperature throughout is 25°C. The oil is ignited by a spark. When equilibrium is again attained, the temperature throughout is 25.3°C. Determine the change in internal energy of the chamber contents, in kcal per mL of cooking oil and in kcal per tablespoon of cooking oil.

Known Data are provided for a constant-volume calorimeter testing cooking oil for calorie value.

Find Determine the change in internal energy of the contents of the calorimeter chamber.

Schematic and Given Data:



Engineering Model

1. The closed system is shown by the dashed line in the accompanying figure.
2. The total volume remains constant, including the chamber, water bath, and the amount of water modeling the metal parts.
3. Water is modeled as incompressible with constant specific heat c.
4. Heat transfer with the surroundings is negligible, and there is no change in kinetic or potential energy.

Analysis With the assumptions listed, the closed system energy balance reads

$$\Delta U + \Delta \cancel{KE}^0 + \Delta \cancel{PE}^0 = \cancel{Q}^0 - \cancel{W}^0$$

or

$$(\Delta U)_{contents} + (\Delta U)_{water} = 0$$

thus

$$(\Delta U)_{contents} = -(\Delta U)_{water} \qquad \text{(a)}$$

The change in internal energy of the contents is equal and opposite to the change in internal energy of the water.

Since water is modeled as incompressible, Eq. 3.20a is used to evaluate the right side of Eq. (a), giving

❶ ❷
$$(\Delta U)_{contents} = -m_w c_w (T_2 - T_1) \qquad \text{(b)}$$

With $m_w = 2.15\text{ kg} + 0.5\text{ kg} = 2.65\text{ kg}$, $(T_2 - T_1) = 0.3\text{ K}$, and $c_w = 4.18\text{ kJ/kg}\cdot\text{K}$ from Table A-19, Eq. (b) gives

$$(\Delta U)_{contents} = -(2.65\text{ kg})(4.18\text{ kJ/kg}\cdot\text{K})(0.3\text{ K}) = -3.32\text{ kJ}$$

Converting to kcal, and expressing the result on a per milliliter of oil basis using the oil volume, 0.1 mL, we get

$$\frac{(\Delta U)_{contents}}{V_{oil}} = \frac{-3.32\text{ kJ}}{0.1\text{ mL}}\left|\frac{1\text{ kcal}}{4.1868\text{ kJ}}\right|$$
$$= -7.9\text{ kcal/mL}$$

The calorie value of the cooking oil is the magnitude—that is, 7.9 kcal/mL. Labels on cooking oil containers usually give calorie value for a serving size of 1 tablespoon (15 mL). Using the calculated value, we get 119 kcal per tablespoon.

---

1. The change in internal energy for water can be found alternatively using Eq. 3.12 together with saturated liquid internal energy data from Table A-2.
2. The change in internal energy of the chamber contents cannot be evaluated using a specific heat because specific heats are defined (Sec. 3.9) only for *pure* substances—that is, substances that are unchanging in composition.

SKILLS DEVELOPED

Ability to…

- define a closed system and identify interactions within it and on its boundary.
- apply the energy balance using the incompressible substance model.

Quick Quiz

Using Eq. 3.12 together with saturated liquid internal energy data from Table A-2, find the change in internal energy of the water, in kJ, and compare with the value obtained assuming water is incompressible. Ans. 3.32 kJ

Example 5 Applying the Mass rate Balance to a Feedwater heater at Steady State

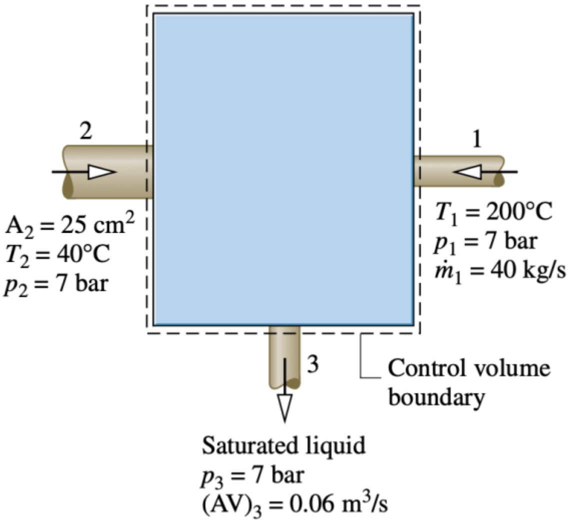
A feedwater heater operating at steady state has two inlets and one exit. At inlet 1, water vapor enters at $p_1 = 7$ bar, $T_1 = 200°C$ with a mass flow rate of 40 kg/s. At inlet 2, liquid water at $p_2 = 7$ bar, $T_2 = 40°C$ enters through an area $A_2 = 25\ cm^2$. Saturated liquid at 7 bar exits at 3 with a volumetric flow rate of 0.06 m^3/s. Determine the mass flow rates at inlet 2 and at the exit, in kg/s, and the velocity at inlet 2, in m/s.

Solution

Known A stream of water vapor mixes with a liquid water stream to produce a saturated liquid stream at the exit. The states at the inlets and exit are specified. Mass flow rate and volumetric flow rate data are given at one inlet and at the exit, respectively.

Find Determine the mass flow rates at inlet 2 and at the exit, and the velocity V_2.

Schematic and Given Data:



Engineering Model The control volume shown on the accompanying figure is at steady state.

Analysis The principal relations to be employed are the mass rate balance (Eq. 4.2) and the expression $\dot{m} = AV/v$ (Eq. 4.4b). At steady state the mass rate balance becomes

❶ $$\frac{dm_{cv}}{dt}^{0} = \dot{m}_1 + \dot{m}_2 - \dot{m}_3$$

Solving for $\dot{m}_2$,

$$\dot{m}_2 = \dot{m}_3 - \dot{m}_1$$

The mass flow rate $\dot{m}_1$ is given. The mass flow rate at the exit can be evaluated from the given volumetric flow rate

$$\dot{m}_3 = \frac{(AV)_3}{v_3}$$

where v_3 is the specific volume at the exit. In writing this expression, one-dimensional flow is assumed. From Table A-3, $v_3 = 1.108 \times 10^{-3}\ m^3/kg$. Hence,

$$\dot{m}_3 = \frac{0.06\ m^3/s}{(1.108 \times 10^{-3}\ m^3/kg)} = 54.15\ kg/s$$

The mass flow rate at inlet 2 is then

$$\dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 54.15 - 40 = 14.15\ kg/s$$

For one-dimensional flow at 2, $\dot{m}_2 = A_2V_2/v_2$, so

$$V_2 = \dot{m}_2 v_2 / A_2$$

State 2 is a compressed liquid. The specific volume at this state can be approximated by $v_2 \approx v_f(T_2)$ (Eq. 3.11). From Table A-2 at 40°C, $v_2 = 1.0078 \times 10^{-3}\ m^3/kg$. So,

$$V_2 = \frac{(14.15\ kg/s)(1.0078 \times 10^{-3}\ m^3/kg)}{25\ cm^2}\left|\frac{10^4\ cm^2}{1\ m^2}\right| = 5.7\ m/s$$

1 In accord with Eq. 4.6, the mass flow rate at the exit equals the sum of the mass flow rates at the inlets. It is left as an exercise to show that the volumetric flow rate at the exit *does not equal* the sum of the volumetric flow rates at the inlets.

SKILLS DEVELOPED

Ability to…

- apply the steady-state mass rate balance.
- apply the mass flow rate expression, Eq. 4.4b.
- retrieve property data for water.

Quick Quiz

Evaluate the volumetric flow rate, in m^3/s, at each inlet. Ans. $(AV)_1 = 12\ m^3/s$, $(AV)_2 = 0.01\ m^3/s$

Example 6 Calculating Exit Area of a Steam Nozzle

Steam enters a converging–diverging nozzle operating at steady state with $p_1 = 40$ bar, $T_1 = 400°C$, and a velocity of 10 m/s. The steam flows through the nozzle with negligible heat transfer and no significant change in potential energy. At the exit, $p_2 = 15$ bar, and the velocity is 665 m/s. The mass flow rate is 2 kg/s. Determine the exit area of the nozzle, in m^2.

Solution

Known Steam flows through a nozzle at steady state with known properties at the inlet and exit, a known mass flow rate, and negligible effects of heat transfer and potential energy.

Find Determine the exit area.

Schematic and Given Data:

Engineering Model

1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer is negligible and $\dot{W}_{cv} = 0$.
3. The change in potential energy from inlet to exit can be neglected.

Analysis The exit area can be determined from the mass flow rate $\dot{m}$ and Eq. 4.4b, which can be arranged to read

$$A_2 = \frac{\dot{m} v_2}{V_2}$$

To evaluate A_2 from this equation requires the specific volume v_2 at the exit, and this requires that the exit state be fixed.

The state at the exit is fixed by the values of two independent intensive properties. One is the pressure p_2, which is known. The other is the specific enthalpy h_2, determined from the steady-state energy rate balance Eq. 4.20a, as follows

$$0 = \dot{Q}_{cv}^{\,0} - \dot{W}_{cv}^{\,0} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

The terms $\dot{Q}_{cv}$ and $\dot{W}_{cv}$ are deleted by assumption 2. The change in specific potential energy drops out in accordance with assumption 3 and $\dot{m}$ cancels, leaving

$$0 = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

From Table A-4, $h_1 = 3213.6$ kJ/kg. The velocities V_1 and V_2 are given. Inserting values and converting the units of the kinetic energy terms to kJ/kg results in

$$h_2 = 3213.6 \text{ kJ/kg} + \left[\frac{(10)^2 - (665)^2}{2}\right]\left(\frac{\text{m}^2}{\text{s}^2}\right)\left|\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}}\right|$$

$$= 3213.6 - 221.1 = 2992.5 \text{ kJ/kg}$$

Finally, referring to Table A-4 at $p_2 = 15$ bar with $h_2 = 2992.5$ kJ/kg, the specific volume at the exit is $v_2 = 0.1627 \text{ m}^3/\text{kg}$. The exit area is then

$$A_2 = \frac{(2 \text{ kg/s})(0.1627 \text{ m}^3/\text{kg})}{665 \text{ m/s}} = 4.89 \times 10^{-4} \text{ m}^2$$

1. Although equilibrium property relations apply at the inlet and exit of the control volume, the intervening states of the steam are not necessarily equilibrium states. Accordingly, the expansion through the nozzle is represented on the T–v diagram as a dashed line.
2. Care must be taken in converting the units for specific kinetic energy to kJ/kg.

SKILLS DEVELOPED

Ability to…

- apply the steady-state energy rate balance to a control volume.
- apply the mass flow rate expression, Eq. 4.4b.
- develop an engineering model.
- retrieve property data for water.

Quick Quiz

Evaluate the nozzle inlet area, in m^2. Ans. 1.47×10^{-2} m^2.

Example 7 Calculating heat transfer from a Steam turbine

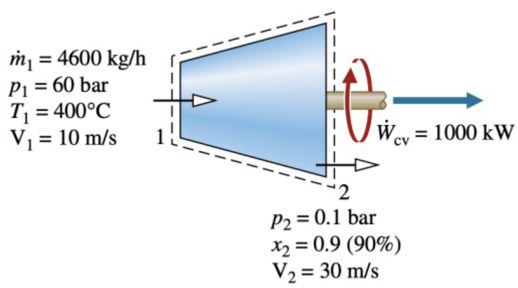
Steam enters a turbine operating at steady state with a mass flow rate of 4600 kg/h. The turbine develops a power output of 1000 kW. At the inlet, the pressure is 60 bar, the temperature is 400°C, and the velocity is 10 m/s. At the exit, the pressure is 0.1 bar, the quality is 0.9 (90%), and the velocity is 30 m/s. Calculate the rate of heat transfer between the turbine and surroundings, in kW.

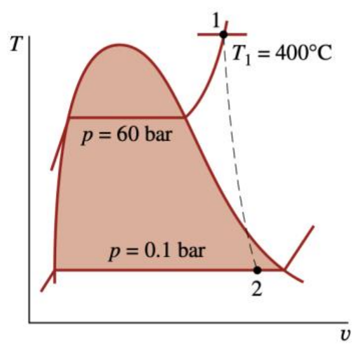
Solution

Known A steam turbine operates at steady state. The mass flow rate, power output, and states of the steam at the inlet and exit are known.

Find Calculate the rate of heat transfer.

Schematic and Given Data:





Engineering Model

1. The control volume shown on the accompanying figure is at steady state.
2. The change in potential energy from inlet to exit can be neglected.

Analysis To calculate the heat transfer rate, begin with the one-inlet, one-exit form of the energy rate balance for a control volume at steady state, Eq. 4.20a. That is,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

where $\dot{m}$ is the mass flow rate. Solving for $\dot{Q}_{cv}$ and dropping the potential energy change from inlet to exit

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}\left[(h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)\right] \tag{a}$$

To compare the magnitudes of the enthalpy and kinetic energy terms, and stress the unit conversions needed, each of these terms is evaluated separately.

First, the specific *enthalpy difference* $h_2 - h_1$ is found. Using Table A-4, $h_1 = 3177.2$ kJ/kg. State 2 is a two-phase liquid–vapor mixture, so with data from Table A-3 and the given quality

$$\begin{aligned} h_2 &= h_{f2} + x_2(h_{g2} - h_{f2}) \\ &= 191.83 + (0.9)(2392.8) = 2345.4 \text{ kJ/kg} \end{aligned}$$

Hence,

$$h_2 - h_1 = 2345.4 - 3177.2 = -831.8 \text{ kJ/kg}$$

Consider next the specific *kinetic energy difference*. Using the given values for the velocities,

$$\left(\frac{V_2^2 - V_1^2}{2}\right) = \left[\frac{(30)^2 - (10)^2}{2}\right]\left(\frac{m^2}{s^2}\right)\left|\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}}\right|$$
$$= 0.4 \text{ kJ/kg}$$

Calculating $\dot{Q}_{cv}$ from Eq. (a),

$$\dot{Q}_{cv} = (1000 \text{ kW}) + \left(4600 \frac{\text{kg}}{\text{h}}\right)(-831.8 + 0.4)\left(\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1 \text{ h}}{3600 \text{ s}}\right|\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$
$$= -62.3 \text{ kW}$$

1. The magnitude of the change in specific kinetic energy from inlet to exit is much smaller than the specific enthalpy change. Note the use of unit conversion factors here and in the calculation of $\dot{Q}_{cv}$ to follow.
2. The negative value of $\dot{Q}_{cv}$ means that there is heat transfer from the turbine to its surroundings, as would be expected. The magnitude of $\dot{Q}_{cv}$ is small relative to the power developed.

SKILLS DEVELOPED

Ability to…

- apply the steady-state energy rate balance to a control volume.
- develop an engineering model.
- retrieve property data for water.

Quick Quiz

If the change in kinetic energy from inlet to exit were neglected, evaluate the heat transfer rate, in kW, keeping all other data unchanged. Comment. Ans. −62.9 kW.

Example 8 Calculating Compressor power

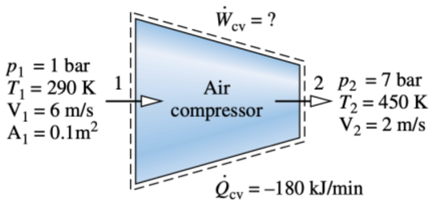
Air enters a compressor operating at steady state at a pressure of 1 bar, a temperature of 290 K, and a velocity of 6 m/s through an inlet with an area of 0.1 m^2. At the exit, the pressure is 7 bar, the temperature is 450 K, and the velocity is 2 m/s. Heat transfer from the compressor to its surroundings occurs at a rate of 180 kJ/min. Employing the ideal gas model, calculate the power input to the compressor, in kW.

Solution

Known An air compressor operates at steady state with known inlet and exit states and a known heat transfer rate.

Find Calculate the power required by the compressor.

Schematic and Given Data:



Engineering Model

1. The control volume shown on the accompanying figure is at steady state.
2. The change in potential energy from inlet to exit can be neglected.
3. ❶ The ideal gas model applies for the air.

Analysis To calculate the power input to the compressor, begin with the one-inlet, one-exit form of the energy rate balance for a control volume at steady state, Eq. 4.20a. That is,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

Solving

$$\dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right]$$

The change in potential energy from inlet to exit drops out by assumption 2.

The mass flow rate $\dot{m}$ can be evaluated with given data at the inlet and the ideal gas equation of state.

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_1 V_1 p_1}{(\bar{R}/M)T_1} = \frac{(0.1\ \text{m}^2)(6\ \text{m/s})(10^5\ \text{N/m}^2)}{\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(290\ \text{K})} = 0.72\ \text{kg/s}$$

The specific enthalpies h_1 and h_2 can be found from Table A-22. At 290 K, $h_1 = 290.16$ kJ/kg. At 450 K, $h_2 = 451.8$ kJ/kg. Substituting values into the expression for $\dot{W}_{cv}$, and applying appropriate unit conversion factors, we get

$$\dot{W}_{cv} = \left(-180\frac{\text{kJ}}{\text{min}}\right)\left|\frac{1\ \text{min}}{60\ \text{s}}\right| + 0.72\frac{\text{kg}}{\text{s}}\left[(290.16 - 451.8)\frac{\text{kJ}}{\text{kg}} + \left(\frac{(6)^2 - (2)^2}{2}\right)\left(\frac{\text{m}^2}{\text{s}^2}\right)\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|\right]$$

$$= -3\frac{\text{kJ}}{\text{s}} + 0.72\frac{\text{kg}}{\text{s}}(-161.64 + 0.02)\frac{\text{kJ}}{\text{kg}}$$

2 $$= -119.4\frac{\text{kJ}}{\text{s}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = -119.4\ \text{kW}$$

1 The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart.

2 In this example $\dot{Q}_{cv}$ and $\dot{W}_{cv}$ have negative values, indicating that the direction of the heat transfer is *from* the compressor and work is done *on* the air passing through the compressor. The magnitude of the power *input* to the compressor is 119.4 kW. The change in kinetic energy does not contribute significantly.

SKILLS DEVELOPED

Ability to…

- apply the steady-state energy rate balance to a control volume.
- apply the mass flow rate expression, Eq. 4.4b.
- develop an engineering model.
- retrieve property data of air modeled as an ideal gas.

Quick Quiz

If the change in kinetic energy from inlet to exit were neglected, evaluate the compressor power, in kW, keeping all other data unchanged. Comment. Ans. −119.4 kW.

Example 9 Analyzing a pump System

A pump steadily draws water from a pond at a volumetric flow rate of 220 gal/min through a pipe having a 5 in. diameter inlet. The water is delivered through a hose terminated by a converging nozzle. The nozzle exit has a diameter of 1 in. and is located 35 ft above the pipe inlet. Water enters at 70°F, 14.7 lbf/in.^{2} and exits with no significant change in temperature or pressure. The magnitude of the rate of heat transfer *from* the pump *to* the surroundings is 5% of the power input. The acceleration of gravity is 32.2 ft/s^2.

Determine **(a)** the velocity of the water at the inlet and exit, each in ft/s, and **(b)** the power required by the pump, in hp.

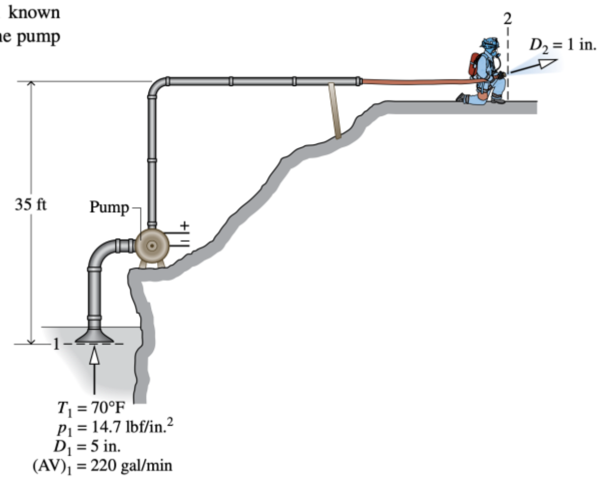
Solution

Known A pump system operates at steady state with known inlet and exit conditions. The rate of heat transfer from the pump is specified as a percentage of the power input.

Find Determine the velocities of the water at the inlet and exit of the pump system and the power required.

Schematic and Given Data:

known
ıe pump



Engineering Model

1. A control volume encloses the pump, inlet pipe, and delivery hose.
2. The control volume is at steady state.
3. The magnitude of the heat transfer from the control volume is 5% of the power input.
4. There is no significant change in temperature or pressure.
5. For liquid water, $v \approx v_f(T)$ (Eq. 3.11) and Eq. 3.13 is used to evaluate specific enthalpy.
6. $g = 32.2\ \text{ft/s}^2$.

Analysis

a. A mass rate balance reduces at steady state to read $\dot{m}_2 = \dot{m}_1$. The common mass flow rate at the inlet and exit, $\dot{m}$, can be evaluated using Eq. 4.4b together with $v \approx v_f(70°\text{F}) = 0.01605\ \text{ft}^3/\text{lb}$ from Table A-2E. That is,

$$\dot{m} = \frac{\text{AV}}{v} = \left(\frac{220\ \text{gal/min}}{0.01605\ \text{ft}^3/\text{lb}}\right)\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\left|\frac{0.13368\ \text{ft}^3}{1\ \text{gal}}\right|$$

$$= 30.54\ \text{lb/s}$$

Thus, the inlet and exit velocities are, respectively,

1 $$\text{V}_1 = \frac{\dot{m}v}{\text{A}_1} = \frac{(30.54\ \text{lb/s})(0.01605\ \text{ft}^3/\text{lb})}{\pi(5\ \text{in.})^2/4}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 3.59\ \text{ft/s}$$

$$\text{V}_2 = \frac{\dot{m}v}{\text{A}_2} = \frac{(30.54\ \text{lb/s})(0.01605\ \text{ft}^3/\text{lb})}{\pi(1\ \text{in.})^2/4}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 89.87\ \text{ft/s}$$

b. To calculate the power input, begin with the one-inlet, one-exit form of the energy rate balance for a control volume at steady state, Eq. 4.20a. That is,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

❷ Introducing $\dot{Q}_{cv} = (0.05)\dot{W}_{cv}$, and solving for $\dot{W}_{cv}$

$$\dot{W}_{cv} = \frac{\dot{m}}{0.95}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right] \quad \text{(a)}$$

Using Eq. 3.13, the enthalpy term is expressed as

$$h_1 - h_2 = [h_f(T_1) + v_f(T_1)[p_1 - p_{sat}(T_1)]] - [h_f(T_2) + v_f(T_2)[p_2 - p_{sat}(T_2)]] \quad \text{(b)}$$

Since there is no significant change in temperature, Eq. (b) reduces to

$$h_1 - h_2 = v_f(T)(p_1 - p_2)$$

As there is also no significant change in pressure, the enthalpy term drops out of the present analysis. Next, evaluating the kinetic energy term

$$\frac{V_1^2 - V_2^2}{2} = \frac{[(3.59)^2 - (89.87)^2]\left(\frac{\text{ft}}{\text{s}}\right)^2}{2}\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| \times \left|\frac{1\ \text{lbf}}{32.174\ \text{lb}\cdot\text{ft/s}^2}\right| = -0.1614\ \text{Btu/lb}$$

Finally, the potential energy term is

$$g(z_1 - z_2) = (32.2\ \text{ft/s}^2)(0 - 35)\text{ft}\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| \times \left|\frac{1\ \text{lbf}}{32.174\ \text{lb}\cdot\text{ft/s}^2}\right| = -0.0450\ \text{Btu/lb}$$

Inserting values into Eq. (a)

$$\dot{W}_{cv} = \left(\frac{30.54\ \text{lb/s}}{0.95}\right)[0 - 0.1614 - 0.0450]\left(\frac{\text{Btu}}{\text{lb}}\right) = -6.64\ \text{Btu/s}$$

Converting to horsepower

$$\dot{W}_{cv} = \left(-6.64\frac{\text{Btu}}{\text{s}}\right)\left|\frac{1\ \text{hp}}{2545\frac{\text{Btu}}{\text{h}}}\right|\left|\frac{3600\ \text{s}}{1\ \text{h}}\right| = -9.4\ \text{hp}$$

where the minus sign indicates that power is provided to the pump.

---

❶ Alternatively, V_1 can be evaluated from the volumetric flow rate at 1. This is left as an exercise.

❷ Since power is required to operate the pump, $\dot{W}_{cv}$ is negative in accord with our sign convention. The energy transfer by heat is from the control volume to the surroundings, and thus $\dot{Q}_{cv}$ is negative as well. Using the value of $\dot{W}_{cv}$ found in part (b), $\dot{Q}_{cv} = (0.05)\ \dot{W}_{cv} = -0.332$ Btu/s (−0.47 hp).

SKILLS DEVELOPED

Ability to…

- apply the steady-state energy rate balance to a control volume.
- apply the mass flow rate expression, Eq. 4.4b.
- develop an engineering model.
- retrieve properties of liquid water.

Quick Quiz

If the nozzle were removed and water exited directly from the hose, whose diameter is 2 in., determine the velocity at the exit, in ft/s, and the power required, in hp, keeping all other data unchanged. Ans. 22.47 ft/s, −2.5 hp.

Example 10 Evaluating performance of a power plant Condenser

Steam enters the condenser of a vapor power plant at 0.1 bar with a quality of 0.95 and condensate exits at 0.1 bar and 45°C. Cooling water enters the condenser in a separate stream as a liquid at 20°C and exits as a liquid at 35°C with no change in pressure. Heat transfer from the outside of the condenser and changes in the kinetic and potential energies of the flowing streams can be ignored. For steady-state operation, determine

a. the ratio of the mass flow rate of the cooling water to the mass flow rate of the condensing steam.

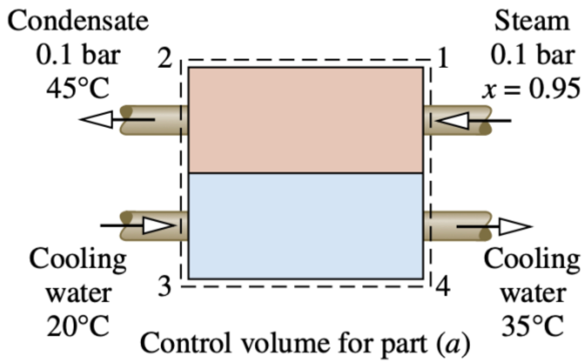
b. the energy transfer from the condensing steam to the cooling water, in kJ per kg of steam passing through the condenser.

Solution

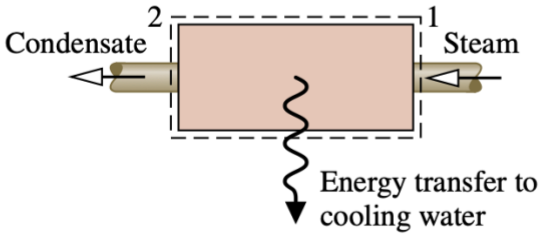
Known Steam is condensed at steady state by interacting with a separate liquid water stream.

Find Determine the ratio of the mass flow rate of the cooling water to the mass flow rate of the steam and the energy transfer from the steam to the cooling water per kg of steam passing through the condenser.

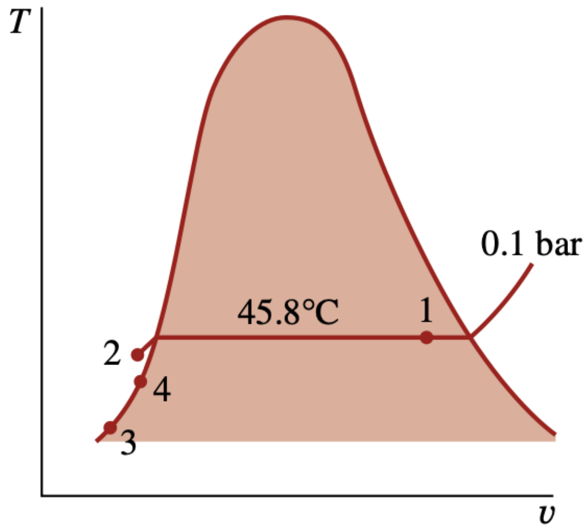
Schematic and Given Data:



Control volume for part (*a*)



Control volume for part (*b*)



Engineering Model

1. Each of the two control volumes shown on the accompanying sketch is at steady state.
2. There is no significant heat transfer between the overall condenser and its surroundings. $\dot{W}_{cv} = 0$.

3. Changes in the kinetic and potential energies of the flowing streams from inlet to exit can be ignored.

4. At states 2, 3, and 4, $h \approx h_f(T)$ (see Eq. 3.14).

Analysis The steam and the cooling water streams do not mix. Thus, the mass rate balances for each of the two streams reduce at steady state to give

$$\dot{m}_1 = \dot{m}_2 \quad \text{and} \quad \dot{m}_3 = \dot{m}_4$$

a. The ratio of the mass flow rate of the cooling water to the mass flow rate of the condensing steam, $\dot{m}_3/\dot{m}_1$, can be found from the steady-state form of the energy rate balance, Eq. 4.18, applied to the overall condenser as follows:

$$0 = \underline{\dot{Q}_{cv}} - \underline{\dot{W}_{cv}} + \dot{m}_1\left(h_1 + \underline{\frac{V_1^2}{2} + gz_1}\right) + \dot{m}_3\left(h_3 + \underline{\frac{V_3^2}{2} + gz_3}\right) - \dot{m}_2\left(h_2 + \underline{\frac{V_2^2}{2} + gz_2}\right) - \dot{m}_4\left(h_4 + \underline{\frac{V_4^2}{2} + gz_4}\right)$$

The underlined terms drop out by assumptions 2 and 3. With these simplifications, together with the above mass flow rate relations, the energy rate balance becomes simply

$$0 = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$$

Solving, we get

$$\frac{\dot{m}_3}{\dot{m}_1} = \frac{h_1 - h_2}{h_4 - h_3}$$

The specific enthalpy h_1 can be determined using the given quality and data from Table A-3. From Table A-3 at 0.1 bar, $h_{f1} = 191.83$ kJ/kg and $h_{g1} = 2584.7$ kJ/kg, so

$$h_1 = 191.83 + 0.95(2584.7 - 191.83) = 2465.1 \text{ kJ/kg}$$

❶ Using assumption 4, the specific enthalpy at 2 is given by $h_2 \approx h_f(T_2) = 188.45$ kJ/kg. Similarly, $h_3 \approx h_f(T_3)$ and $h_4 \approx h_f(T_4)$, giving $h_4 - h_3 = 62.7$ kJ/kg. Thus,

$$\frac{\dot{m}_3}{\dot{m}_1} = \frac{2465.1 - 188.45}{62.7} = 36.3$$

b. For a control volume enclosing the steam side of the condenser only, begin with the steady-state form of energy rate balance, Eq. 4.20a.

❷ $$0 = \dot{Q}_{cv} - \underline{\dot{W}_{cv}} + \dot{m}_1\left[(h_1 - h_2) + \underline{\frac{(V_1^2 - V_2^2)}{2}} + \underline{g(z_1 - z_2)}\right]$$

The underlined terms drop out by assumptions 2 and 3. The following expression for the rate of energy transfer between the condensing steam and the cooling water results:

$$\dot{Q}_{cv} = \dot{m}_1(h_2 - h_1)$$

Dividing by the mass flow rate of the steam, $\dot{m}_1$, and inserting values

$$\frac{\dot{Q}_{cv}}{\dot{m}_1} = h_2 - h_1 = 188.45 - 2465.1 = -2276.7 \text{ kJ/kg}$$

where the minus sign signifies that energy is transferred *from* the condensing steam *to* the cooling water.

---

1. Alternatively, $(h_4 - h_3)$ can be evaluated using the incompressible liquid model via Eq. 3.20b.
2. Depending on where the boundary of the control volume is located, two different formulations of the energy rate balance are obtained. In part (a), both streams are included in the control volume. Energy transfer between them occurs internally and not across the boundary of the control volume, so the term $\dot{Q}_{cv}$ drops out of the energy rate balance. With the control volume of part (b), however, the term $\dot{Q}_{cv}$ must be included.

SKILLS DEVELOPED

Ability to…

- apply the steady-state mass and energy rate balances to a control volume.
- develop an engineering model.
- retrieve property data for water.

Quick Quiz

If the mass flow rate of the condensing steam is 125 kg/s, determine the mass flow rate of the cooling water, in kg/s. Ans. 4538 kg/s.

Example 10 Cooling Computer Components

The electronic components of a computer are cooled by air flowing through a fan mounted at the inlet of the electronics enclosure. At steady state, air enters at 20°C, 1 atm. For noise control, the velocity of the entering air cannot exceed 1.3 m/s. For temperature control, the temperature of the air at the exit cannot exceed 32°C. The electronic components and fan receive, respectively, 80 W and 18 W of electric power. Determine the smallest fan inlet area, in cm^2, for which the limits on the entering air velocity and exit air temperature are met.

Solution

Known The electronic components of a computer are cooled by air flowing through a fan mounted at the inlet of the electronics enclosure. Conditions are specified for the air at the inlet and exit. The power required by the electronics and the fan is also specified.

Find Determine the smallest fan area for which the specified limits are met.

Schematic and Given Data:

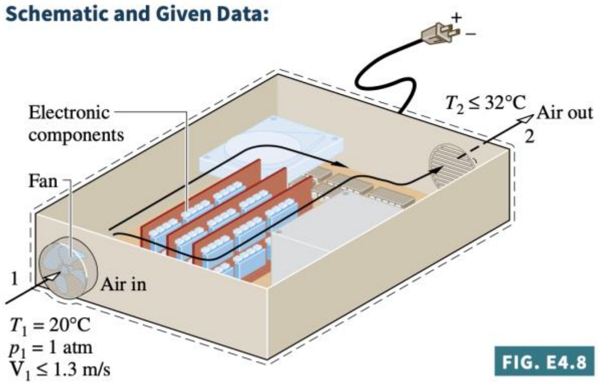


FIG. E4.8

Engineering Model

1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer from the *outer* surface of the electronics enclosure to the surroundings is negligible. Thus, $\dot{Q}_{cv} = 0$.

❶ 3. Changes in kinetic and potential energies can be ignored.

❷ 4. Air is modeled as an ideal gas with $c_p = 1.005$ kJ/kg · K.

Analysis The inlet area A_1 can be determined from the mass flow rate $\dot{m}$ and Eq. 4.4b, which can be rearranged to read

$$A_1 = \frac{\dot{m} v_1}{V_1} \quad \text{(a)}$$

The mass flow rate can be evaluated, in turn, from the steady-state energy rate balance, Eq. 4.20a.

$$0 = \underline{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \underline{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \underline{g(z_1 - z_2)}\right]$$

The underlined terms drop out by assumptions 2 and 3, leaving

$$0 = -\dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

where $\dot{W}_{cv}$ accounts for the *total* electric power provided to the electronic components and the fan: $\dot{W}_{cv} = (-80 \text{ W}) + (-18 \text{ W}) = -98$ W. Solving for $\dot{m}$, and using assumption 4 with Eq. 3.51 to evaluate $(h_1 - h_2)$

$$\dot{m} = \frac{(-\dot{W}_{cv})}{c_p(T_2 - T_1)}$$

Introducing this into the expression for A_1, Eq. (a), and using the ideal gas model to evaluate the specific volume v_1

$$A_1 = \frac{1}{V_1}\left[\frac{(-\dot{W}_{cv})}{c_p(T_2 - T_1)}\right]\left(\frac{RT_1}{p_1}\right)$$

From this expression we see that A_1 *increases* when V_1 and/or T_2 *decrease*. Accordingly, since $V_1 \leq 1.3$ m/s and $T_2 \leq 305$ K (32°C), the inlet area must satisfy

$$A_1 \geq \frac{1}{1.3 \text{ m/s}}\left[\frac{98 \text{ W}}{\left(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(305 - 293)\text{K}}\left|\frac{1 \text{ kJ}}{10^3 \text{ J}}\right|\left|\frac{1 \text{ J/s}}{1 \text{ W}}\right|\right]$$
$$\times \left(\frac{\left(\frac{8314 \text{ N} \cdot \text{m}}{28.97 \text{ kg} \cdot \text{K}}\right)293 \text{ K}}{1.01325 \times 10^5 \text{ N/m}^2}\right)\left|\frac{10^4 \text{ cm}^2}{1 \text{ m}^2}\right|$$
$$\geq 52 \text{ cm}^2$$

For the specified conditions, the smallest fan area is 52 cm^2.

1. Cooling air typically enters and exits electronic enclosures at low velocities, and thus kinetic energy effects are insignificant.
2. The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart. Since the temperature of the air increases by no more than 12°C, the specific heat c_p is nearly constant (Table A-20).

SKILLS DEVELOPED

Ability to…

- apply the steady-state energy rate balance to a control volume.
- apply the mass flow rate expression, Eq. 4.4b.
- develop an engineering model.
- retrieve property data of air modeled as an ideal gas.

Quick Quiz

If heat transfer occurs at a rate of 11 W from the outer surface of the computer case to the surroundings, determine the smallest fan inlet area for which the limits on entering air velocity and exit air temperature are met if the total power input remains at 98 W. Ans. 46 cm^2.

Example 11 Evaluating performance of a Waste heat recovery System

An industrial process discharges 2×10^5 ft^3/min of gaseous combustion products at 400°F, 1 atm. As shown in Fig. E4.10, a proposed system for utilizing the combustion products combines a heat-recovery steam generator with a turbine. At steady state, combustion products exit the steam generator at 260°F, 1 atm and a separate stream of water enters at 40 lbf/in.^{2}, 102°F with a mass flow rate of 275 lb/min. At the exit of the turbine, the pressure is 1 lbf/in.^{2} and the quality is 93%. Heat transfer from the outer surfaces of the steam generator and turbine can be ignored, as can the changes in kinetic and potential energies of the flowing streams. There is no significant pressure drop for the water flowing through the steam generator. The combustion products can be modeled as air as an ideal gas.

a. Determine the power developed by the turbine, in Btu/min.

b. Determine the turbine inlet temperature, in °F.

c. Evaluating the work developed by the turbine at \$0.115 per kW · h, determine the value, in \$/year, for 8000 hours of operation annually.

Solution

Known Steady-state operating data are provided for a system consisting of a heat-recovery steam generator and a turbine.

Find Determine the power developed by the turbine and the turbine inlet temperature. Evaluate the annual value of the power developed.

Schematic and Given Data:

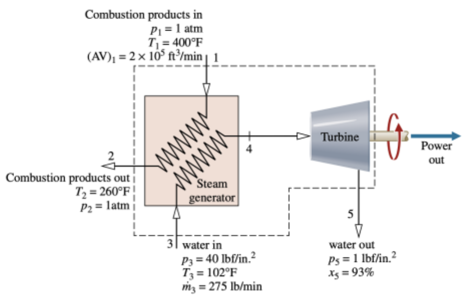


FIG. E4.10

Engineering Model

1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer is negligible, and changes in kinetic and potential energy can be ignored.
3. There is no pressure drop for water flowing through the steam generator.
4. The combustion products are modeled as air as an ideal gas.

Analysis:

a. The power developed by the turbine is determined from a control volume enclosing both the steam generator and the turbine. Since the gas and water streams do not mix, mass rate balances for each of the streams reduce, respectively, to give

$$\dot{m}_1 = \dot{m}_2, \qquad \dot{m}_3 = \dot{m}_5$$

For this control volume, the appropriate form of the steady-state energy rate balance is Eq. 4.18, which reads

$$0 = \underline{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_1\left(h_1 + \underline{\frac{V_1^2}{2} + gz_1}\right) + \dot{m}_3\left(h_3 + \underline{\frac{V_3^2}{2} + gz_3}\right)$$
$$- \dot{m}_2\left(h_2 + \underline{\frac{V_2^2}{2} + gz_2}\right) - \dot{m}_5\left(h_5 + \underline{\frac{V_5^2}{2} + gz_5}\right)$$

The underlined terms drop out by assumption 2. With these simplifications, together with the above mass flow rate relations, the energy rate balance becomes

$$\dot{W}_{cv} = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_5)$$

The mass flow rate $\dot{m}_1$ can be evaluated with given data at inlet 1 and the ideal gas equation of state

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{(AV)_1 p_1}{(\bar{R}/M)T_1} = \frac{(2 \times 10^5 \text{ ft}^3/\text{min})(14.7 \text{ lbf/in.}^2)}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot °\text{R}}\right)(860°\text{R})}\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|$$

$$= 9230.6 \text{ lb/min}$$

The specific enthalpies h_1 and h_2 can be found from Table A-22E: At 860°R, $h_1 = 206.46$ Btu/lb, and at 720°R $h_2 = 172.39$ Btu/lb. At state 3, water is a liquid. Using Eq. 3.14 and saturated liquid data from Table A-2E, $h_3 \approx h_f(T_3) = 70$ But/lb. State 5 is a two-phase liquid–vapor mixture. With data from Table A-3E and the given quality

$$h_5 = h_{f5} + x_5(h_{g5} - h_{f5})$$
$$= 69.74 + 0.93(1036.0) = 1033.2 \text{ Btu/lb}$$

Substituting values into the expression for $\dot{W}_{cv}$

$$\dot{W}_{cv} = \left(9230.6\frac{\text{lb}}{\text{min}}\right)(206.46 - 172.39)\frac{\text{Btu}}{\text{lb}}$$
$$+ \left(275\frac{\text{lb}}{\text{min}}\right)(70 - 1033.2)\frac{\text{Btu}}{\text{lb}}$$
$$= 49610\frac{\text{Btu}}{\text{min}}$$

b. To determine T_4, it is necessary to fix the state at 4. This requires two independent property values. With assumption 3, one of these properties is pressure, $p_4 = 40$ lbf/in.^{2} The other is the specific enthalpy h_4, which can be found from an energy rate balance for a control volume enclosing just the steam generator. Mass rate balances for each of the two streams give $\dot{m}_1 = \dot{m}_2$ and $\dot{m}_3 = \dot{m}_4$. With assumption 2 and these mass flow rate relations, the steady-state form of the energy rate balance reduces to

$$0 = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$$

Solving for h_4

❶ $$h_4 = h_3 + \frac{\dot{m}_1}{\dot{m}_3}(h_1 - h_2)$$
$$= 70\frac{\text{Btu}}{\text{lb}} + \left(\frac{9230.6 \text{ lb/min}}{275 \text{ lb/min}}\right)(206.46 - 172.39)\frac{\text{Btu}}{\text{lb}}$$
$$= 1213.6\frac{\text{Btu}}{\text{lb}}$$

Interpolating in Table A-4E at $p_4 = 40$ lbf/in.^{2} with h_4, we get $T_4 = 354°$F.

Interpolating in Table A-4E at $p_4 = 40$ lbf/in.^{2} with h_4, we get $T_4 = 354°F$.

c. Using the result of part (a), together with the given economic data and appropriate conversion factors, the value of the power developed for 8000 hours of operation annually is

$$\text{annual value} = \left(49610 \frac{\text{Btu}}{\text{min}}\left|\frac{60 \text{ min}}{1 \text{ h}}\right|\left|\frac{1 \text{ kW}}{3413 \text{ Btu/h}}\right|\right) \times \left(8000 \frac{\text{h}}{\text{year}}\right)\left(0.115 \frac{\$}{\text{kW} \cdot \text{h}}\right)$$

❷
$$= 802{,}000 \frac{\$}{\text{year}}$$

---

❶ Alternatively, to determine h_4 a control volume enclosing just the turbine can be considered.

❷ The decision about implementing this solution to the problem of utilizing the hot combustion products discharged from an industrial process would necessarily rest on the outcome of a detailed economic evaluation, including the cost of purchasing and operating the steam generator, turbine, and auxiliary equipment.

SKILLS DEVELOPED

Ability to…

- apply the steady-state mass and energy rate balances to a control volume.
- apply the mass flow rate expression, Eq. 4.4b.
- develop an engineering model.
- retrieve property data for water and for air modeled as an ideal gas.
- conduct an elementary economic evaluation.

Quick Quiz

Taking a control volume enclosing just the turbine, evaluate the turbine inlet temperature, in °F. Ans. 354°F.

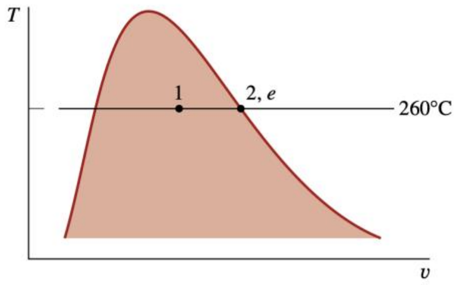
Example 12 Evaluating heat transfer for a partially Emptying tank

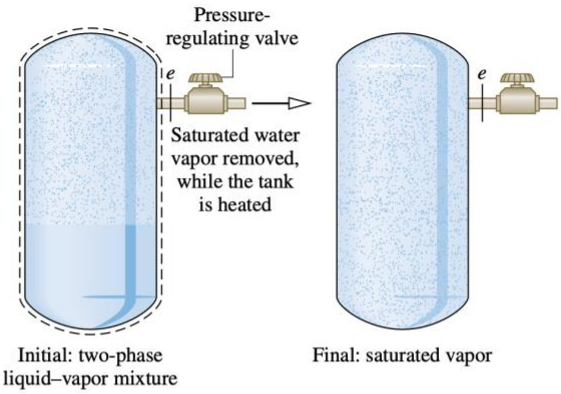
A tank having a volume of 0.85 m^3 initially contains water as a two-phase liquid–vapor mixture at 260°C and a quality of 0.7. Saturated water vapor at 260°C is slowly withdrawn through a pressure-regulating valve at the top of the tank as energy is transferred by heat to maintain constant pressure in the tank. This continues until the tank is filled with saturated vapor at 260°C. Determine the amount of heat transfer, in kJ. Neglect all kinetic and potential energy effects.

Solution

Known A tank initially holding a two-phase liquid–vapor mixture is heated while saturated water vapor is slowly removed. This continues at constant pressure until the tank is filled only with saturated vapor.

Schematic and Given Data:





Engineering Model

1. The control volume is defined by the dashed line on the accompanying diagram.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects can be neglected.
3. At the exit the state remains constant.
4. ❶ The initial and final states of the mass within the vessel are equilibrium states.

Analysis Since there is a single exit and no inlet, the mass rate balance Eq. 4.2 takes the form

$$\frac{dm_{cv}}{dt} = -\dot{m}_e$$

With assumption 2, the energy rate balance Eq. 4.15 reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Combining the mass and energy rate balances results in

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + h_e \frac{dm_{cv}}{dt}$$

By assumption 3, the specific enthalpy at the exit is constant. Accordingly, integration of the last equation gives

$$\Delta U_{cv} = Q_{cv} + h_e \, \Delta m_{cv}$$

Solving for the heat transfer Q_{cv},

$$Q_{cv} = \Delta U_{cv} - h_e \Delta m_{cv}$$

or

❷ $$Q_{cv} = (m_2 u_2 - m_1 u_1) - h_e(m_2 - m_1)$$

where m_1 and m_2 denote, respectively, the initial and final amounts of mass within the tank.

The terms u_1 and m_1 of the foregoing equation can be evaluated with property values from Table A-2 at 260°C and the given value for quality. Thus,

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1})$$
$$= 1128.4 + (0.7)(2599.0 - 1128.4) = 2157.8 \text{ kJ/kg}$$

Also,

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1})$$
$$= 1.2755 \times 10^{-3} + (0.7)(0.04221 - 1.2755 \times 10^{-3})$$
$$= 29.93 \times 10^{-3} \text{ m}^3/\text{kg}$$

Using the specific volume v_1, the mass initially contained in the tank is

$$m_1 = \frac{V}{v_1} = \frac{0.85 \text{ m}^3}{(29.93 \times 10^{-3} \text{ m}^3/\text{kg})} = 28.4 \text{ kg}$$

The final state of the mass in the tank is saturated vapor at 260°C so Table A-2 gives

$$u_2 = u_g(260°\text{C}) = 2599.0 \text{ kJ/kg},$$
$$v_2 = v_g(260°\text{C}) = 42.21 \times 10^{-3} \text{ m}^3/\text{kg}$$

The mass contained within the tank at the end of the process is

$$m_2 = \frac{V}{v_2} = \frac{0.85 \text{ m}^3}{(42.21 \times 10^{-3} \text{ m}^3/\text{kg})} = 20.14 \text{ kg}$$

Table A-2 also gives $h_e = h_g$ (260°C) = 2796.6 kJ/kg.

Substituting values into the expression for the heat transfer yields

$$Q_{cv} = (20.14)(2599.0) - (28.4)(2157.8) - 2796.6(20.14 - 28.4)$$
$$= 14{,}162 \text{ kJ}$$

---

❶ In this case, idealizations are made about the state of the vapor exiting *and* the initial and final states of the mass contained within the tank.

❷ This expression for Q_{cv} can be obtained by applying Eqs. 4.23, 4.25, and 4.27. The details are left as an exercise.

SKILLS DEVELOPED

Ability to…

- apply the time-dependent mass and energy rate balances to a control volume.
- develop an engineering model.
- retrieve property data for water.

Quick Quiz

If the initial quality were 90%, determine the heat transfer, in kJ, keeping all other data unchanged. Ans. 3707 kJ.

Example 13 Using Steam for Emergency power Generation

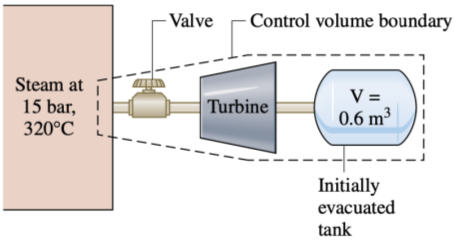
Steam at a pressure of 15 bar and a temperature of 320°C is contained in a large vessel. Connected to the vessel through a valve is a turbine followed by a small initially evacuated tank with a volume of 0.6 m^3. When emergency power is required, the valve is opened and the tank fills with steam until the pressure is 15 bar. The temperature in the tank is then 400°C. The filling process takes place adiabatically and kinetic and potential energy effects are negligible. Determine the amount of work developed by the turbine, in kJ.

Solution

Known Steam contained in a large vessel at a known state flows from the vessel through a turbine into a small tank of known volume until a specified final condition is attained in the tank.

Find Determine the work developed by the turbine.

Schematic and Given Data:



Engineering Model

1. The control volume is defined by the dashed line on the accompanying diagram.
2. For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.
3. (1) The state of the steam within the large vessel remains constant. The final state of the steam in the smaller tank is an equilibrium state.
4. The amount of mass stored within the turbine and the interconnecting piping at the end of the filling process is negligible.

Analysis Since the control volume has a single inlet and no exits, the mass rate balance, Eq. 4.2, reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

The energy rate balance, Eq. 4.15, reduces with assumption 2 to

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + \dot{m}_i h_i$$

Combining the mass and energy rate balances gives

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + h_i \frac{dm_{cv}}{dt}$$

Integrating

$$\Delta U_{cv} = -W_{cv} + h_i \Delta m_{cv}$$

In accordance with assumption 3, the specific enthalpy of the steam entering the control volume is constant at the value corresponding to the state in the large vessel.

Solving for W_{cv}

$$W_{cv} = h_i \Delta m_{cv} - \Delta U_{cv}$$

ΔU_{cv} and Δm_{cv} denote, respectively, the changes in internal energy and mass of the control volume. With assumption 4, these terms can be identified with the small tank only.

Since the tank is initially evacuated, the terms ΔU_{cv} and Δm_{cv} reduce to the internal energy and mass within the tank at the end of the process. That is,

$$\Delta U_{cv} = (m_2 u_2) - \cancelto{0}{(m_1 u_1)}, \quad \Delta m_{cv} = m_2 - \cancelto{0}{m_1}$$

where 1 and 2 denote the initial and final states within the tank, respectively.

Collecting results yields

❷❸
$$W_{cv} = m_2(h_i - u_2) \qquad \text{(a)}$$

The mass within the tank at the end of the process can be evaluated from the known volume and the specific volume of steam at 15 bar and 400°C from Table A-4

$$m_2 = \frac{V}{v_2} = \frac{0.6 \text{ m}^3}{(0.203 \text{ m}^3/\text{kg})} = 2.96 \text{ kg}$$

The specific internal energy of steam at 15 bar and 400°C from Table A-4 is 2951.3 kJ/kg. Also, at 15 bar and 320°C, $h_i = 3081.9$ kJ/kg.

Substituting values into Eq. (a)

$$W_{cv} = 2.96 \text{ kg } (3081.9 - 2951.3)\text{kJ/kg} = 386.6 \text{ kJ}$$

---

❶ In this case idealizations are made about the state of the steam entering the tank and the final state of the steam in the tank. These idealizations make the transient analysis manageable.

❷ A significant aspect of this example is the energy transfer into the control volume by flow work, incorporated in the pv term of the specific enthalpy at the inlet.

❸ This result can also be obtained by reducing Eq. 4.28. The details are left as an exercise.

SKILLS DEVELOPED

Ability to…

- apply the time-dependent mass and energy rate balances to a control volume.
- develop an engineering model.
- retrieve property data for water.

Quick Quiz

If the turbine were removed, and the steam allowed to flow adiabatically into the small tank until the pressure in the tank is 15 bar, determine the final steam temperature in the tank, in °C. Ans. 477°C.

Example 14 Storing Compressed Air in a Tank

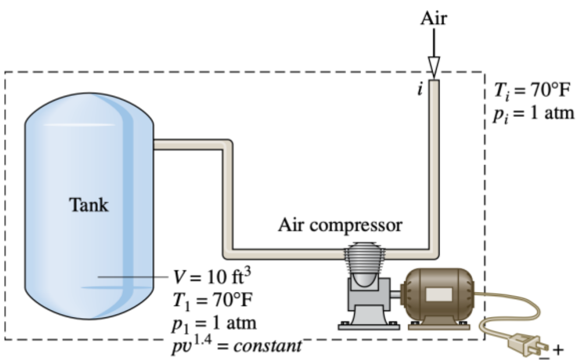
An air compressor rapidly fills a 10-ft^3 tank, initially containing air at 70°F, 1 atm, with air drawn from the atmosphere at 70°F, 1 atm. During filling, the relationship between the pressure and specific volume of the air in the tank is $pv^{1.4} = constant$. The ideal gas model applies for the air, and kinetic and potential energy effects are negligible. Plot the pressure, in atm, and the temperature, in °F, of the air within the tank, each versus the ratio m/m_1, where m_1 is the initial mass in the tank and m is the mass in the tank at time $t > 0$. Also, plot the compressor work input, in Btu, versus m/m_1. Let m/m_1 vary from 1 to 3.

Solution

Known An air compressor rapidly fills a tank having a known volume. The initial state of the air in the tank and the state of the entering air are known.

Find Plot the pressure and temperature of the air within the tank, and plot the air compressor work input, each versus m/m_1 ranging from 1 to 3.

Schematic and Given Data:



Engineering Model

1. The control volume is defined by the dashed line on the accompanying diagram.
2. Because the tank is filled rapidly, $\dot{Q}_{cv}$ is ignored.
3. Kinetic and potential energy effects are negligible.
4. The state of the air entering the control volume remains constant.
5. The air stored within the air compressor and interconnecting pipes can be ignored.
6. (1) The relationship between pressure and specific volume for the air in the tank is $pv^{1.4} = constant$.
7. The ideal gas model applies for the air.

Analysis The required plots are developed using *Interactive Thermodynamics: IT*. The *IT* program is based on the following analysis. The pressure p in the tank at time $t > 0$ is determined from

$$pv^{1.4} = p_1 v_1^{1.4}$$

where the corresponding specific volume v is obtained using the known tank volume V and the mass m in the tank at that time. That is, $v = V/m$. The specific volume of the air in the tank initially, v_1, is calculated from the ideal gas equation of state and the known initial temperature, T_1, and pressure, p_1. That is,

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot {}^\circ\text{R}}\right)(530^\circ\text{R})}{(14.7 \text{ lbf/in.}^2)}\left|\frac{1 \text{ ft}^2}{144 \text{ in.}^2}\right| = 13.35\frac{\text{ft}^3}{\text{lb}}$$

Once the pressure p is known, the corresponding temperature T can be found from the ideal gas equation of state, $T = pv/R$.

To determine the work, begin with the mass rate balance Eq. 4.2, which reduces for the single-inlet control volume to

$$\frac{dm_{\text{cv}}}{dt} = \dot{m}_i$$

Then, with assumptions 2 and 3, the energy rate balance Eq. 4.15 reduces to

$$\frac{dU_{\text{cv}}}{dt} = -\dot{W}_{\text{cv}} + \dot{m}_i h_i$$

Combining the mass and energy rate balances and integrating using assumption 4 give

$$\Delta U_{\text{cv}} = -W_{\text{cv}} + h_i \Delta m_{\text{cv}}$$

Denoting the work *input* to the compressor by $W_{in} = -W_{cv}$ and using assumption 5, this becomes

2
$$W_{in} = mu - m_1u_1 - (m - m_1)h_i \qquad \text{(a)}$$

where m_1 is the initial amount of air in the tank, determined from

$$m_1 = \frac{V}{\upsilon_1} = \frac{10 \text{ ft}^3}{13.35 \text{ ft}^3/\text{lb}} = 0.75 \text{ lb}$$

As a *sample* calculation to validate the *IT* program below, consider the case $m = 1.5$ lb, which corresponds to $m/m_1 = 2$. The specific volume of the air in the tank at that time is

$$\upsilon = \frac{V}{m} = \frac{10 \text{ ft}^3}{1.5 \text{ lb}} = 6.67 \frac{\text{ft}^3}{\text{lb}}$$

The corresponding pressure of the air is

$$p = p_1\left(\frac{\upsilon_1}{\upsilon}\right)^{1.4} = (1 \text{ atm})\left(\frac{13.35 \text{ ft}^3/\text{lb}}{6.67 \text{ ft}^3/\text{lb}}\right)^{1.4}$$
$$= 2.64 \text{ atm}$$

and the corresponding temperature of the air is

$$T = \frac{p\upsilon}{R} = \left(\frac{(2.64 \text{ atm})(6.67 \text{ ft}^3/\text{lb})}{\left(\frac{1545 \text{ ft} \cdot \text{lbf}}{28.97 \text{ lb} \cdot {}^\circ\text{R}}\right)}\right)\left|\frac{14.7 \text{ lbf/in.}^2}{1 \text{ atm}}\right|\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|$$
$$= 699^\circ\text{R } (239^\circ\text{F})$$

Evaluating u_1, u, and h_i at the appropriate temperatures from Table A-22E, $u_1 = 90.3$ Btu/lb, $u = 119.4$ Btu/lb, $h_i = 126.7$ Btu/lb. Using Eq. (a), the required work input is

$$W_{in} = mu - m_1u_1 - (m - m_1)h_i$$
$$= (1.5 \text{ lb})\left(119.4 \frac{\text{Btu}}{\text{lb}}\right) - (0.75 \text{ lb})\left(90.3 \frac{\text{Btu}}{\text{lb}}\right)$$
$$- (0.75 \text{ lb})\left(126.7 \frac{\text{Btu}}{\text{lb}}\right)$$
$$= 16.4 \text{ Btu}$$

IT Program Choosing English units from the **Units** menu, and selecting Air from the **Properties** menu, the *IT* program for solving the problem is

```
//Given Data
p1 = 1//atm
T1 = 70//°F
Ti = 70//°F
V = 10//ft³
n = 1.4
```

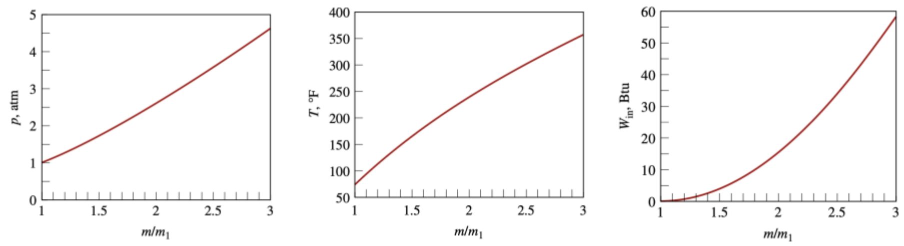
```
// Determine the pressure and temperature
for t > 0

v1 = v_TP("Air", T1, p1)
v = V/m
p * v ^n = p1 * v1 ^n
v = v_TP("Air", T, p)

// Specify the mass and mass ratio r
v1 = V/m1
r = m/m1
r = 2

// Calculate the work using Eq. (a)
Win = m * u - m1 * u1 - hi * (m - m1)
u1 = u_T("Air", T1)
u = u_T("Air", T)
hi = h_T("Air", Ti)
```

Using the **Solve** button, obtain a solution for the sample case $r = m/m_1 = 2$ considered above to validate the program. Good agreement is obtained, as can be verified. Once the program is validated, use the **Explore** button to vary the ratio m/m_1 from 1 to 3 in steps of 0.01. Then, use the **Graph** button to construct the required plots. The results follow:



We conclude from the first two plots that the pressure and temperature each increase as the tank fills. The work required to fill the tank increases as well. These results are as expected.

1. This pressure-specific volume relationship is in accord with what might be measured. The relationship is also consistent with the uniform state idealization, embodied by Eqs. 4.26 and 4.27.
2. This expression can also be obtained by reducing Eq. 4.28. The details are left as an exercise.

SKILLS DEVELOPED

Ability to…

- apply the time-dependent mass and energy rate balances to a control volume.
- develop an engineering model.
- retrieve property data for air modeled as an ideal gas.
- solve iteratively and plot the results using IT.

Quick Quiz

As a *sample* calculation, for the case m = 2.25 lb, evaluate p, in atm. Compare with the value read from the plot of Fig. E4.13b. Ans. 4.67 atm.

Example 15 Determining temperature-time Variation in a Well-Stirred tank

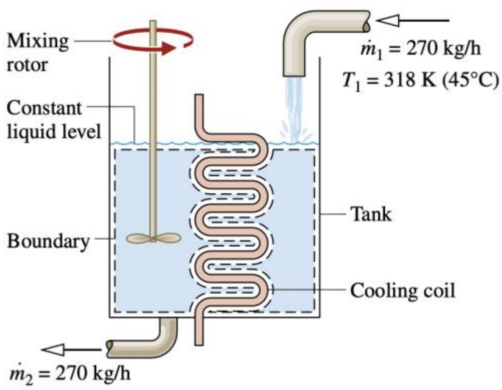
A tank containing 45 kg of liquid water initially at 45°C has one inlet and one exit with equal mass flow rates. Liquid water enters at 45°C and a mass flow rate of 270 kg/h. A cooling coil immersed in the water removes energy at a rate of 7.6 kW. The water is well mixed by a paddle wheel so that the water temperature is uniform throughout. The power input to the water from the paddle wheel is 0.6 kW. The pressures at the inlet and exit are equal and all kinetic and potential energy effects can be ignored. Plot the variation of water temperature with time.

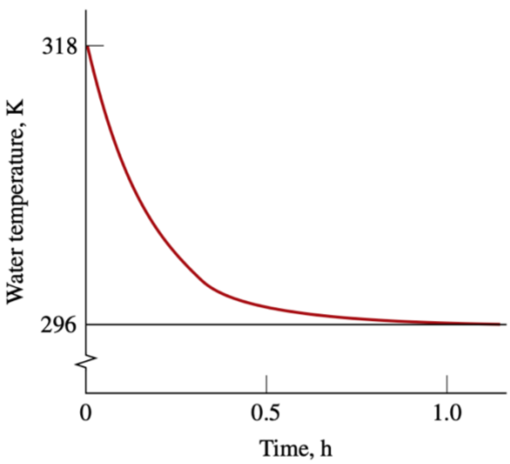
Solution

Known Liquid water flows into and out of a well-stirred tank with equal mass flow rates as the water in the tank is cooled by a cooling coil.

Find Plot the variation of water temperature with time.

Schematic and Given Data:





Engineering Model

1. The control volume is defined by the dashed line on the accompanying diagram.
2. For the control volume, the only significant heat transfer is with the cooling coil. Kinetic and potential energy effects can be neglected.
3. (1) The water temperature is uniform with position throughout and varies only with time: $T = T(t)$.
4. The water in the tank is incompressible, and there is no change in pressure between inlet and exit.

Analysis The energy rate balance, Eq. 4.15, reduces with assumption 2 to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

where $\dot{m}$ denotes the mass flow rate.

The mass contained within the control volume remains constant with time, so the term on the left side of the energy rate balance can be expressed as

$$\frac{dU_{cv}}{dt} = \frac{d(m_{cv}u)}{dt} = m_{cv}\frac{du}{dt}$$

Since the water is assumed incompressible, the specific internal energy depends on temperature only. Hence, the chain rule can be used to write

$$\frac{du}{dt} = \frac{du}{dT}\frac{dT}{dt} = c\frac{dT}{dt}$$

where c is the specific heat. Collecting results

$$\frac{dU_{cv}}{dt} = m_{cv}c\frac{dT}{dt}$$

With Eq. 3.20b the enthalpy term of the energy rate balance can be expressed as

$$h_1 - h_2 = c(T_1 - T_2) + v(\cancelto{0}{p_1 - p_2})$$

where the pressure term is dropped by assumption 4. Since the water is well mixed, the temperature at the exit equals the temperature of the overall quantity of liquid in the tank, so

$$h_1 - h_2 = c(T_1 - T)$$

where T represents the uniform water temperature at time t.

With the foregoing considerations the energy rate balance becomes

$$m_{cv}c\frac{dT}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}c(T_1 - T)$$

As can be verified by direct substitution, the solution of this first-order, ordinary differential equation is

$$T = C_1 \exp\left(-\frac{\dot{m}}{m_{cv}}t\right) + \left(\frac{\dot{Q}_{cv} - \dot{W}_{cv}}{\dot{m}c}\right) + T_1$$

The constant C_1 is evaluated using the initial condition: at $t = 0$, $T = T_1$. Finally,

$$T = T_1 + \left(\frac{\dot{Q}_{cv} - \dot{W}_{cv}}{\dot{m}c}\right)\left[1 - \exp\left(-\frac{\dot{m}}{m_{cv}}t\right)\right]$$

Substituting given numerical values together with the specific heat c for liquid water from Table A-19

$$T = 318\text{ K} + \left[\frac{[-7.6 - (-0.6)]\text{kJ/s}}{\left(\frac{270}{3600}\frac{\text{kg}}{\text{s}}\right)\left(4.2\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\right]\left[1 - \exp\left(-\frac{270\text{ kg/h}}{45\text{ kg}}t\right)\right]$$

$$= 318 - 22[1 - \exp(-6t)]$$

where t is in hours. Using this expression, we construct the accompanying plot showing the variation of temperature with time.

1. In this case idealizations are made about the state of the mass contained within the system and the states of the liquid entering and exiting. These idealizations make the transient analysis manageable.

SKILLS DEVELOPED

Ability to…

- apply the time-dependent mass and energy rate balances to a control volume.
- develop an engineering model.
- apply the incompressible substance model for water.
- solve an ordinary differential equation and plot the solution.

Quick Quiz

What is the water temperature, in °C, when *steady state* is achieved? Ans. 23°C.