ELEC-C5220
Lecture 2:
Tensors for data representation

Machine learning in information technology

Aalto University
School of Electrical
Engineering

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About programming exercises

- Exercise sessions on Mondays 14-16
- First session had plenty of room available
- Deadline for exercises is Monday evening on the following week
- You can still get help for the Exercise 01 in next week's session
- Who has already returned the Exercise for Week 1?
- How much time did it take?

Question from the exercise

- What are Tensors and why do we need them?
- In the first week exercise, everything was vectors and matrices

Lecture overview

- What are Tensors?
- Tensors for representing images
- Multi-class classifiers and one-hot encoding
- Tensors for representing audio
- Audio as 1D vector (waveform)
- Audio as 2D matrix (spectrogram)

What are tensors?

- **Tensors are n-dimensional rectangular arrays**
- **Topic for this lecture: how to use tensors to represent structure in data?**
- **Examples with**
  - Images
  - Audio
  - Classes (categorical)
  - Text is also categorical, more on this later

Tensor notation

- **Scalar – 0D Tensor** $x \in \mathbb{R}$ $()$
- **Vector – 1D Tensor** $\mathbf{x} \in \mathbb{R}^D$ (D)
- **Matrix – 2D Tensor** $\mathbf{X} \in \mathbb{R}^{N \times M}$ (N, M)

Tensor notation

- **3D Tensor, e.g., multi-channel audio**

$$x \in \mathbb{R}^{B \times C \times T} \qquad (B, C, T)$$

- **4D Tensor, e.g., color images**

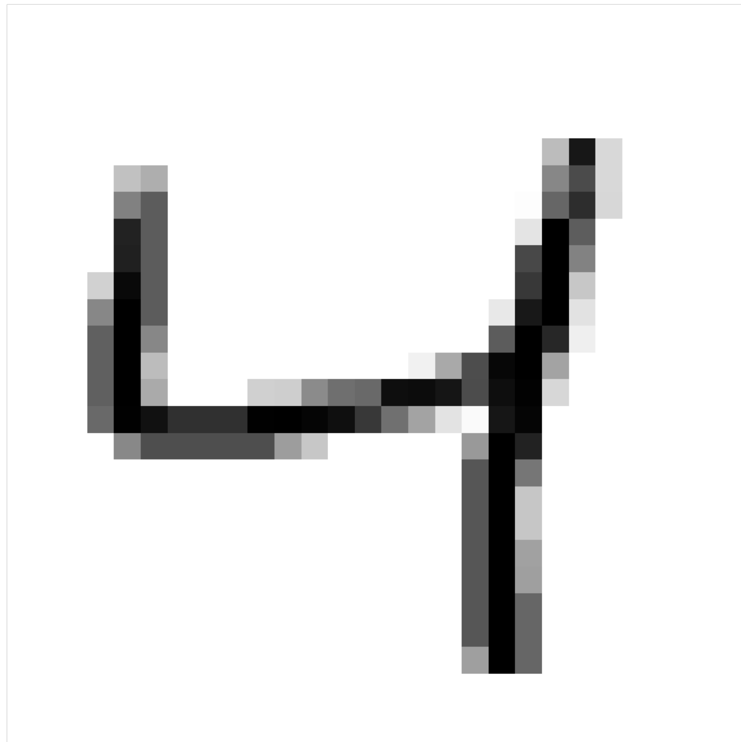
$$x \in \mathbb{R}^{B \times C \times H \times W} \qquad (B, C, H, W)$$

- **5D Tensor, e.g, video**

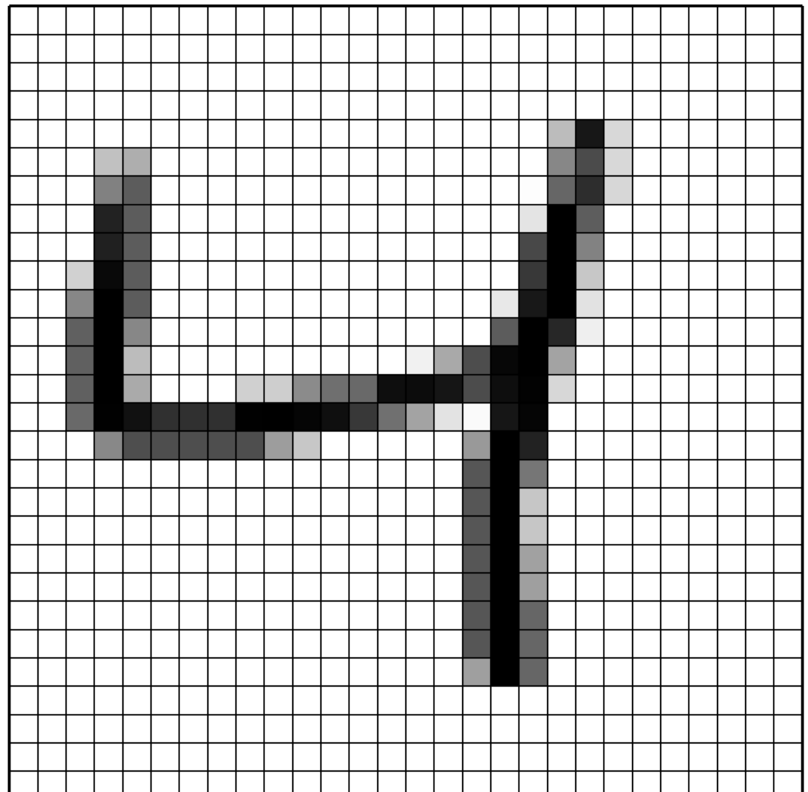
$$x \in \mathbb{R}^{B \times C \times H \times W \times T} \qquad (B, C, H, W, T)$$

Images - monochrome

MNIST hand-written digits

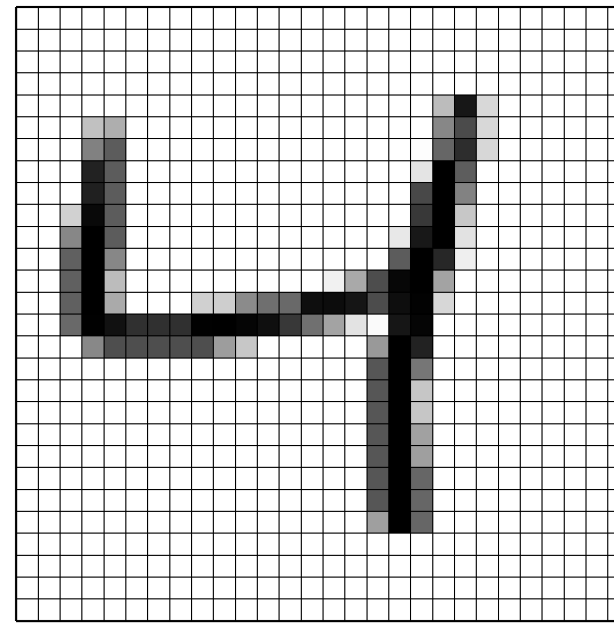


MNIST hand-written digits



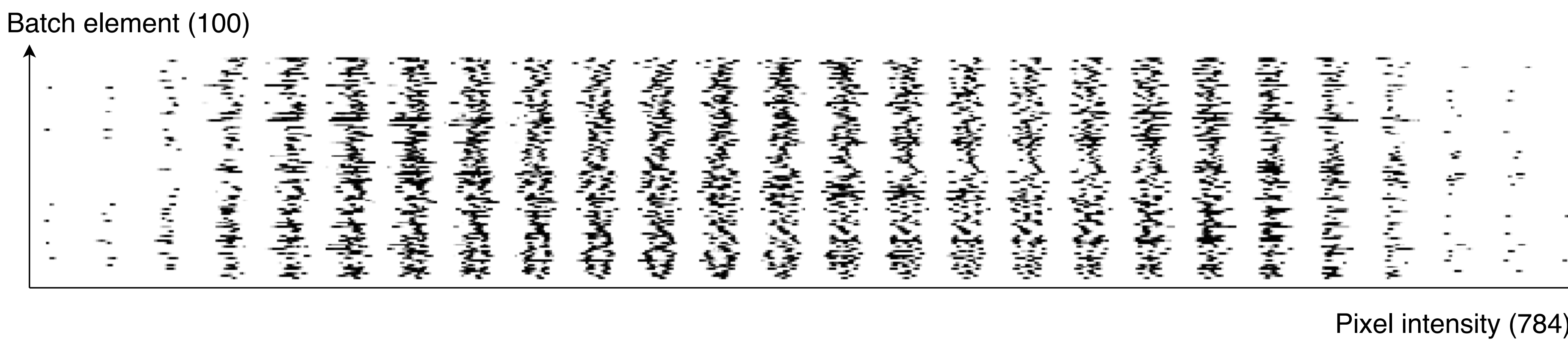
MNIST hand-written digits

- **28 x 28 pixel grid**
- **What if our model can only handle flat vector inputs, how to vectorise?**
- **Idea 1:**
  - Take every pixel value as a dimension
  - Rasterise the image to a 28 x 28 = 784 dimensional vector

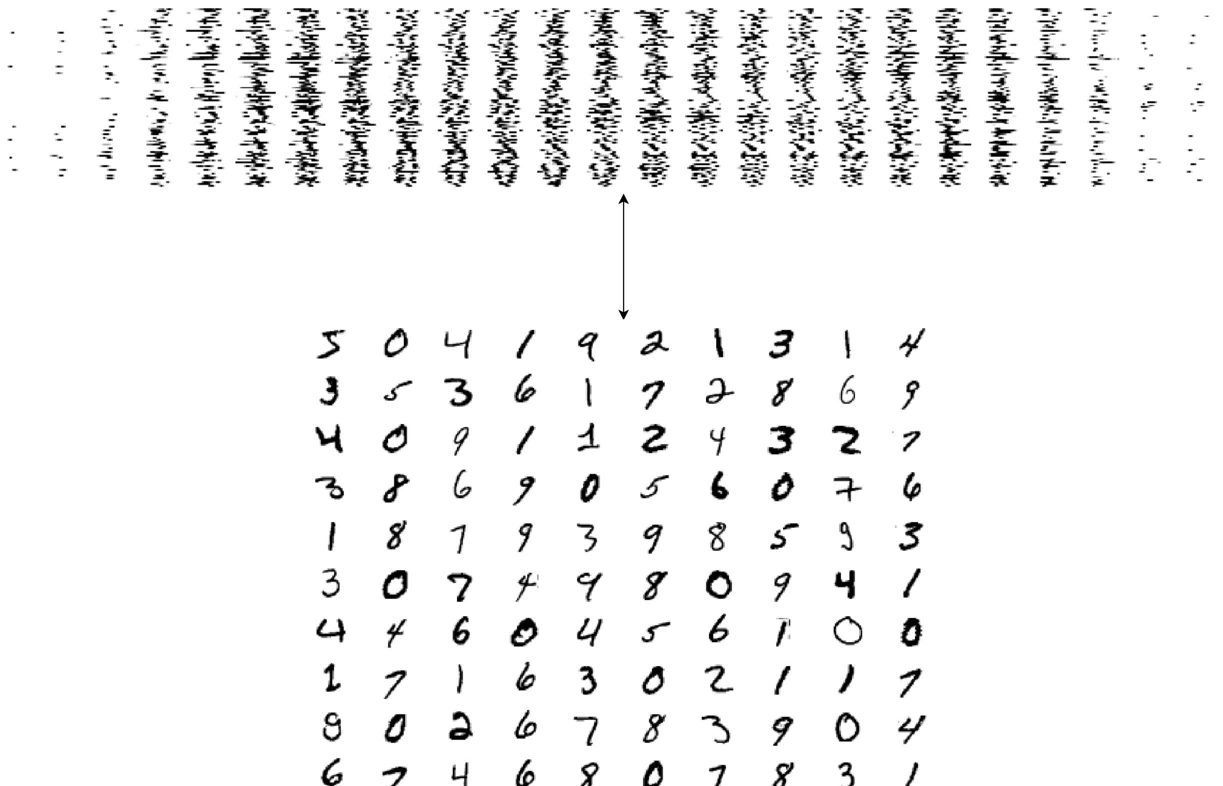


Flattened handwritten digits

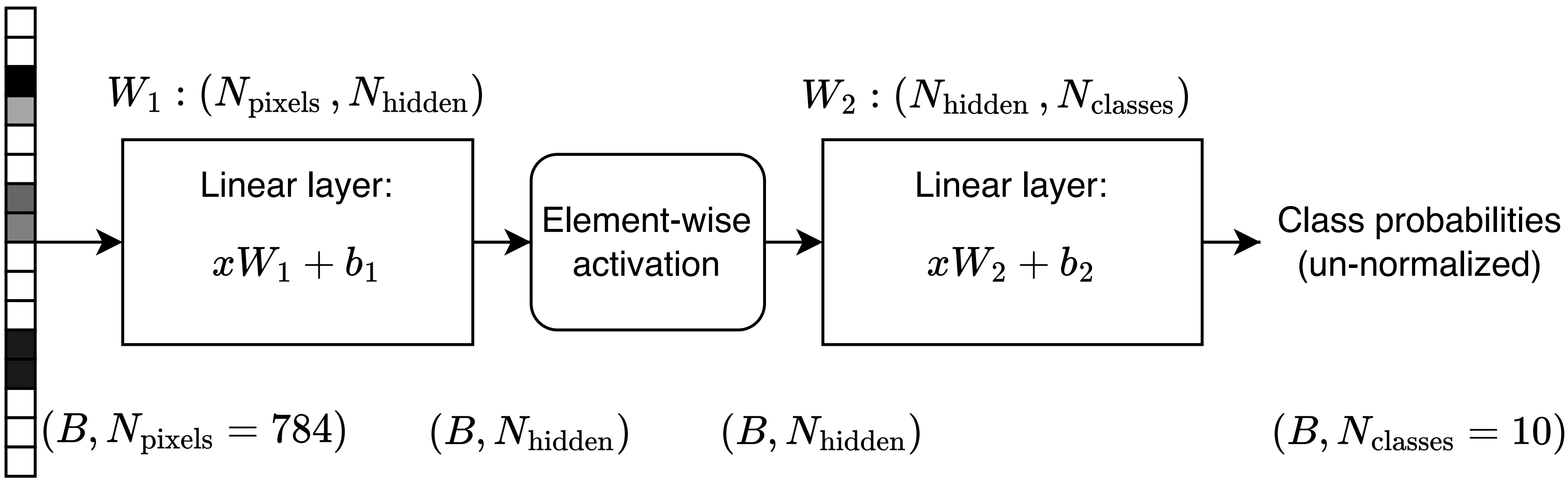
- **Pros: easy to do matrix multiplication to apply linear or DNN classifiers**
- **Cons: structure was lost, no notion of neighbouring pixels in vertical and horzontal directions**



Same data, different representations



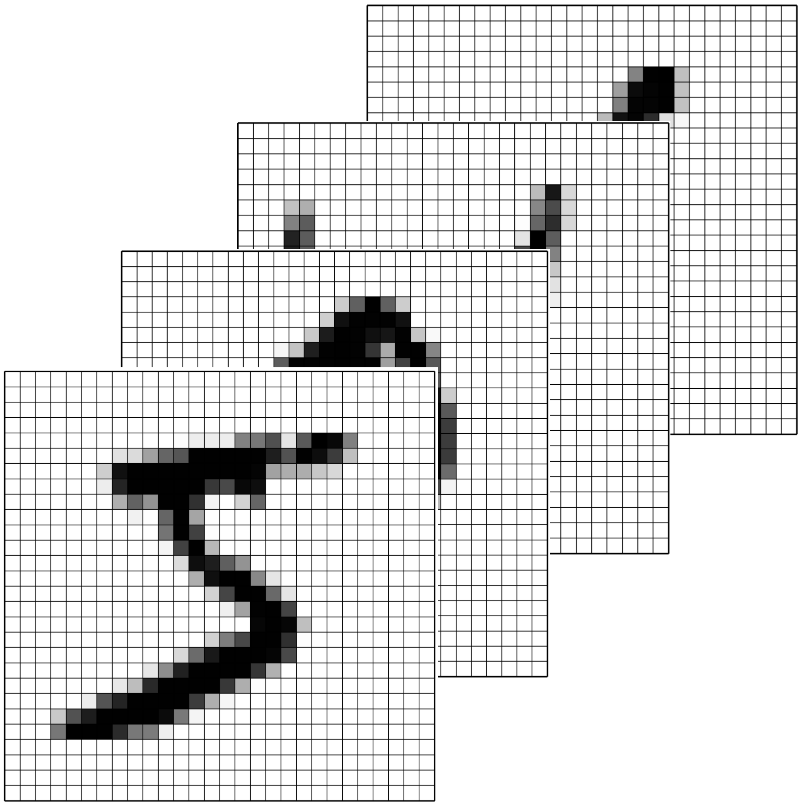
DNN Classifier for MNIST digits

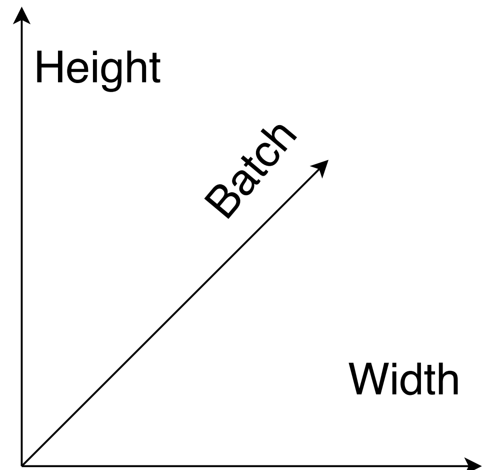


DNN Classifier for MNIST digits

- **Works fine for MNIST but,**
    - What if we want to work on other image sizes?
    - What about color
    - Very annoying for humans to inspect learned representations for debugging
    - Fragile and overparameterised
- **Try it yourself in Exercise 2!**

Tensors for representing images

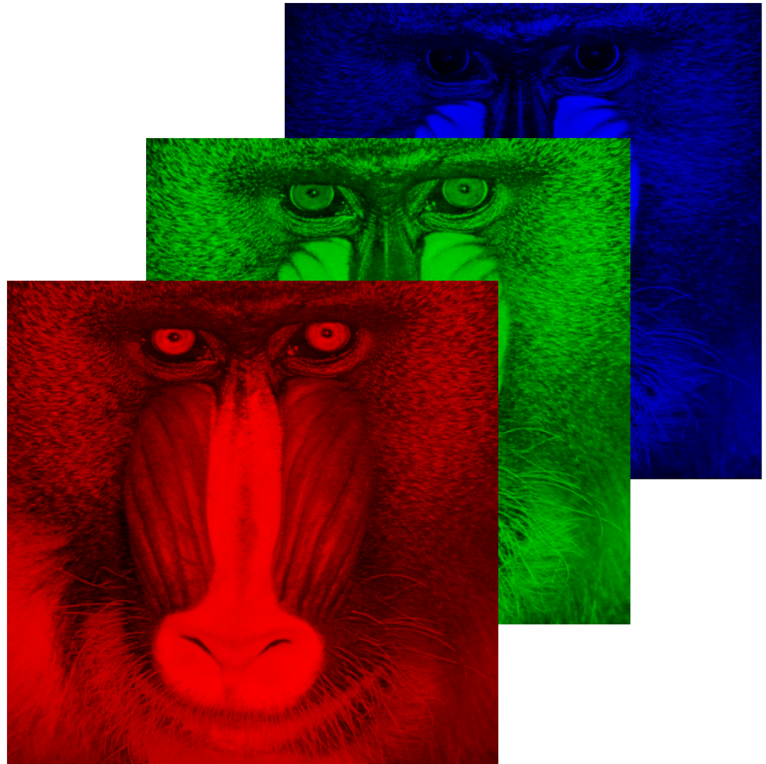


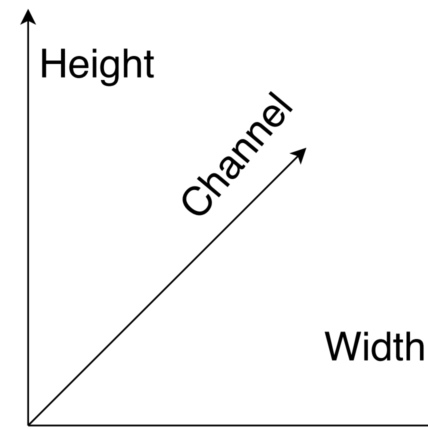


3D: (Batch, Height, Width)

4D: (Batch, Channels=1, Height, Width)

Tensors for RGB color images

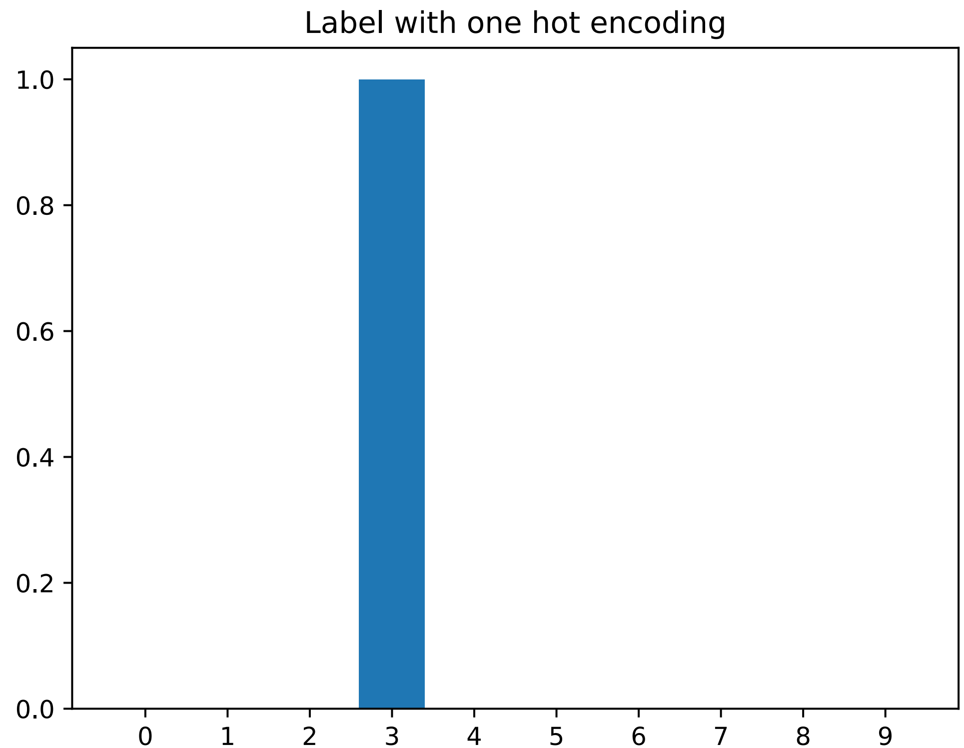




(Channel, Height, Width)

Categorigal distribution for classifiers

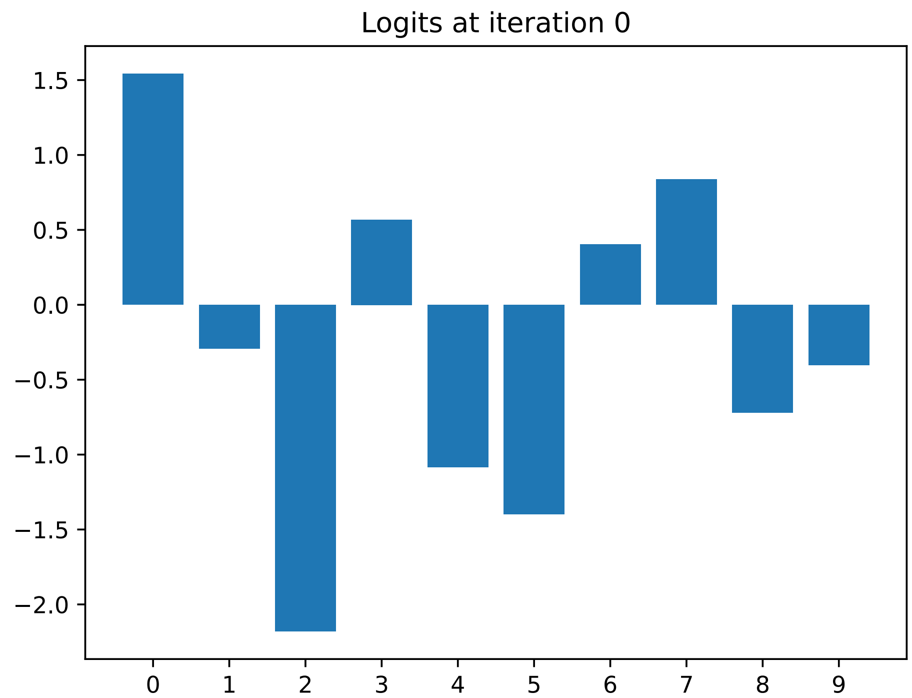
- **Let's look at one example of digit "3"**
- **Probability 1 for the correct class**
- **Probability 0 for the other classes**
- **This is also called a *one-hot embedding***



Logits and Softmax

- **Classifier initially outputs a vector of random numbers**
- **Normalize these to probability distribution with the Softmax function**

$$\text{softmax}(\mathbf{z}) = \frac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}}$$



Cross-entropy loss for categorical distribution

- **Binary cross-entropy (Lecture 1)**

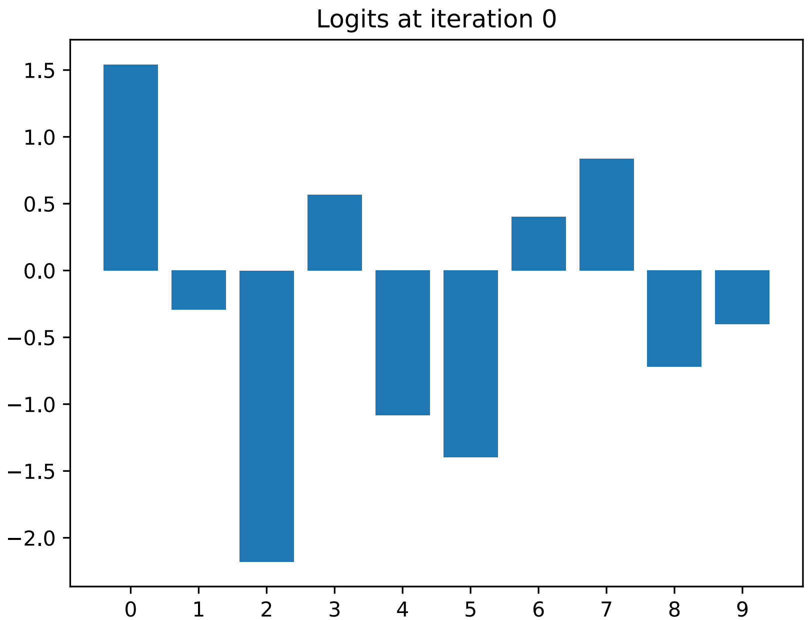
$$L_{\text{BCE}} = -\mathbb{E}[\mathbf{y} \log \hat{\mathbf{y}} + (1 - \mathbf{y}) \log(1 - \hat{\mathbf{y}})]$$

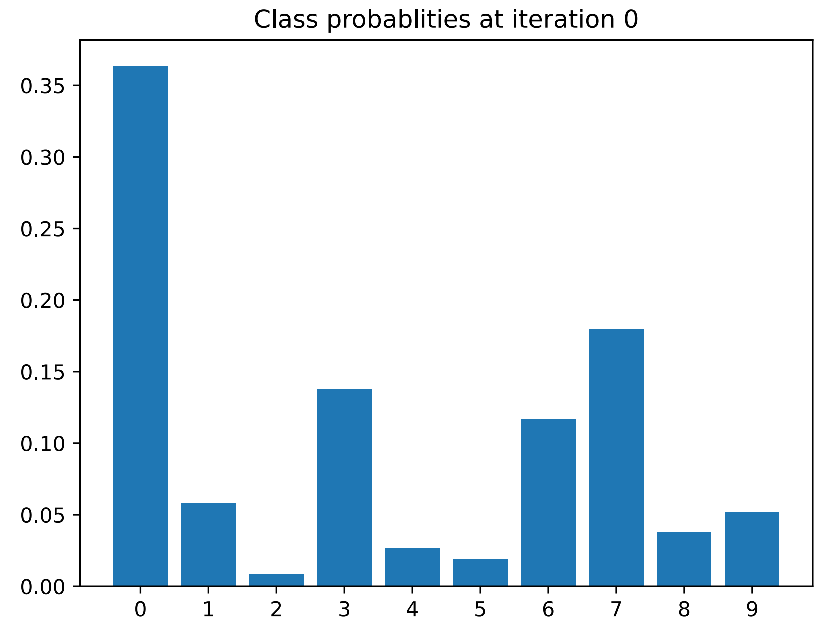
- **Categorical cross-entropy**

$$L_{\text{CCE}} = -\mathbb{E}\left[\sum_{i=1}^{K} \mathbf{y}_i \log \hat{\mathbf{y}}_i\right]$$

- **Probability of other classes is zero!**

Fitting softmax for one example





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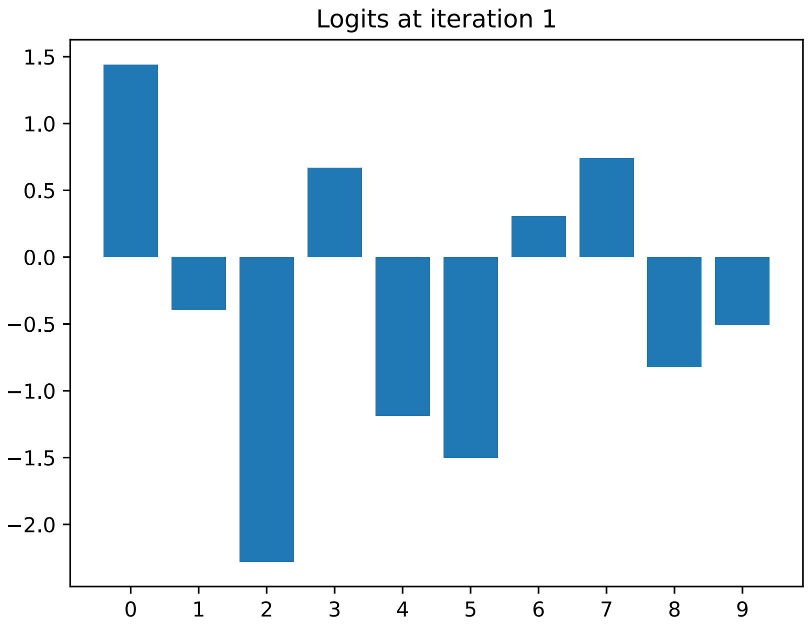
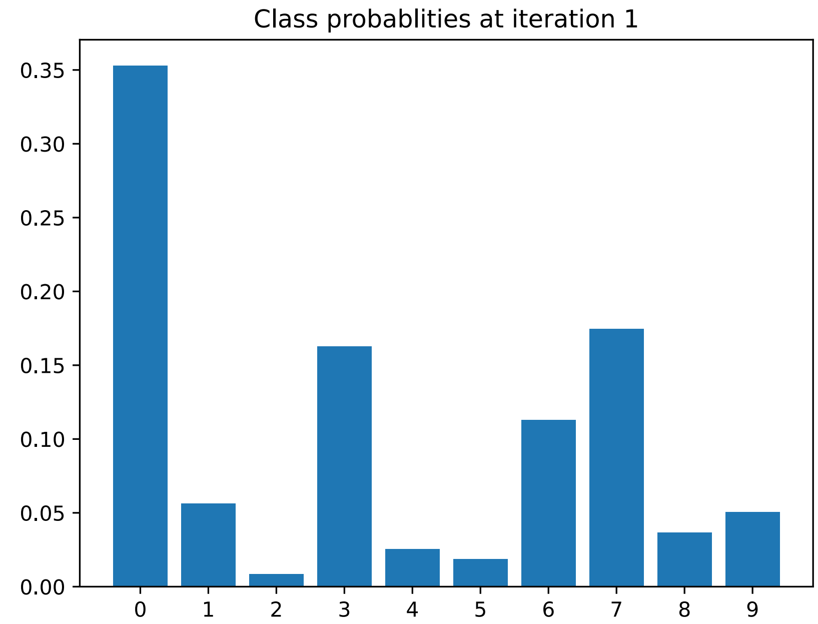
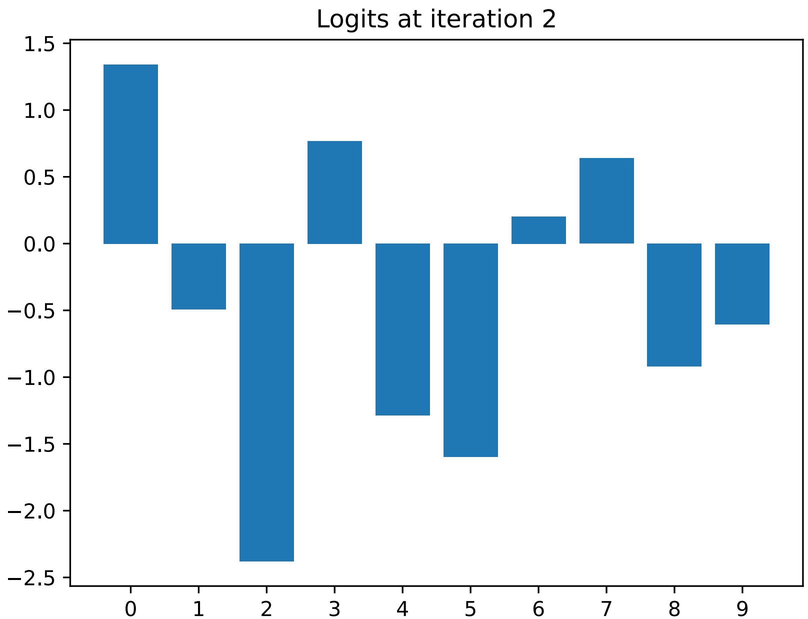
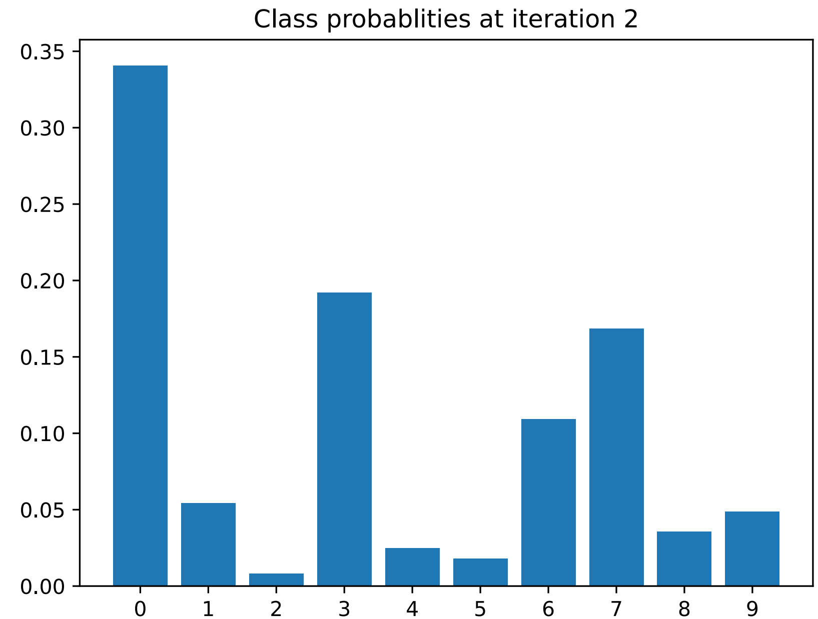
Fitting softmax for one example



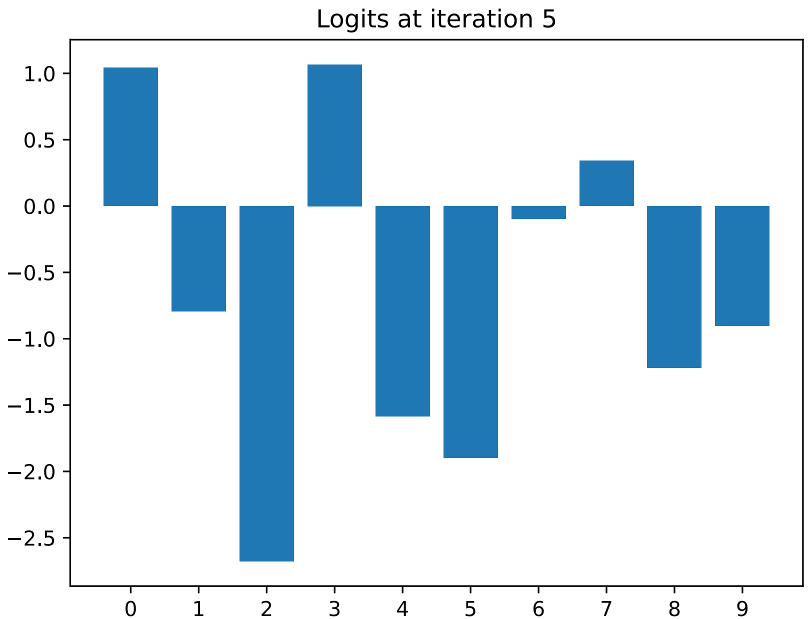


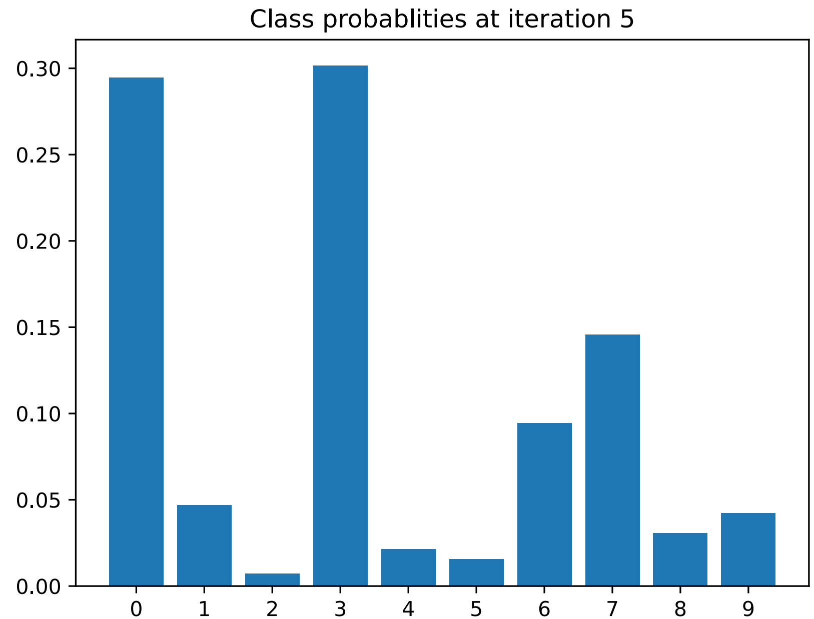
Fitting softmax for one example



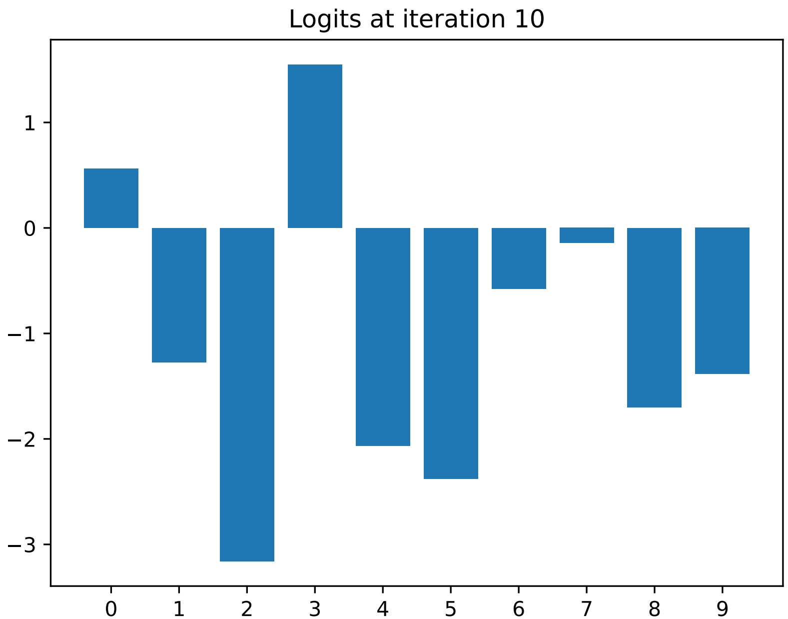


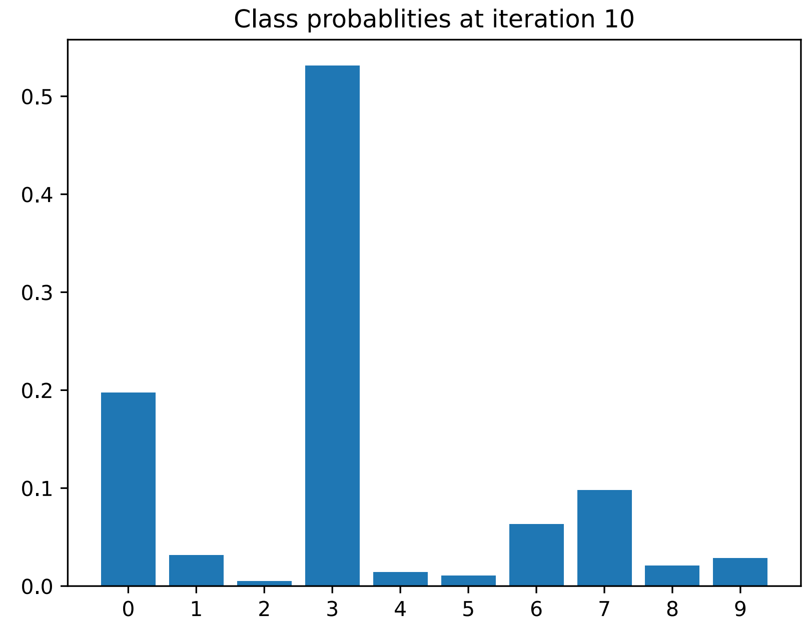
Fitting softmax for one example



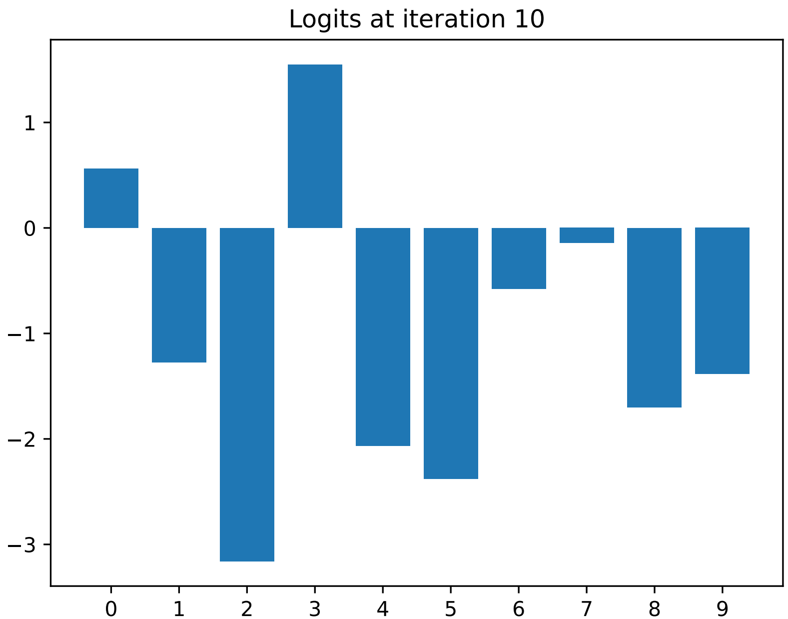


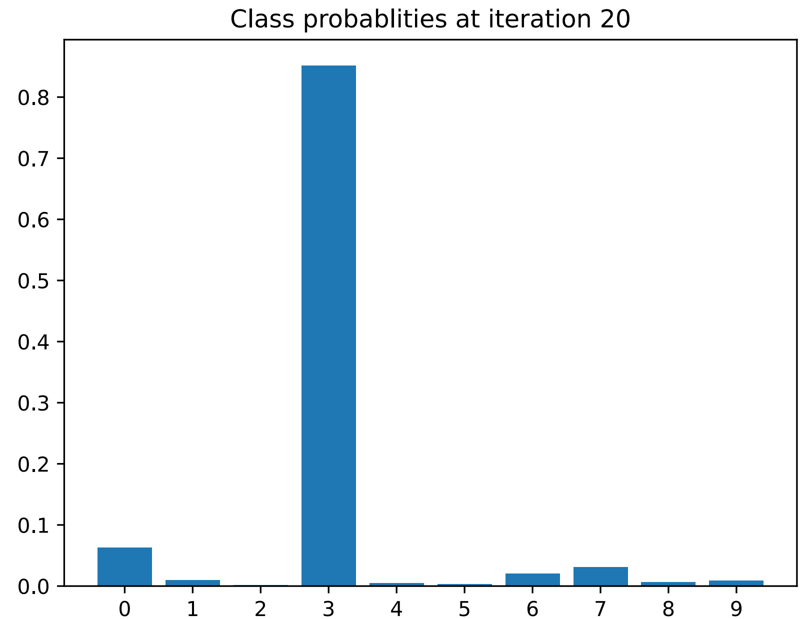
Fitting softmax for one example



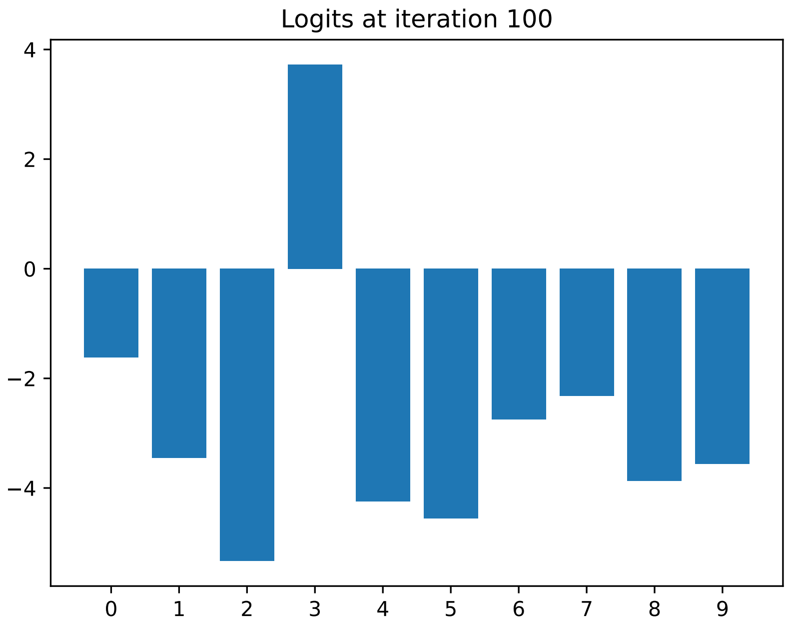


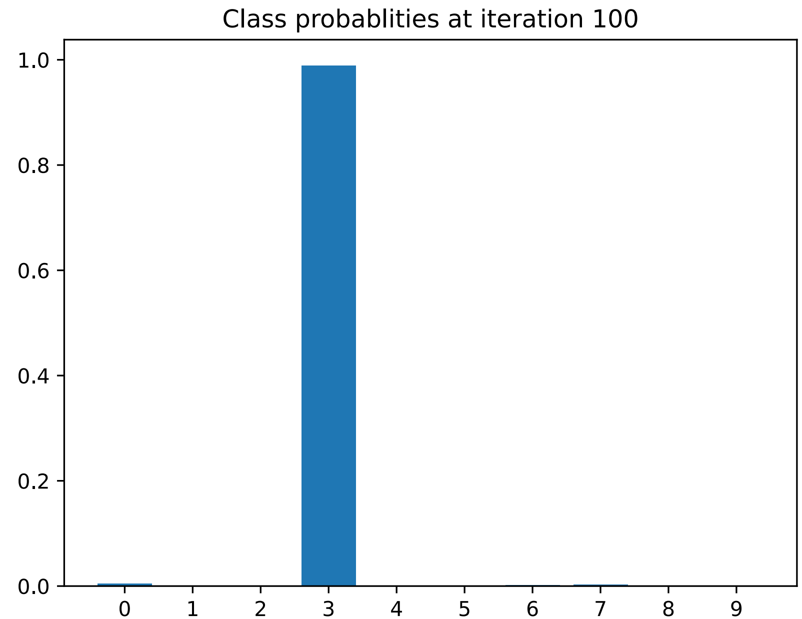
Fitting softmax for one example





Fitting softmax for one example





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Categorical distribution for text

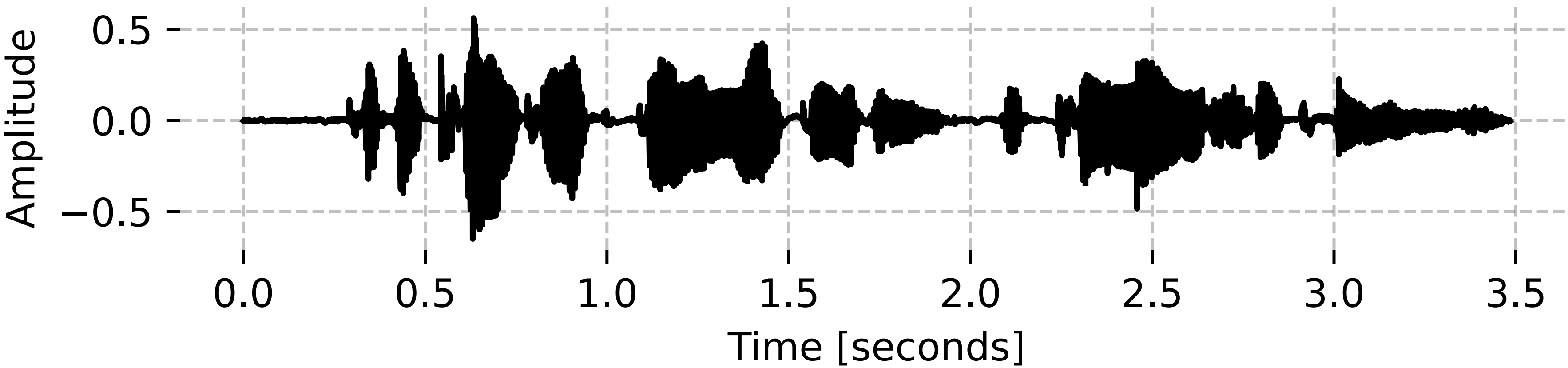
- You can use similar one-hot embeddings to encode text or other symbols
- More about this later on the language modeling Lecture 6

Break

- Questions?
- Next: audio representations

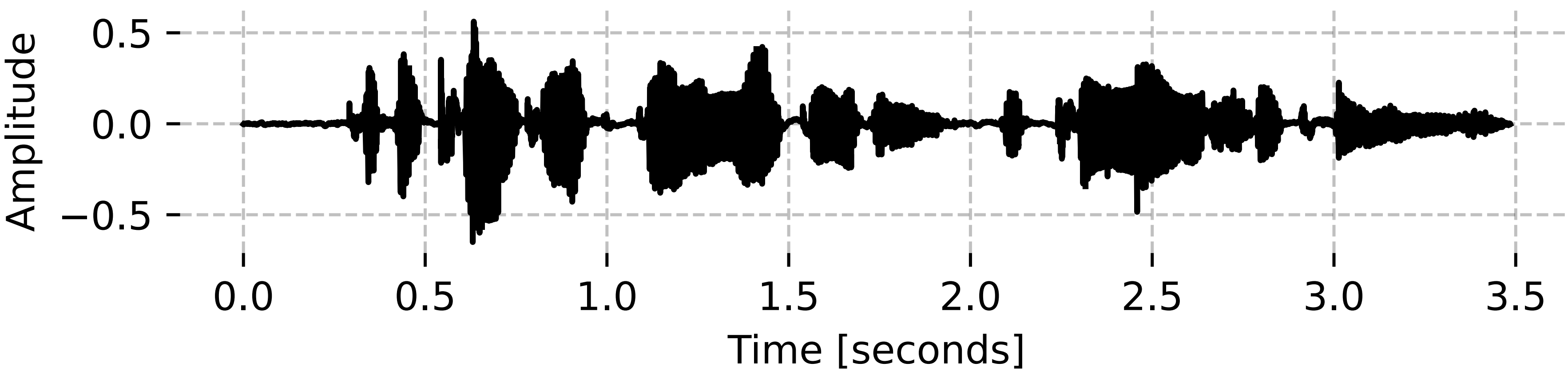
Audio - waveform

- **Sequence of amplitude values sampled at regular intervals (e.g., 48 kHz or 16 kHz)**
- **Monophonic (single channel) audio is a vector, where time corresponds to dimension**



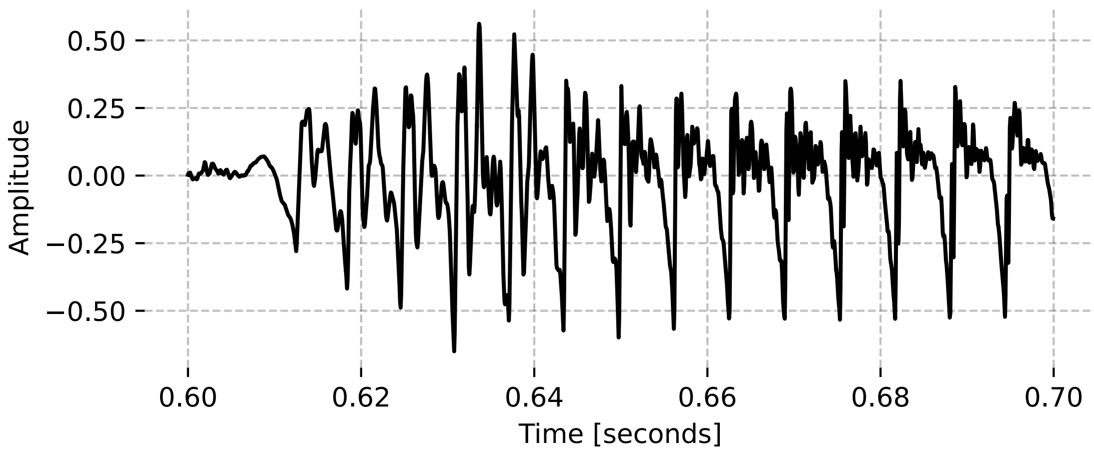
Audio - waveform

- For example, one second of audio at 16 kHz would be a 16 000 dimensional vector (remember the curse of dimensionality?)
- How to work on continous streams of audio?
- Hard to see anything on this zoom-level



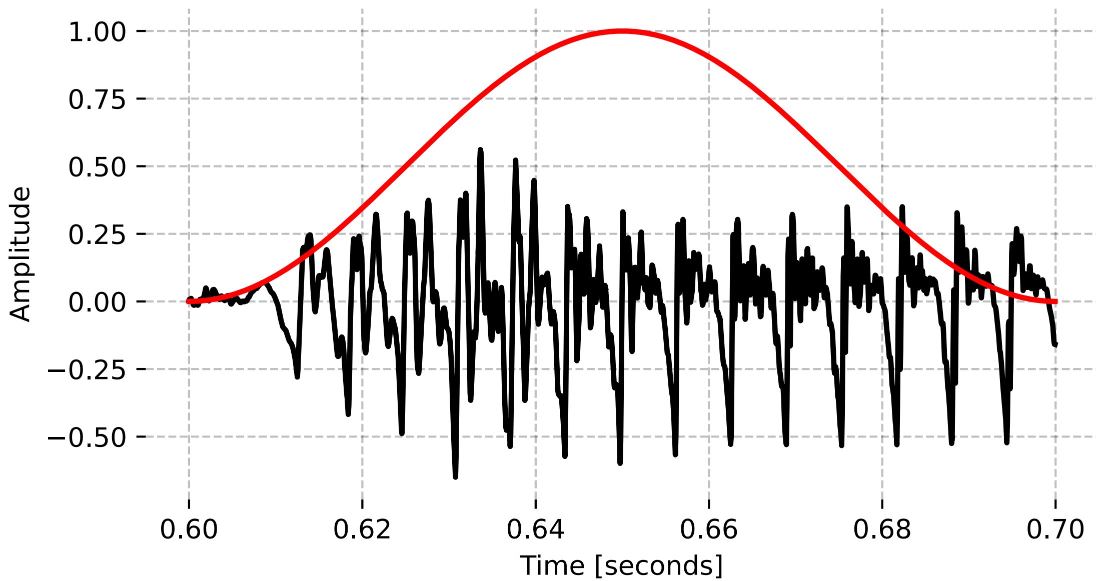
Audio and short-time frames

- **Zoom in on 100ms of audio**
- **In this example, voiced speech is (quasi) periodic, we can do Fourier analysis!**



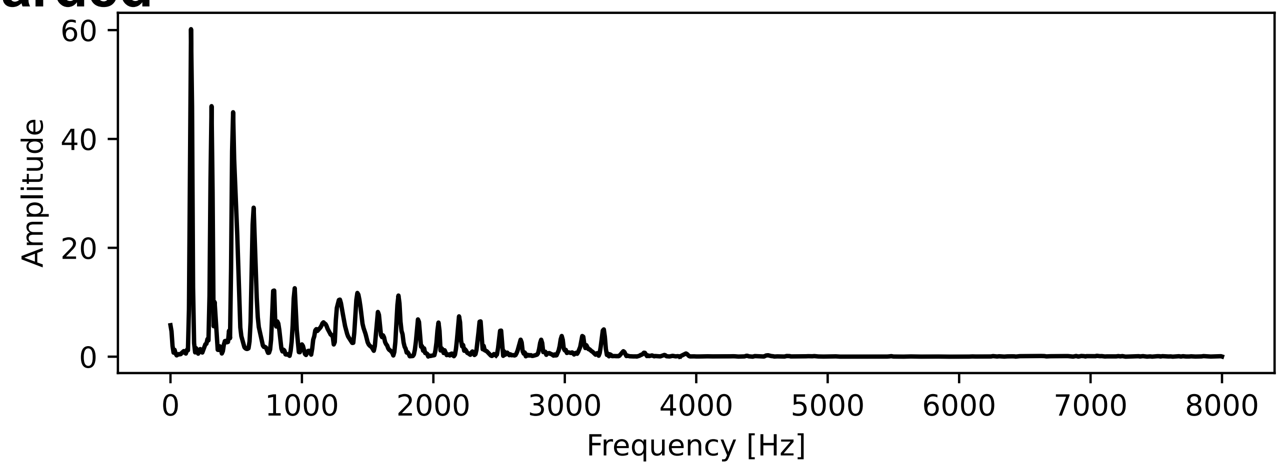
Audio and short-time frames

- Multiply with a tapered window (half cosine, Hann, etc.)
- Apply Discrete Fourier Transform (DFT)
- In practice, Fast Fourier Transform (FFT)



Audio and short-time spectrum

- **Plot frequency magnitudes on linear scale - harmonic overtone series shows up!**
- **Remember: FFTs are complex-valued, phase information was discarded**



Fourier transform and complex numbers

- **Fourier transform outputs sinusoids (i.e., complex numbers on the unit circle)**
- **Remember phase when processing signals (synthesis, coding, enhancement)**
- **Ignore phase for detection tasks (speech recognition, etc.)**

Real, Imaginary

$$z = x + iy$$

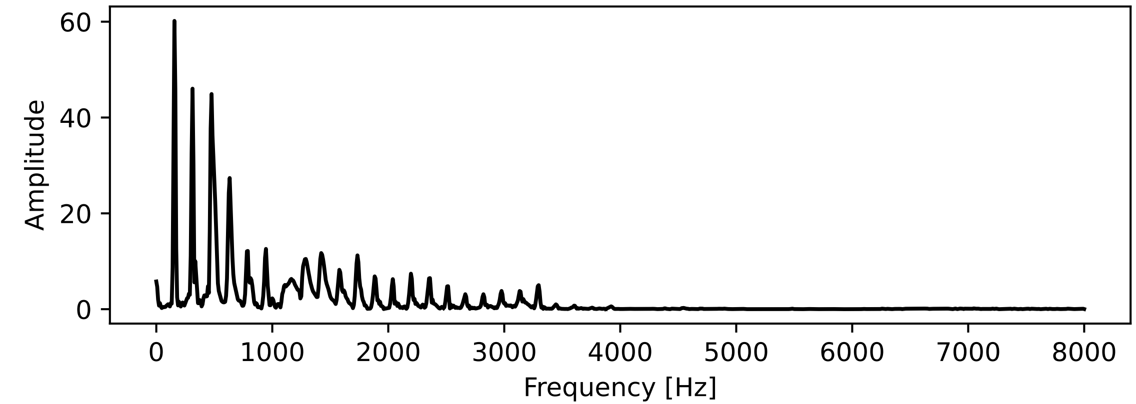
Amplitude, Phase

$$z = Ae^{i\theta}$$

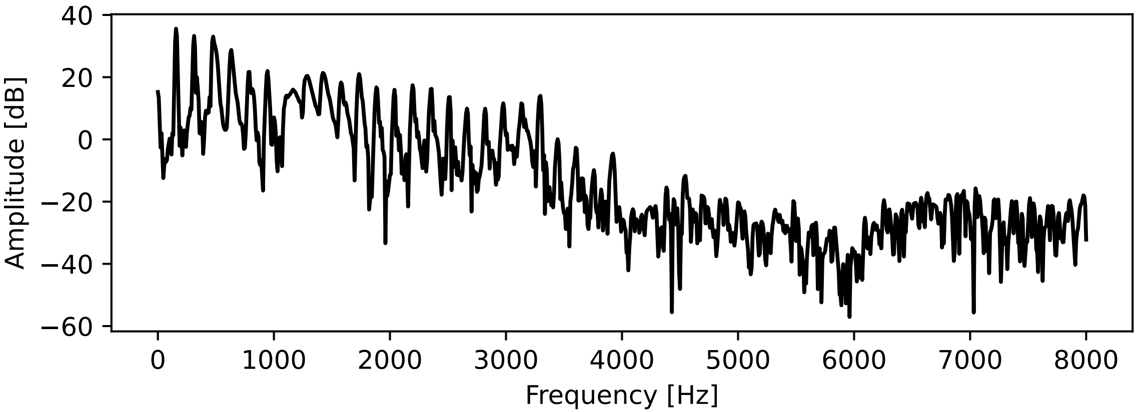
Frequency, Initial phase

$$z = Ae^{i(2\pi f + \theta_0)}$$

Decibels and log-scale spectrum



$$A = |z|$$



$$A_{\mathrm{dB}} = 20 \log_{10}(|z| + \varepsilon)$$

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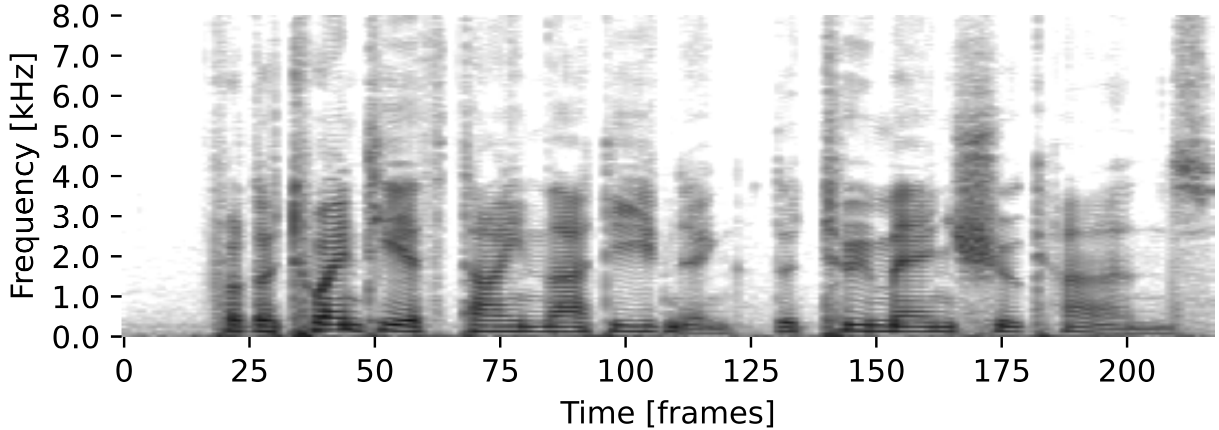
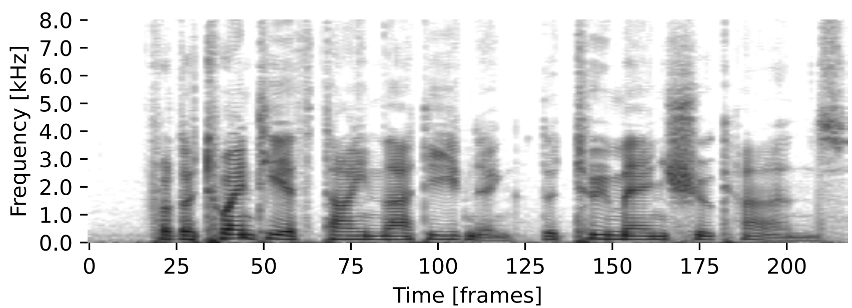
Audio spectrograms

- **Short-Time Fourier Transform (STFT)**
- **Sliding window, apply FFT to each frame**
- **Log-magnitudes (dB-scale) are usually best for visualisation**



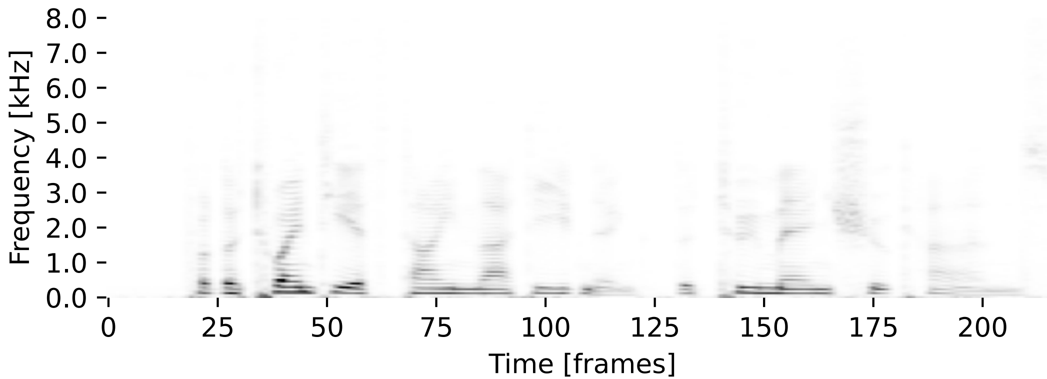
Audio spectrograms

- Spectrogram is a matrix, shape (T, F)
- T = Number of time frames
- F = Number of frequency bins
- Spectrogram is an image (you are looking at it)
- For machine learning models: 4D tensor $(B, C = 1, F, T)$



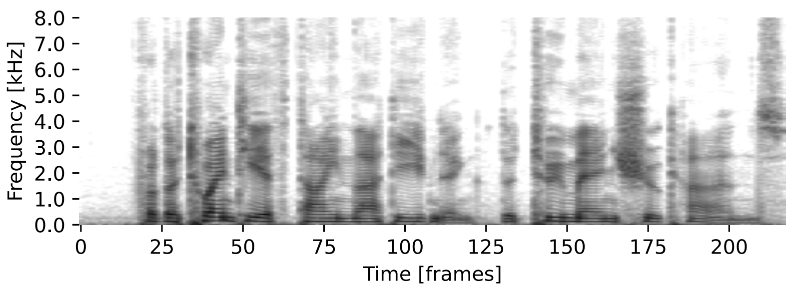
Audio spectrograms - pitfalls

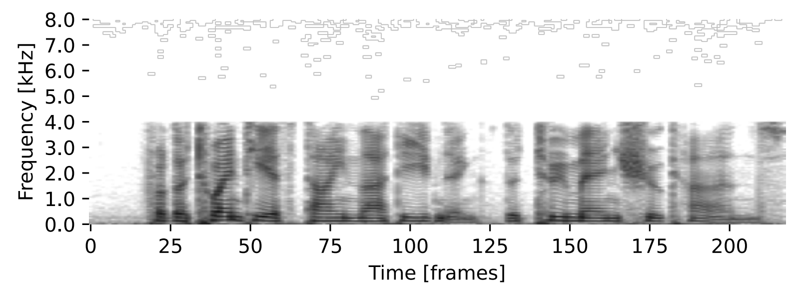
- **Linear amplitude spectrogram (below) are correct shape, but the dynamic range is too large to see much**
- **Logarithms of zeros are numerical trouble (remember to add a small value)**



Filtering in frequency domain

- **Filtering in the frequency domain corresponds to multiplying STFT amplitudes**
- **Exercise 2: implement a low-pass filter**





Filtering in frequency domain

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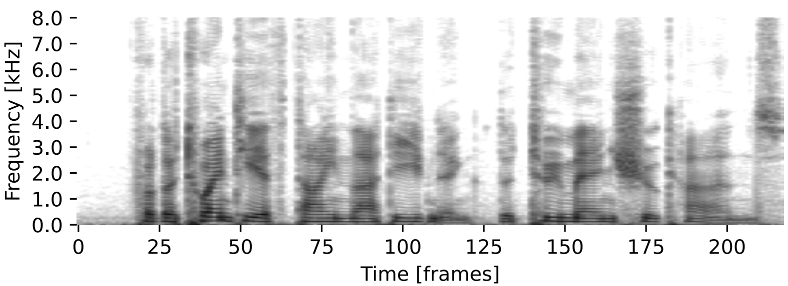

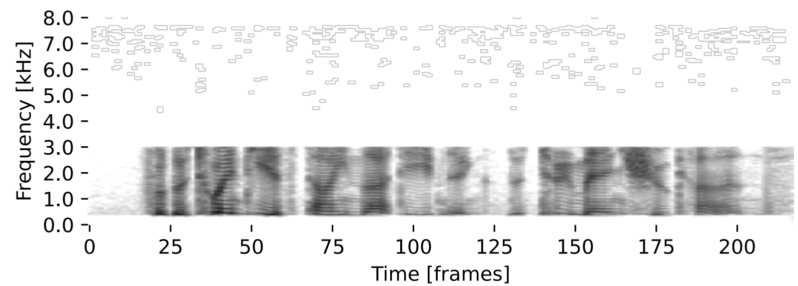

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End of Lecture 2

- Questions?