In class -exercises 22. - 23.1.2024 Round 3

The In class -exercises are to be done in the exercise session and the assistant will give advice on how to do them if necessary. The correct solutions to the problems will be discussed together. To obtain points for these exercises, you only need to be present.

- 1. (a) Calcutate the Jacobian matrix for $f(x, y) = (x^2y, y^3 + \cos(4x))$.
 - (b) Give an example of a function $f \colon \mathbb{R}^2 \to \mathbb{R}^2$ such that its Jacobian matrix at the point (0,0) is $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$.
- 2. Calculate using the chain rule $\partial f/\partial t$ ja $\partial f/\partial s$, when $f(x,y) = \sqrt{x^2 + y^2}$, $x(s,t) = e^{st}$ and $y(s,t) = 1 + s^2 \cos(t)$.
- 3. The dimensions of a rectangular box change according to the local temperature fluctuations. Assume that at a given moment, the length of the box increases at a rate of 0.01 cm/s, width decreases at a rate of 0.02 cm/s and height increases at a rate of 0.005 cm/s. If, at the same instant, the length of the box is 15 cm, the width 8 cm and height 5 cm, does the volume of the box increase or decrease?
- 4. Use linear approximation to calculate the value of the function

$$f(x, y, z) = \sqrt{x + 2y + 3z^2}$$

at the point (2.2, 1.9, 0.9) without calculator.