In class -exercises 15.-17.1.2024

The In class -exercises are to be done in the exercise session and the assistant will give advice on how to do them if necessary. The correct solutions to the problems will be discussed together. To obtain points for these exercises, you only need to be present.

1. Do exercise 14.1.7 in Guichard's Calculus text.
   https://www.whitman.edu/mathematics/calculus_online/section14.01.html

2. The graph $z = f(x, y)$ for function $f(x, y) = x^2/a^2 + y^2/b^2$ represents *elliptic paraboloid*. Write the equations of the following curves and modify the equations to identify which curve is involved.

   (a) What is the intersection curve of elliptic paraboloid intersecting with the xz-plane? What about yz-planes?

   (b) What curves are the level curves $f(x, y) = c^2$ for the constant $c > 0$?

3. We will next investigate if there exists the following limit

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}.$$

   (a) In Calculus 1 we learned the very usuful tool: l'Hospital. Is there a way to apply it to the calculation of this limit?

   (b) Calculate what is the limit when (x, y) approaches the origin along a any straight line.

   (c) Does the limit exists? And if so, what it is?

4. (a) Decide the *possible* limit of the function

$$f(x, y) = \frac{x^2 y^2}{x^2 + y^2},\ (x, y) \neq (0, 0),$$

   at the origin by examining its behaviour on the lines $y = kx$.

   (b) Prove using polar coordinates $x = r\cos\theta$, $y = r\sin\theta$, that the number obtained in (a) is the limit of the function at the origin.
   (Hint: The condition $(x, y) \to (0, 0)$ with polar coordinates is the same as $r \to 0+$.)

   (c) Prove using inequalities that the number obtained in (a) is the limit of the function at the origin.