

**Homework -exercises 25.-26.1.2024**

## Round 3

To get points from these exercises do them at home before the second exercise session of the week and at the beginning of the class mark them on the list.

- Let  $z = f(x, y) = \frac{2y}{x^2+y^2}$ ,  $x(s) = \cos(s)$  and  $y(s) = \sin(s)$ . Calculate the derivative  $\frac{dz}{ds}$  in two different ways:
  - by using the chain rule,
  - by substituting the expressions of  $x$  and  $y$  to  $z$ .
- Calculate the gradient for the function  $f(x, y, z) = xy + 2xz + 3yz$  at the point  $(2, -1, 3)$ .

- The base of the pyramid is a square of side  $s$  and the height of the pyramid is  $h$ . Use the differential (i.e. linear approximation) to estimate the relative error  $|\Delta V|/V$  of the volume of the pyramid

$$V = V(h, s) = \frac{1}{3}hs^2$$

when  $s$  is measured with 1 % relative accuracy and  $h$  (harder to measure!) with 3 % relative accuracy.

**Hint:** To begin with, the general formula to be used in this exercise is

$$|\Delta f(x, y)| \lesssim |f_x| \cdot |\Delta x| + |f_y| \cdot |\Delta y|.$$

The notation  $\lesssim$  denotes an approximate inequality whose (relative) accuracy improves when  $|\Delta x|, |\Delta y| \rightarrow 0$ , but  $\leq$  is not necessary valid, because the estimate only includes the linear part of the change, i.e. the differential without error terms. There would be equality only in the form

$$\Delta f := f(x + \Delta x, y + \Delta y) - f(x, y) = f_x \Delta x + f_y \Delta y + \text{error term},$$

from which follows

$$|\Delta f| \leq |f_x| \cdot |\Delta x| + |f_y| \cdot |\Delta y| + |\text{error term}|.$$