Example 1 Evaluating Work and heat transfer for an Internally Reversible process of Water

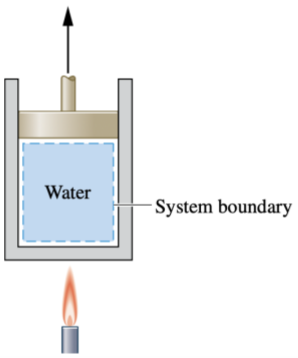
Water, initially a saturated liquid at 150°C (423.15 K), is contained in a piston–cylinder assembly. The water undergoes a process to the corresponding saturated vapor state, during which the piston moves freely in the cylinder. If the change of state is brought about by heating the water as it undergoes an internally reversible process at constant pressure and temperature, determine the work and heat transfer per unit of mass, each in kJ/kg.

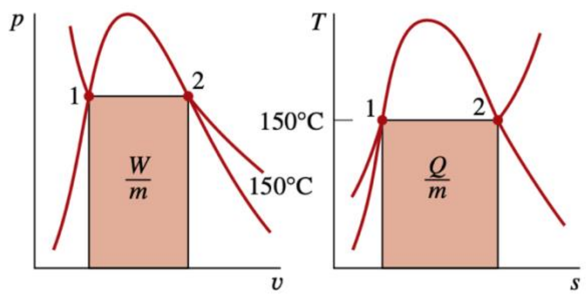
Solution

Known Water contained in a piston–cylinder assembly undergoes an internally reversible process at 150°C from saturated liquid to saturated vapor.

Find Determine the work and heat transfer per unit mass.

Schematic and Given Data:





Engineering Model

1. The water in the piston–cylinder assembly is a closed system.
2. The process is internally reversible.
3. Temperature and pressure are constant during the process.
4. There is no change in kinetic or potential energy between the two end states.

Analysis At constant pressure the work is

$$\frac{W}{m} = \int_1^2 p\,d\upsilon = p(\upsilon_2 - \upsilon_1)$$

With values from Table A-2 at 150°C

$$\frac{W}{m} = (4.758 \text{ bar})(0.3928 - 1.0905 \times 10^{-3})\left(\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|$$

$$= 186.38 \text{ kJ/kg}$$

Since the process is internally reversible and at constant temperature, Eq. 6.23 gives

$$Q = \int_1^2 T\,dS = m\int_1^2 T\,dS$$

or

$$\frac{Q}{m} = T(s_2 - s_1)$$

1 $$\frac{Q}{m} = (423.15 \text{ K})(6.8379 - 1.8418) \text{ kJ/kg}\cdot\text{K} = 2114.1 \text{ kJ/kg}$$

As shown in the accompanying figure, the work and heat transfer can be represented as areas on p–υ and T–s diagrams, respectively.

1 The heat transfer can be evaluated alternatively from an energy balance written on a unit mass basis as

$$u_2 - u_1 = \frac{Q}{m} - \frac{W}{m}$$

Introducing $W/m = p(\upsilon_2 - \upsilon_1)$ and solving

$$\frac{Q}{m} = (u_2 - u_1) + p(\upsilon_2 - \upsilon_1)$$
$$= (u_2 + p\upsilon_2) - (u_1 + p\upsilon_1)$$
$$= h_2 - h_1$$

From Table A-2 at 150°C, $h_2 - h_1 = 2114.3$ kJ/kg, which agrees with the value for Q/m obtained in the solution.

SKILLS DEVELOPED

Ability to…

- evaluate work and heat transfer for an internally reversible process and represent them as areas on p–υ and T–s diagrams, respectively.
- retrieve entropy data for water.

Quick Quiz

If the initial and final states were saturation states at 100°C (373.15 K), determine the work and heat transfer per unit of mass, each in kJ/kg. Ans. 170 kJ/kg, 2257 kJ/kg.

Example 2 Determining Work and Entropy production for an Irreversible process of Water

Water, initially a saturated liquid at 150°C, is contained within a piston–cylinder assembly. The water undergoes a process to the corresponding saturated vapor state, during which the piston moves freely in the cylinder. There is no heat transfer with the surroundings. If the change of state is brought about by the action of a paddle wheel, determine the net work per unit mass, in kJ/kg, and the amount of entropy produced per unit mass, in kJ/kg · K.

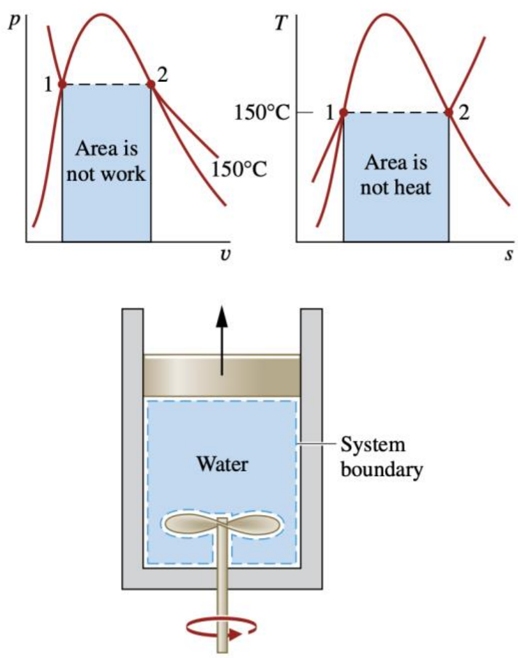
Solution

Known Water contained in a piston–cylinder assembly undergoes an adiabatic process from saturated liquid to saturated vapor at 150°C. During the process, the piston moves freely, and the water is rapidly stirred by a paddle wheel.

Find Determine the net work per unit mass and the entropy produced per unit mass.

Schematic and Given Data:

1



Engineering Model

1. The water in the piston–cylinder assembly is a closed system.
2. There is no heat transfer with the surroundings.
3. The system is at an equilibrium state initially and finally. There is no change in kinetic or potential energy between these two states.

Analysis As the volume of the system increases during the process, there is an energy transfer by work from the system during the expansion, as well as an energy transfer by work to the system via the paddle wheel. The *net* work can be evaluated from an energy balance, which reduces with assumptions 2 and 3 to

$$\Delta U + \cancel{\Delta \mathrm{KE}}^{0} + \cancel{\Delta \mathrm{PE}}^{0} = \cancel{Q}^{0} - W$$

On a unit mass basis, the energy balance is then

$$\frac{W}{m} = -(u_2 - u_1)$$

With specific internal energy values from Table A-2 at 150°C, $u_1 = 631.68$ kJ/kg, $u_2 = 2559.5$ kJ/kg, we get

$$\frac{W}{m} = -1927.82 \frac{\mathrm{kJ}}{\mathrm{kg}}$$

The minus sign indicates that the work input by stirring is greater in magnitude than the work done by the water as it expands.

The amount of entropy produced is evaluated by applying the entropy balance Eq. 6.24. Since there is no heat transfer, the term accounting for entropy transfer vanishes:

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma$$

On a unit mass basis, this becomes on rearrangement

$$\frac{\sigma}{m} = s_2 - s_1$$

With specific entropy values from Table A-2 at 150°C, $s_1 = 1.8418$ kJ/kg · K, $s_2 = 6.8379$ kJ/kg · K, we get

2 $$\frac{\sigma}{m} = 4.9961 \frac{\mathrm{kJ}}{\mathrm{kg} \cdot \mathrm{K}}$$

1. Although each end state is an equilibrium state at the same pressure and temperature, the pressure and temperature are not necessarily uniform throughout the system at *intervening* states, nor are they necessarily constant in value during the process. Accordingly, there is no well-defined "path" for the process. This is emphasized by the use of dashed lines to represent the process on these p–v and T–s diagrams. The dashed lines indicate only that a process has taken place, and no "area" should be associated with them. In particular, note that the process is adiabatic, so the "area" below the dashed line on the T–s diagram can have no significance as heat transfer. Similarly, the work cannot be associated with an area on the p–v diagram.

2. The change of state is the same in the present example as in Example 6.1. However, in Example 6.1 the change of state is brought about by heat transfer while the system undergoes an internally reversible process. Accordingly, the value of entropy production for the process of Example 6.1 is zero. Here, fluid friction is present during the process and the entropy production is positive in value. Accordingly, different values of entropy production are obtained for two processes between the *same* end states. This demonstrates that entropy production is not a property.

SKILLS DEVELOPED

Ability to…

- apply the closed system energy and entropy balances.
- retrieve property data for water.

Quick Quiz

If the initial and final states were saturation states at 100°C, determine the net work, in kJ/kg, and the amount of entropy produced, in kJ/kg · K. Ans. −2087.56 kJ/kg, 6.048 kJ/kg · K.

Example 3 Evaluating Minimum theoretical Compression Work

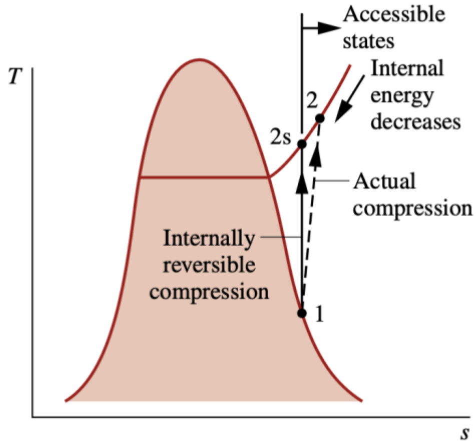
Refrigerant 134a is compressed adiabatically in a piston–cylinder assembly from saturated vapor at 10°F to a final pressure of 120 lbf/in.^{2} Determine the minimum theoretical work input required per unit mass of refrigerant, in Btu/lb.

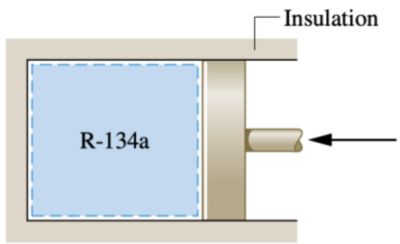
Solution

Known Refrigerant 134a is compressed without heat transfer from a specified initial state to a specified final pressure.

Find Determine the minimum theoretical work input required per unit of mass.

Schematic and Given Data:





Engineering Model

1. The Refrigerant 134a is a closed system.
2. There is no heat transfer with the surroundings.
3. The initial and final states are equilibrium states. There is no change in kinetic or potential energy between these states.

Analysis An expression for the work can be obtained from an energy balance. By applying assumptions 2 and 3, we get

$$\Delta U + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} = \cancel{Q}^{0} - W$$

When written on a unit mass basis, the work *input* is then

$$\left(-\frac{W}{m}\right) = u_2 - u_1$$

The specific internal energy u_1 can be obtained from Table A-10E as $u_1 = 94.68$ Btu/lb. Since u_1 is known, the value for the work input depends on the specific internal energy u_2. The minimum work input corresponds to the smallest allowed value for u_2, determined using the second law as follows.

Applying the entropy balance, Eq. 6.24, we get

$$\Delta S = \int_1^2 \cancelto{0}{\left(\frac{\delta Q}{T}\right)_b} + \sigma$$

where the entropy transfer term is set equal to zero because the process is adiabatic. Thus, the *allowed* final states must satisfy

$$s_2 - s_1 = \frac{\sigma}{m} \geq 0$$

The restriction indicated by the foregoing equation can be interpreted using the accompanying *T–s* diagram. Since σ cannot be negative, states with $s_2 < s_1$ are not accessible adiabatically. When irreversibilities are present during the compression, entropy is produced, so $s_2 > s_1$. The state labeled 2s on the diagram would be attained in the limit as irreversibilities are reduced to zero. This state corresponds to an *isentropic* compression.

By inspection of Table A-12E, we see that when pressure is fixed, the specific internal energy decreases as specific entropy decreases. Thus, the smallest allowed value for u_2 corresponds to state 2s. Interpolating in Table A-12E at 120 lb/in.^{2}, with $s_{2s} = s_1 = 0.2214$ Btu/lb · °R, we find that $u_{2s} = 107.46$ Btu/lb, which corresponds to a temperature at state 2s of about 101°F. Finally, the *minimum* work input is

❶ $$\left(-\frac{W}{m}\right)_{min} = u_{2s} - u_1 = 107.46 - 94.68 = 12.78 \text{ Btu/lb}$$

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❶ The effect of irreversibilities exacts a penalty on the work input required: A greater work input is needed for the actual adiabatic compression process than for an internally reversible adiabatic process between the same initial state and the same final pressure. See the Quick Quiz to follow.

SKILLS DEVELOPED

Ability to…

- apply the closed system energy and entropy balances.
- retrieve property data for Refrigerant 134a.

Quick Quiz

If the refrigerant were compressed adiabatically to a final state where $p_2 = 120$ lbf/in.^{2}, $T_2 = 120$°F, determine the work input, in Btu/lb, and the amount of entropy produced, in Btu/lb · °R. Ans. 17.16 Btu/lb, 0.0087 Btu/lb · °R.

Example 4 pinpointing Irreversibilities

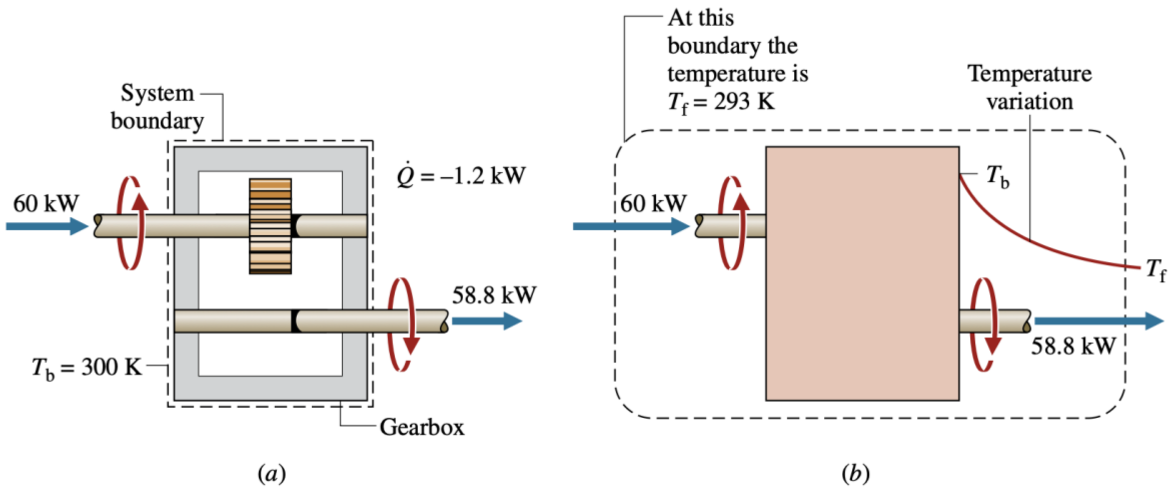
Referring to Example 2.4, evaluate the rate of entropy production $\dot{\sigma}$, in kW/K, for **(a)** the gearbox as the system and **(b)** an enlarged system consisting of the gearbox and enough of its surroundings that heat transfer occurs at the temperature of the surroundings away from the immediate vicinity of the gearbox, $T_f = 293$ K (20°C).

Solution

Known A gearbox operates at steady state with known values for the power input through the high-speed shaft, power output through the low-speed shaft, and heat transfer rate. The temperature on the outer surface of the gearbox and the temperature of the surroundings away from the gearbox are also known.

Find Evaluate the entropy production rate $\dot{\sigma}$ for each of the two specified systems shown in the schematic.

Schematic and Given Data:



Engineering Model

1. In part (a), the gearbox is taken as a closed system operating at steady state, as shown on the accompanying sketch labeled with data from Example 2.4.
2. In part (b) the gearbox and a portion of its surroundings are taken as a closed system, as shown on the accompanying sketch labeled with data from Example 2.4.
3. The temperature of the outer surface of the gearbox and the temperature of the surroundings do not vary.

Analysis

a. To obtain an expression for the entropy production rate, begin with the entropy balance for a closed system on a time rate basis: Eq. 6.28. Since heat transfer takes place only at temperature T_b, the entropy rate balance reduces at steady state to

$$\frac{d\cancel{S}^0}{\cancel{dt}} = \frac{\dot{Q}}{T_b} + \dot{\sigma}$$

Solving

$$\dot{\sigma} = -\frac{\dot{Q}}{T_b}$$

Introducing the known values for the heat transfer rate $\dot{Q}$ and the surface temperature T_b

$$\dot{\sigma} = -\frac{(-1.2 \text{ kW})}{(300 \text{ K})} = 4 \times 10^{-3} \text{ kW/K}$$

b. Since heat transfer takes place at temperature T_f for the enlarged system, the entropy rate balance reduces at steady state to

$$\frac{d\cancel{S}^0}{\cancel{dt}} = \frac{\dot{Q}}{T_f} + \dot{\sigma}$$

Solving

$$\dot{\sigma} = -\frac{\dot{Q}}{T_f}$$

Introducing the known values for the heat transfer rate $\dot{Q}$ and the temperature T_f

❶ $$\dot{\sigma} = -\frac{(-1.2 \text{ kW})}{(293 \text{ K})} = 4.1 \times 10^{-3} \text{ kW/K}$$

❶ The value of the entropy production rate calculated in part (a) gauges the significance of irreversibilities associated with friction and heat transfer *within* the gearbox. In part (b), an additional source of irreversibility is included in the enlarged system, namely, the irreversibility associated with the heat transfer from the outer surface of the gearbox at T_b to the surroundings at T_f. In this case, the irreversibilities within the gearbox are dominant, accounting for about 98% of the total rate of entropy production.

SKILLS DEVELOPED

Ability to…

- apply the closed system entropy rate balance.
- develop an engineering model.

Quick Quiz

If the power delivered were 59.32 kW, evaluate the outer surface temperature, in K, and the rate of entropy production, in kW/K, for the gearbox as the system, keeping input power, h, and A from Example 2.4 the same. Ans. 297 K, 2.3×10^{-3} kW/K.

Example 5 Quenching a hot Metal Bar

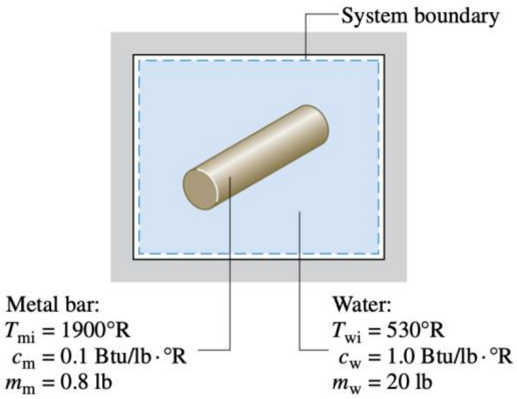
A 0.8-lb metal bar initially at 1900°R is removed from an oven and quenched by immersing it in a closed tank containing 20 lb of water initially at 530°R. Each substance can be modeled as incompressible. An appropriate constant specific heat value for the water is $c_w = 1.0$ Btu/lb · °R, and an appropriate value for the metal is $c_m = 0.1$ Btu/lb · °R. Heat transfer from the tank contents can be neglected. Determine **(a)** the final equilibrium temperature of the metal bar and the water, in °R, and **(b)** the amount of entropy produced, in Btu/°R.

Solution

Known A hot metal bar is quenched by immersing it in a closed tank containing water.

Find Determine the final equilibrium temperature of the metal bar and the water and the amount of entropy produced.

Schematic and Given Data:



Engineering Model

1. The metal bar and the water within the tank form a system, as shown on the accompanying sketch.
2. There is no energy transfer by heat or work: The system is isolated.
3. There is no change in kinetic or potential energy.
4. The water and metal bar are each modeled as incompressible with known specific heats.

Analysis

a. The final equilibrium temperature can be evaluated from an energy balance for the isolated system

$$\Delta \cancel{KE}^{0} + \Delta \cancel{PE}^{0} + \Delta U = \cancel{Q}^{0} - \cancel{W}^{0}$$

where the indicated terms vanish by assumptions 2 and 3. Since internal energy is an extensive property, its value for the isolated system is the sum of the values for the water and metal, respectively. Thus, the energy balance becomes

$$\Delta U]_{\text{water}} + \Delta U]_{\text{metal}} = 0$$

Using Eq. 3.20a to evaluate the internal energy changes of the water and metal in terms of the constant specific heats,

$$m_w c_w (T_f - T_{wi}) + m_m c_m (T_f - T_{mi}) = 0$$

where T_f is the final equilibrium temperature, and T_{wi} and T_{mi} are the initial temperatures of the water and metal, respectively. Solving for T_f and inserting values

$$\begin{aligned} T_f &= \frac{m_w (c_w/c_m) T_{wi} + m_m T_{mi}}{m_w (c_w/c_m) + m_m} \\ &= \frac{(20\text{ lb})(10)(530°\text{R}) + (0.8\text{ lb})(1900°\text{R})}{(20\text{ lb})(10) + (0.8\text{ lb})} \\ &= 535°\text{R} \end{aligned}$$

b. The amount of entropy production can be evaluated from an entropy balance. Since no heat transfer occurs between the isolated system and its surroundings, there is no accompanying entropy transfer, and an entropy balance for the isolated system reduces to

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_b^{\nearrow 0} + \sigma$$

Entropy is an extensive property, so its value for the isolated system is the sum of its values for the water and the metal, respectively, and the entropy balance becomes

$$\Delta S]_{\text{water}} + \Delta S]_{\text{metal}} = \sigma$$

Evaluating the entropy changes using Eq. 6.13 for incompressible substances, the foregoing equation can be written as

$$\sigma = m_w c_w \ln \frac{T_f}{T_{wi}} + m_m c_m \ln \frac{T_f}{T_{mi}}$$

Inserting values

$$\sigma = (20\text{ lb})\left(1.0 \frac{\text{Btu}}{\text{lb} \cdot °\text{R}}\right) \ln \frac{535}{530} + (0.8\text{ lb})\left(0.1 \frac{\text{Btu}}{\text{lb} \cdot °\text{R}}\right) \ln \frac{535}{1900}$$

❶ ❷ $$= \left(0.1878 \frac{\text{Btu}}{°\text{R}}\right) + \left(-0.1014 \frac{\text{Btu}}{°\text{R}}\right) = 0.0864 \frac{\text{Btu}}{°\text{R}}$$

❶ The metal bar experiences a *decrease* in entropy. The entropy of the water *increases*. In accord with the increase of entropy principle, the entropy of the isolated system *increases*.

❷ The value of σ is sensitive to roundoff in the value of T_f.

SKILLS DEVELOPED

Ability to…

- apply the closed system energy and entropy balances.
- apply the incompressible substance model.

Quick Quiz

If the mass of the metal bar were 0.45 lb, determine the final equilibrium temperature, in °R, and the amount of entropy produced, in Btu/°R, keeping all other given data the same. Ans. 533°R, 0.0557 Btu/°R.

Example 5 Determining Entropy production in a Steam turbine

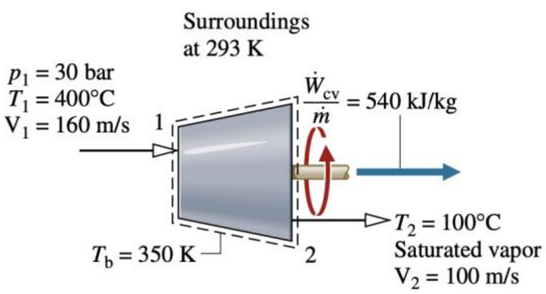
Steam enters a turbine with a pressure of 30 bar, a temperature of 400°C, and a velocity of 160 m/s. Saturated vapor at 100°C exits with a velocity of 100 m/s. At steady state, the turbine develops work equal to 540 kJ per kg of steam flowing through the turbine. Heat transfer between the turbine and its surroundings occurs at an average outer surface temperature of 350 K. Determine the rate at which entropy is produced within the turbine per kg of steam flowing, in kJ/kg · K. Neglect the change in potential energy between inlet and exit.

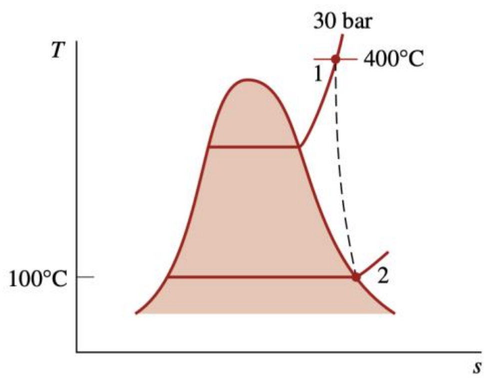
Solution

Known Steam expands through a turbine at steady state for which data are provided.

Find Determine the rate of entropy production per kg of steam flowing.

Schematic and Given Data:





Engineering Model

1. The control volume shown on the accompanying sketch is at steady state.
2. Heat transfer from the turbine to the surroundings occurs at a specified average outer surface temperature.
3. The change in potential energy between inlet and exit can be neglected.

Analysis To determine the entropy production per unit mass flowing through the turbine, begin with mass and entropy rate balances for the one-inlet, one-exit control volume at steady state:

$$\dot{m}_1 = \dot{m}_2$$

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}_1 s_1 - \dot{m}_2 s_2 + \dot{\sigma}_{cv}$$

Since heat transfer occurs only at $T_b = 350$ K, the first term on the right side of the entropy rate balance reduces to $\dot{Q}_{cv}/T_b$. Combining the mass and entropy rate balances

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

where $\dot{m}$ is the mass flow rate. Solving for $\dot{\sigma}_{cv}/\dot{m}$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{\dot{Q}_{cv}/\dot{m}}{T_b} + (s_2 - s_1)$$

The heat transfer rate, $\dot{Q}_{cv}/\dot{m}$, required by this expression is evaluated next.

Reduction of the mass and energy rate balances results in

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)$$

where the potential energy change from inlet to exit is dropped by assumption 3. From Table A-4 at 30 bar, 400°C, $h_1 = 3230.9$ kJ/kg, and from Table A-2, $h_2 = h_g(100°C) = 2676.1$ kJ/kg. Thus,

$$\frac{\dot{Q}_{cv}}{\dot{m}} = 540\frac{\text{kJ}}{\text{kg}} + (2676.1 - 3230.9)\left(\frac{\text{kJ}}{\text{kg}}\right) + \left[\frac{(100)^2 - (160)^2}{2}\right]\left(\frac{\text{m}^2}{\text{s}^2}\right)\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= 540 - 554.8 - 7.8 = -22.6\text{ kJ/kg}$$

From Table A-2, $s_2 = 7.3549$ kJ/kg · K, and from Table A-4, $s_1 = 6.9212$ kJ/kg · K. Inserting values into the expression for entropy production

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{(-22.6\text{ kJ/kg})}{350\text{ K}} + (7.3549 - 6.9212)\left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)$$

$$= 0.0646 + 0.4337 = 0.498\text{ kJ/kg}\cdot\text{K}$$

SKILLS DEVELOPED

Ability to…

- apply the control volume, mass, energy, and entropy rate balances.
- retrieve property data for water.

Quick Quiz

If the boundary were located to include the turbine and a portion of the immediate surroundings so heat transfer occurs at the temperature of the surroundings, 293 K, determine the rate at which entropy is produced within the enlarged control volume, in kJ/K per kg of steam flowing, keeping all other given data the same. Ans. 0.511 kJ/kg · K.

Example 6 Evaluating a performance Claim

An inventor claims to have developed a device requiring no energy transfer by work, $\dot{W}_{cv}$, or heat transfer, yet able to produce hot and cold streams of air from a single stream of air at an intermediate temperature. The inventor provides steady-state test data indicating that when air enters at a temperature of 70°F and a pressure of 5.1 atm, separate streams of air exit at temperatures of 0 and 175°F, respectively, and each at a pressure of 1 atm. Sixty percent of the mass entering the device exits at the lower temperature. Evaluate the inventor's claim, employing the ideal gas model for air and ignoring changes in the kinetic and potential energies of the streams from inlet to exit.

Solution

Known Data are provided for a device that at steady state produces hot and cold streams of air from a single stream of air at an intermediate temperature without energy transfers by work or heat.

Find Evaluate whether the device can operate as claimed.

Schematic and Given Data:

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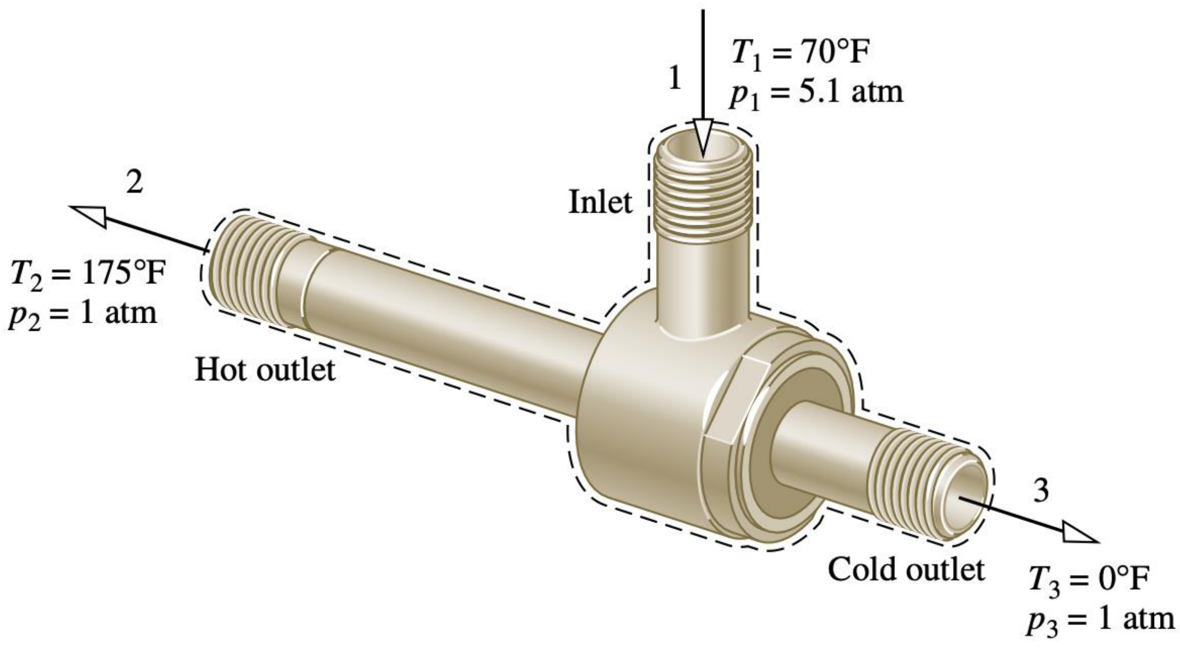


Engineering Model

1. The control volume shown on the accompanying sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and $\dot{Q}_{cv} = 0$.
3. Changes in the kinetic and potential energies from inlet to exit can be ignored.
4. (1) The air is modeled as an ideal gas with constant $c_p = 0.24$ Btu/lb · °R.

Analysis For the device to operate as claimed, the conservation of mass and energy principles must be satisfied. The second law of thermodynamics also must be satisfied, and in particular the rate of entropy production cannot be negative. Accordingly, the mass, energy, and entropy rate balances are considered in turn.

With assumptions 1–3, the mass and energy rate balances reduce, respectively, to

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

$$0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Since $\dot{m}_3 = 0.6\dot{m}_1$, it follows from the mass rate balance that $\dot{m}_3 = 0.4\dot{m}_1$. By combining the mass and energy rate balances and evaluating changes in specific enthalpy using constant c_p, the energy rate balance is also satisfied. That is,

$$\begin{aligned} 0 &= (\dot{m}_2 + \dot{m}_3)h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \\ &= \dot{m}_2(h_1 - h_2) + \dot{m}_3(h_1 - h_3) \\ &= 0.4\dot{m}_1[c_p(T_1 - T_2)] + 0.6\dot{m}_1[c_p(T_1 - T_3)] \\ &= 0.4(T_1 - T_2) + 0.6(T_1 - T_3) \quad \text{②} \\ &= 0.4(-105) + 0.6(70) \\ &= 0 \end{aligned}$$

Accordingly, with the given data the conservation of mass and energy principles are satisfied.

Since no significant heat transfer occurs, the entropy rate balance at steady state reads

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{0} + \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

Combining the mass and entropy rate balances

$$\begin{aligned} 0 &= (\dot{m}_2 + \dot{m}_3)s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv} \\ &= \dot{m}_2(s_1 - s_2) + \dot{m}_3(s_1 - s_3) + \dot{\sigma}_{cv} \\ &= 0.4\dot{m}_1(s_1 - s_2) + 0.6\dot{m}_1(s_1 - s_3) + \dot{\sigma}_{cv} \end{aligned}$$

Solving for $\dot{\sigma}_{cv}/\dot{m}_1$ and using Eq. 6.22 to evaluate changes in specific entropy

③
$$\frac{\dot{\sigma}_{cv}}{\dot{m}_1} = 0.4\left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right] + 0.6\left[c_p \ln\frac{T_3}{T_1} - R\ln\frac{p_3}{p_1}\right]$$

$$= 0.4\left[\left(0.24\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{635}{530} - \left(\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{1}{5.1}\right]$$

④
$$+\,0.6\left[\left(0.24\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{460}{530} - \left(\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{1}{5.1}\right]$$

$$= 0.1086\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

Thus, the second law of thermodynamics is also satisfied.

⑤ On the basis of this evaluation, the inventor's claim does not violate principles of thermodynamics.

Thus, the second law of thermodynamics is also satisfied.

⑤ On the basis of this evaluation, the inventor's claim does not violate principles of thermodynamics.

1. Since the specific heat c_p of air varies little over the temperature interval from 0 to 175°F, c_p can be taken as constant. From Table A-20E, $c_p = 0.24$ Btu/lb · °R.
2. Since temperature *differences* are involved in this calculation, the temperatures can be either in °R or °F.
3. In this calculation involving temperature *ratios*, the temperatures are in °R. Temperatures in °F should not be used.
4. If the value of the rate of entropy production had been negative or zero, the claim would be rejected. A negative value is impossible by the second law and a zero value would indicate operation without irreversibilities.
5. Such devices *do* exist. They are known as *vortex tubes* and are used in industry for *spot cooling*.

SKILLS DEVELOPED

Ability to…

- apply the control volume, mass, energy, and entropy rate balances.
- apply the ideal gas model with constant c_p.

Quick Quiz

If the inventor would claim that the hot and cold streams exit the device at 5.1 atm, evaluate the revised claim, keeping all other given data the same. Ans. Claim Invalid.

Example 7 Determining Entropy production in heat pump Components

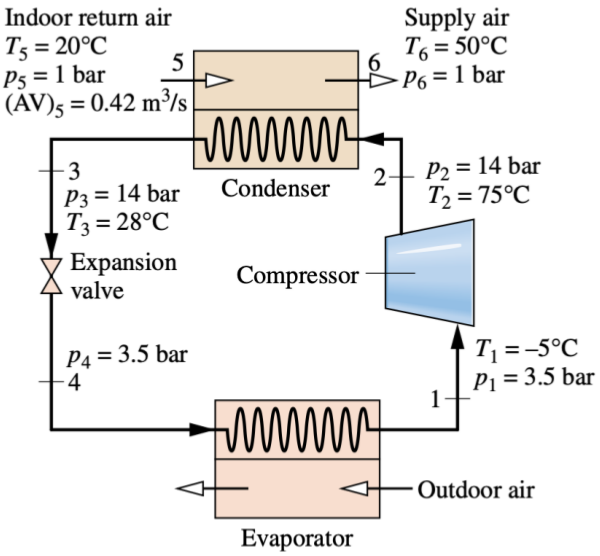
Components of a heat pump for supplying heated air to a dwelling are shown in the schematic below. At steady state, Refrigerant 22 enters the compressor at −5°C, 3.5 bar and is compressed adiabatically to 75°C, 14 bar. From the compressor, the refrigerant passes through the condenser, where it condenses to liquid at 28°C, 14 bar. The refrigerant then expands through a throttling valve to 3.5 bar. The states of the refrigerant are shown on the accompanying *T–s* diagram. Return air from the dwelling enters the condenser at 20°C, 1 bar with a volumetric flow rate of 0.42 m^3/s and exits at 50°C with a negligible change in pressure. Using the ideal gas model for the air and neglecting kinetic and potential energy effects, **(a)** determine the rates of entropy production, in kW/K, for control volumes enclosing the condenser, compressor, and expansion valve, respectively. **(b)** Discuss the sources of irreversibility in the components considered in part (a).

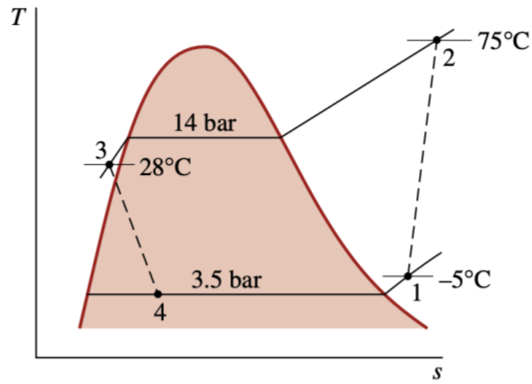
Solution

Known Refrigerant 22 is compressed adiabatically, condensed by heat transfer to air passing through a heat exchanger, and then expanded through a throttling valve. Steady-state operating data are known.

Find Determine the entropy production rates for control volumes enclosing the condenser, compressor, and expansion valve, respectively, and discuss the sources of irreversibility in these components.

Schematic and Given Data:





Engineering Model

1. Each component is analyzed as a control volume at steady state.
2. The compressor operates adiabatically, and the expansion across the valve is a *throttling process.*
3. For the control volume enclosing the condenser, $\dot{W}_{cv} = 0$ and $\dot{Q}_{cv} = 0$.
4. Kinetic and potential energy effects can be neglected.
5. (1) The air is modeled as an ideal gas with constant $c_p = 1.005$ kJ/kg · K.

Analysis

a. Let us begin by obtaining property data at each of the principal refrigerant states located on the accompanying schematic and T–s diagram. At the inlet to the compressor, the refrigerant is a superheated vapor at –5°C, 3.5 bar, so from Table A-9, $s_1 = 0.9572$ kJ/kg · K. Similarly, at state 2, the refrigerant is a superheated vapor at 75°C, 14 bar, so interpolating in Table A-9 gives $s_2 = 0.98225$ kJ/kg · K and $h_2 = 294.17$ kJ/kg.

State 3 is compressed liquid at 28°C, 14 bar. From Table A-7, $s_3 \approx s_f(28°C) = 0.2936$ kJ/kg · K and $h_3 \approx h_f(28°C) = 79.05$ kJ/kg. The expansion through the valve is a *throttling process*, so $h_3 = h_4$. Using data from Table A-8, the quality at state 4 is

$$x_4 = \frac{(h_4 - h_{f4})}{(h_{fg})_4} = \frac{(79.05 - 33.09)}{(212.91)} = 0.216$$

and the specific entropy is

$$s_4 = s_{f4} + x_4(s_{g4} - s_{f4}) = 0.1328 + 0.216(0.9431 - 0.1328) = 0.3078 \text{ kJ/kg} \cdot \text{K}$$

Condenser

Consider the control volume enclosing the condenser. With assumptions 1 and 3, the entropy rate balance reduces to

$$0 = \dot{m}_{\text{ref}}(s_2 - s_3) + \dot{m}_{\text{air}}(s_5 - s_6) + \dot{\sigma}_{\text{cond}}$$

To evaluate $\dot{\sigma}_{\text{cond}}$ requires the two mass flow rates, $\dot{m}_{\text{air}}$ and $\dot{m}_{\text{ref}}$, and the change in specific entropy for the air. These are obtained next.

Evaluating the mass flow rate of air using the ideal gas model and the known volumetric flow rate

$$\dot{m}_{\text{air}} = \frac{(\text{AV})_5}{v_5} = (\text{AV})_5 \frac{p_5}{RT_5}$$

$$= \left(0.42 \frac{\text{m}^3}{\text{s}}\right) \frac{(1 \text{ bar})}{\left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(293 \text{ K})} \left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}}\right|$$

$$= 0.5 \text{ kg/s}$$

The refrigerant mass flow rate is determined using an energy balance for the control volume enclosing the condenser together with assumptions 1, 3, and 4 to obtain

$$\dot{m}_{\text{ref}} = \frac{\dot{m}_{\text{air}}(h_6 - h_5)}{(h_2 - h_3)}$$

With assumption 5, $h_6 - h_5 = c_p(T_6 - T_5)$. Inserting values

2 $$\dot{m}_{\text{ref}} = \frac{\left(0.5 \frac{\text{kg}}{\text{s}}\right)\left(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(323 - 293)\text{K}}{(294.17 - 79.05) \text{ kJ/kg}} = 0.07 \text{ kg/s}$$

Using Eq. 6.22, the change in specific entropy of the air is

$$s_6 - s_5 = c_p \ln \frac{T_6}{T_5} - R \ln \frac{p_6}{p_5}$$

$$= \left(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right) \ln\left(\frac{323}{293}\right) - R \ln\left(\frac{1.0}{1.0}\right)^{0} = 0.098 \text{ kJ/kg} \cdot \text{K}$$

Finally, solving the entropy balance for $\dot{\sigma}_{\text{cond}}$ and inserting values

$$\dot{\sigma}_{\text{cond}} = \dot{m}_{\text{ref}}(s_3 - s_2) + \dot{m}_{\text{air}}(s_6 - s_5)$$

$$= \left[\left(0.07 \frac{\text{kg}}{\text{s}}\right)(0.2936 - 0.98225) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} + (0.5)(0.098)\right] \left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 7.95 \times 10^{-4} \frac{\text{kW}}{\text{K}}$$

Compressor

For the control volume enclosing the compressor, the entropy rate balance reduces with assumptions 1 and 3 to

$$0 = \dot{m}_{\text{ref}}(s_1 - s_2) + \dot{\sigma}_{\text{comp}}$$

or

$$\dot{\sigma}_{\text{comp}} = \dot{m}_{\text{ref}}(s_2 - s_1)$$

$$= \left(0.07 \frac{\text{kg}}{\text{s}}\right)(0.98225 - 0.9572)\left(\frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right) \left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 17.5 \times 10^{-4} \text{ kW/K}$$

Valve

Finally, for the control volume enclosing the throttling valve, the entropy rate balance reduces to

$$0 = \dot{m}_{\text{ref}}(s_3 - s_4) + \dot{\sigma}_{\text{valve}}$$

Solving for $\dot{\sigma}_{\text{valve}}$ and inserting values

$$\dot{\sigma}_{\text{valve}} = \dot{m}_{\text{ref}}(s_4 - s_3) = \left(0.07\frac{\text{kg}}{\text{s}}\right)(0.3078 - 0.2936)\left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= 9.94 \times 10^{-4}\text{ kW/K}$$

b. The following table summarizes, in rank order, the calculated entropy production rates:

3

Component	$\dot{\sigma}_{\text{cv}}$ (kW/K)
compressor	17.5×10^{-4}
valve	9.94×10^{-4}
condenser	7.95×10^{-4}

b. The following table summarizes, in rank order, the calculated entropy production rates:

3

Component	$\dot{\sigma}_{\text{cv}}$ (kW/K)
compressor	17.5×10^{-4}
valve	9.94×10^{-4}
condenser	7.95×10^{-4}

Entropy production in the compressor is due to fluid friction, mechanical friction of the moving parts, and internal heat transfer. For the valve, the irreversibility is primarily due to fluid friction accompanying the expansion across the valve. The principal source of irreversibility in the condenser is the temperature difference between the air and refrigerant streams. In this example, there are no pressure drops for either stream passing through the condenser, but slight pressure drops due to fluid friction would normally contribute to the irreversibility of condensers. The evaporator shown in Fig. E6.8 has not been analyzed.

1 Due to the relatively small temperature change of the air, the specific heat c_p can be taken as constant at the average of the inlet and exit air temperatures.

2 Temperatures in K are used to evaluate $\dot{m}_{\text{ref}}$, but since a temperature *difference* is involved the same result would be obtained if temperatures in °C were used. Temperatures in K, and not °C, are required when a temperature *ratio* is involved, as in Eq. 6.22 used to evaluate $s_6 - s_5$.

3 By focusing attention on reducing irreversibilities at the sites with the highest entropy production rates, *thermodynamic* improvements may be possible. However, costs and other constraints must be considered and can be overriding.

SKILLS DEVELOPED

Ability to...

- apply the control volume, mass, energy, and entropy rate balances.
- develop an engineering model.
- retrieve property data for Refrigerant 22.
- apply the ideal gas model with constant c_p.

Quick Quiz

If the compressor operated adiabatically *and* without internal irreversibilities, determine the temperature of the refrigerant at the compressor exit, in °C, keeping the compressor inlet state and exit pressure the same. Ans. 65°C.

Example 8 Determining turbine Work Using the Isentropic Efficiency

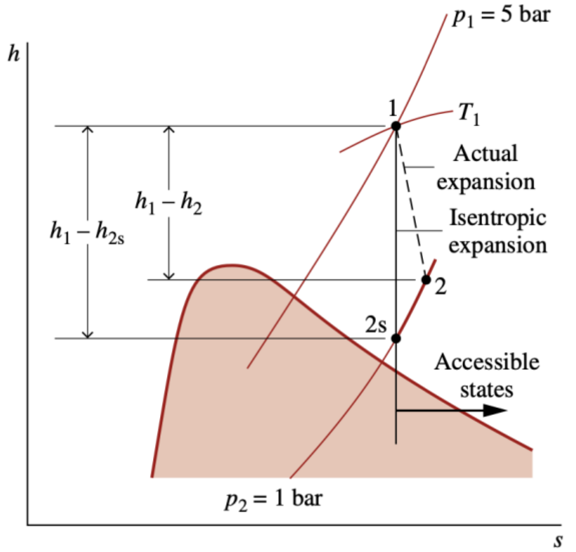
A steam turbine operates at steady state with inlet conditions of $p_1 = 5$ bar, $T_1 = 320°C$. Steam leaves the turbine at a pressure of 1 bar. There is no significant heat transfer between the turbine and its surroundings, and kinetic and potential energy changes between inlet and exit are negligible. If the isentropic turbine efficiency is 75%, determine the work developed per unit mass of steam flowing through the turbine, in kJ/kg.

Solution

Known Steam expands through a turbine operating at steady state from a specified inlet state to a specified exit pressure. The turbine efficiency is known.

Find Determine the work developed per unit mass of steam flowing through the turbine.

Schematic and Given Data:



Engineering Model

1. A control volume enclosing the turbine is at steady state.
2. The expansion is adiabatic and changes in kinetic and potential energy between the inlet and exit can be neglected.

Analysis The work developed can be determined using the isentropic turbine efficiency, Eq. 6.46, which on rearrangement gives

$$\frac{\dot{W}_{cv}}{\dot{m}} = \eta_t \left(\frac{\dot{W}_{cv}}{\dot{m}} \right)_s = \eta_t (h_1 - h_{2s})$$

From Table A-4, $h_1 = 3105.6$ kJ/kg and $s_1 = 7.5308$ kJ/kg · K. The exit state for an isentropic expansion is fixed by $p_2 = 1$ and

1 $s_{2s} = s_1$. Interpolating with specific entropy in Table A-4 at 1 bar gives $h_{2s} = 2743.0$ kJ/kg. Substituting values

2 $$\frac{\dot{W}_{cv}}{\dot{m}} = 0.75(3105.6 - 2743.0) = 271.95 \text{ kJ/kg}$$

1 At 2s, the temperature is about 133°C.

2 The effect of irreversibilities is to exact a penalty on the work output of the turbine. The work is only 75% of what it would be for an isentropic expansion between the given inlet state and the turbine exhaust pressure. This is clearly illustrated in terms of enthalpy differences on the accompanying *h–s* diagram.

SKILLS DEVELOPED

Ability to…

- apply the isentropic turbine efficiency, Eq. 6.46.
- retrieve *steam table* data.

Quick Quiz

Determine the temperature of the steam at the turbine exit, in °C. Ans. 179°C.