# Quantifying Light

J. Lehtinen (Tofino, BC, Canada)

# Today

- Intuitive (?) behavior of light

- Quantifying light under ray optics
  - Solid angle
  - Irradiance
  - Radiance
  - Radiosity

- Application: soft shadows from area light sources

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# Rendering <=> what is the radiance hitting my sensor?

# Rendering <=> what is the radiance hitting my sensor?

## “Radiance”? That’s what we’ll see today.

# Assumptions

- We assume the Ray Optics Model
  - Also called geometric optics
  - Disregard quantum phenomena like diffraction
    - Rendering optical disks is hard :)
    - Basically, assume scene features are "large" w.r.t. wavelength
  - Assume wavelengths are separate
    - No energy transfer between frequencies (fluorescence)
      => a photon does not change its energy, only gets scattered and absorbed
    - In principle: carry out computations separately for each wavelength
    - Usually in practice: do separate calculations for R, G, B
  - Usually, don't care about much polarization
  - For simplicity, we only treat opaque (non-transparent) surfaces



J. Lehtinen

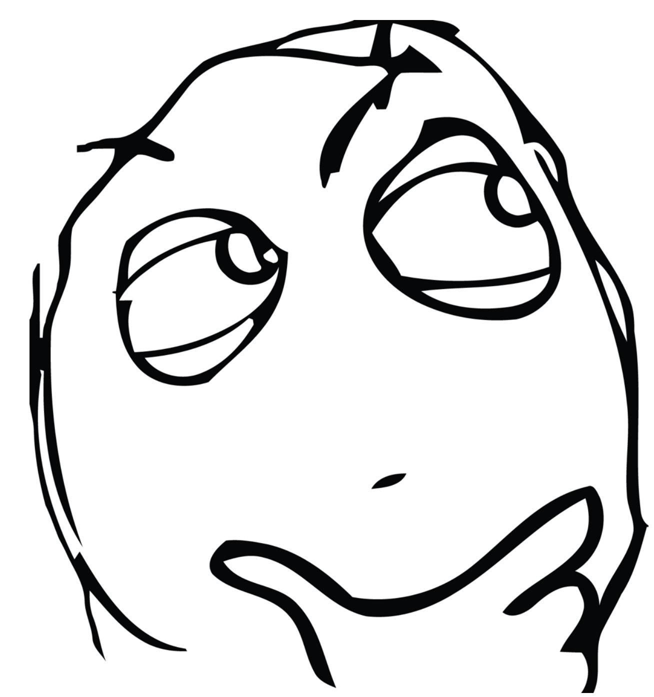

beurer

# Properties of Light, Intuitively

- How “*bright*” something is doesn’t directly tell you how brightly it *illuminates* something
  - The lamp appears just as bright from across the room and when you stick your nose to it (“intensity does not attenuate”)
  - Also, the lamp’s apparent brightness does not change much with the angle of exitance

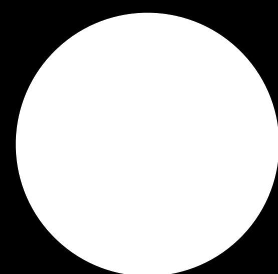
# Properties of Light, Intuitively

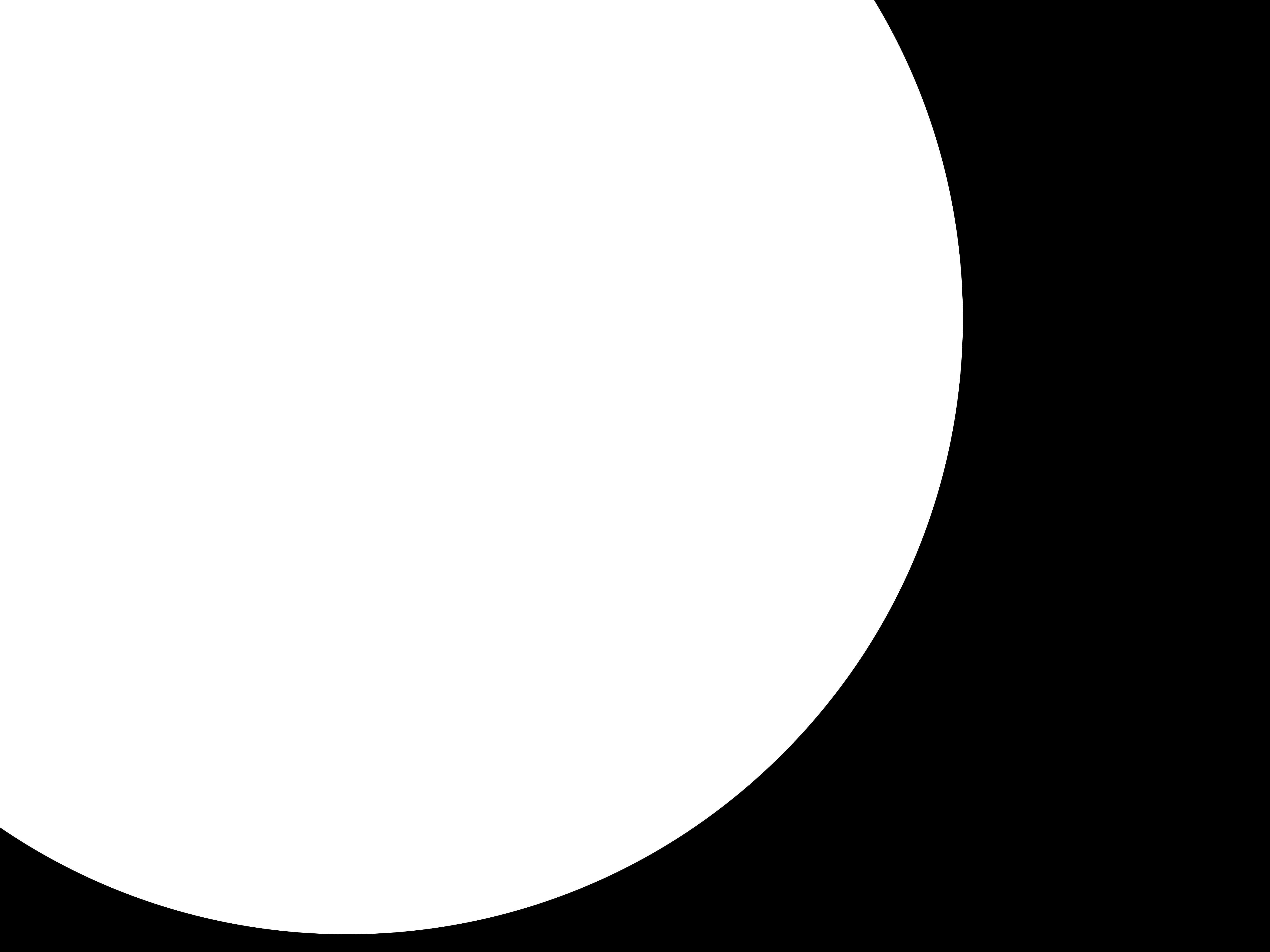
- How "*bright*" something is doesn't directly tell you how brightly it *illuminates* something
  - The lamp appears just as bright from across the room and when you stick your nose to it ("intensity does not attenuate")
  - Also, the lamp's apparent brightness does not change much with the angle of exitance
- **However**
  - if you take the receiving surface further away, it will reflect less light and appear darker
  - If you tilt the receiving surface, it will reflect less light and appear darker



# What’s Going On?

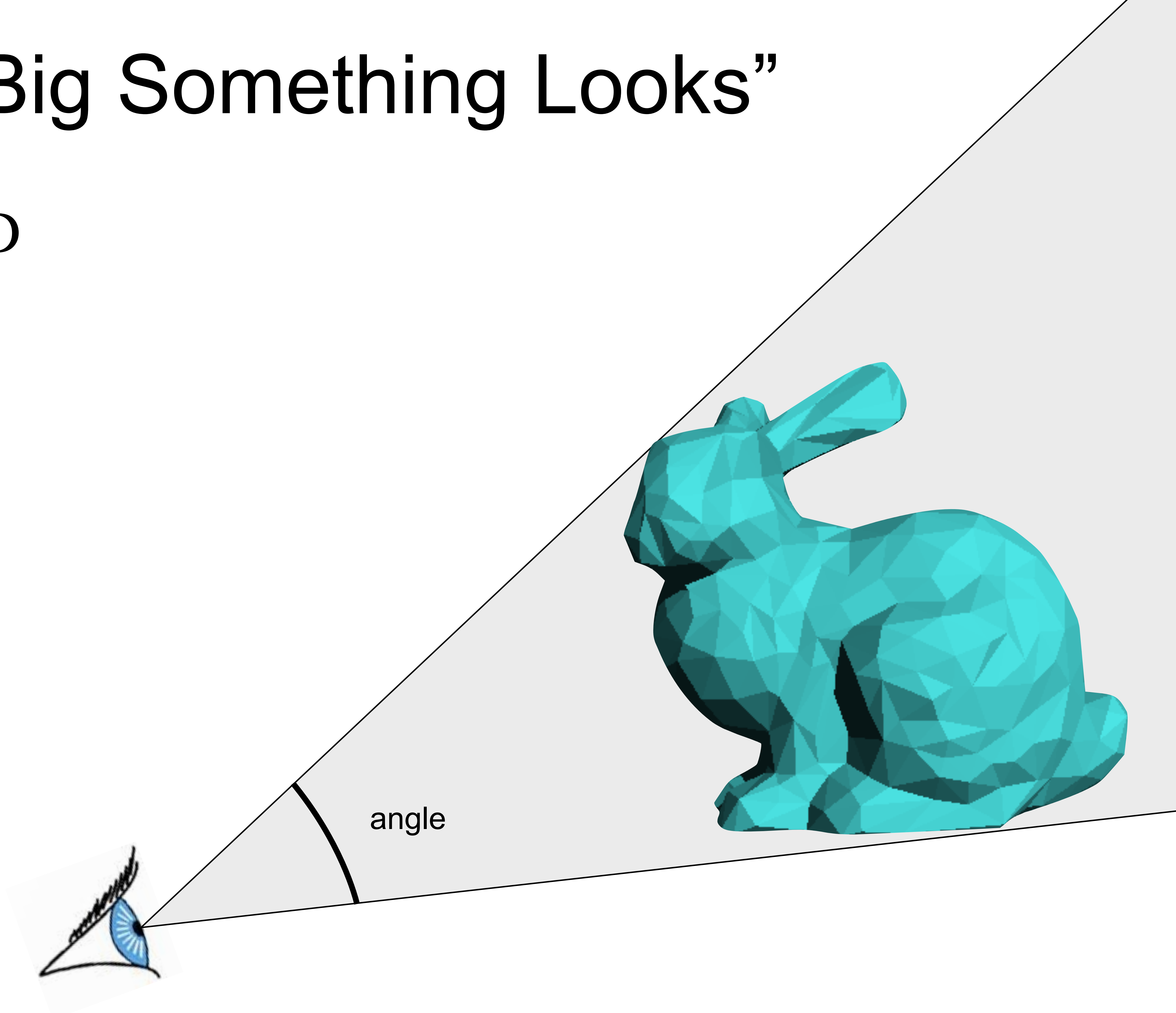
- “Illumination power” determined by **solid angle** subtended by the light source
  - Simple: “how big something looks”
  - Remember this well!
  - (Receiver orientation also has a role: a little later)





# "How Big Something Looks"

- First, 2D



# "How Big Something Looks"

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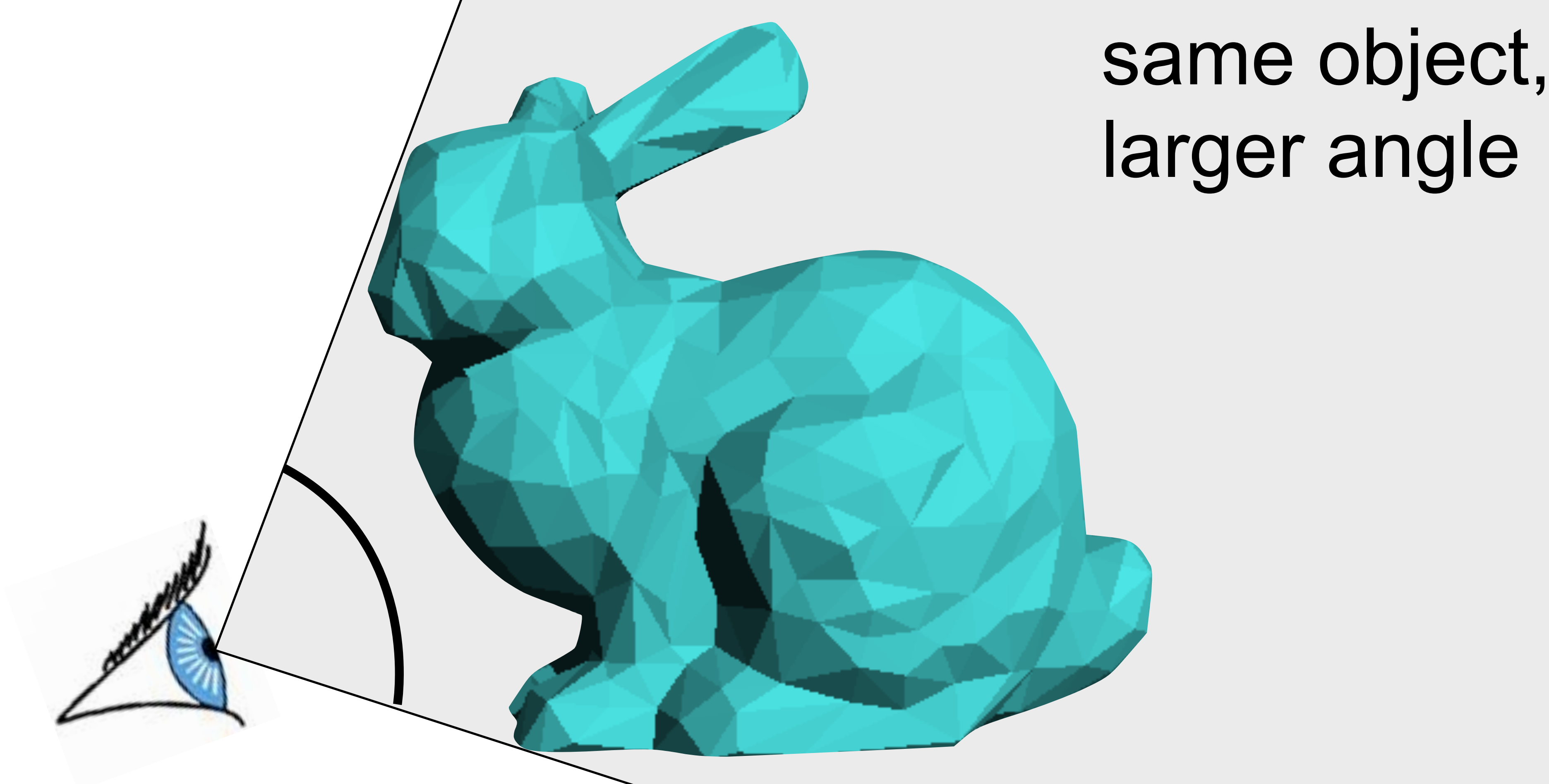
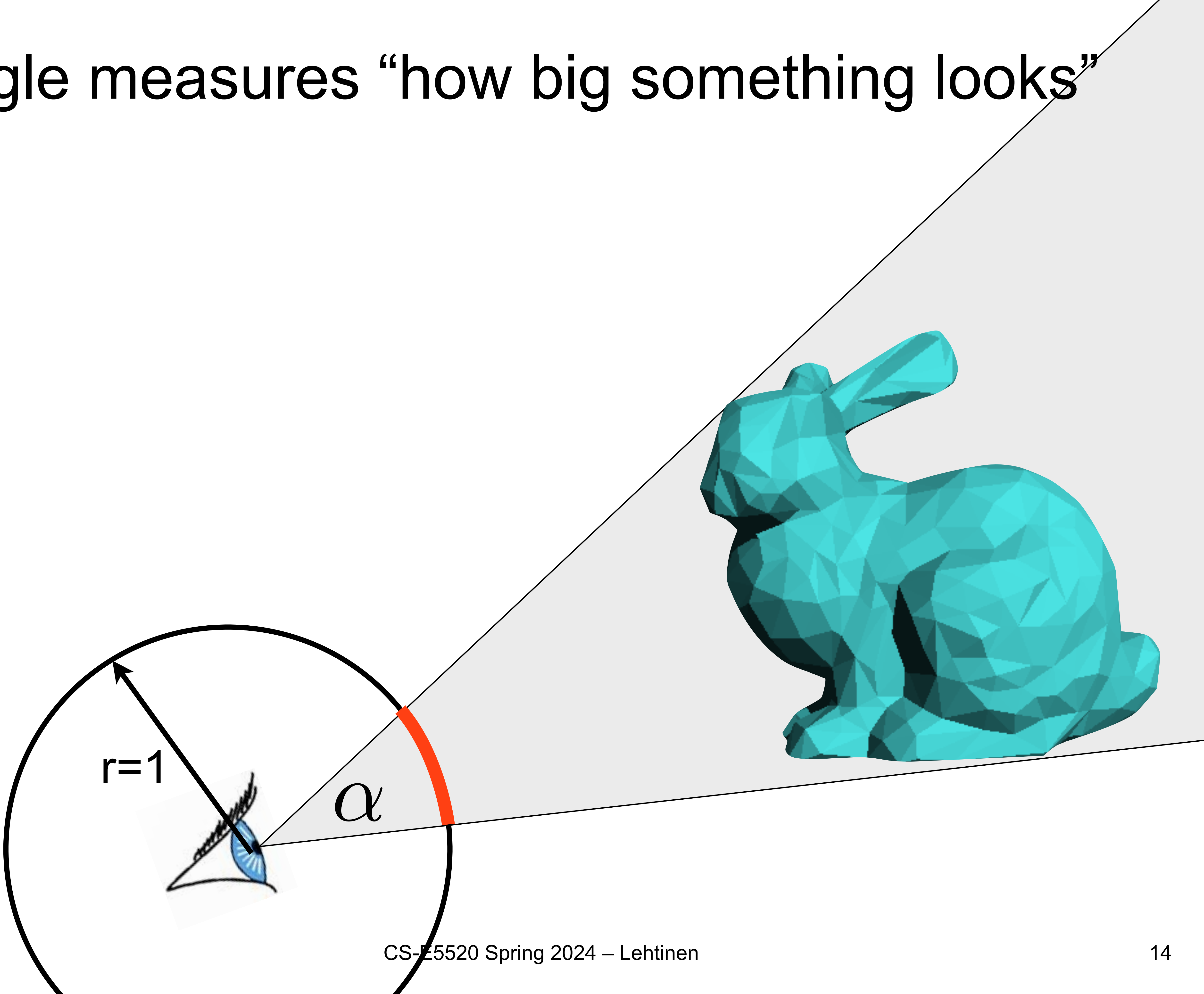
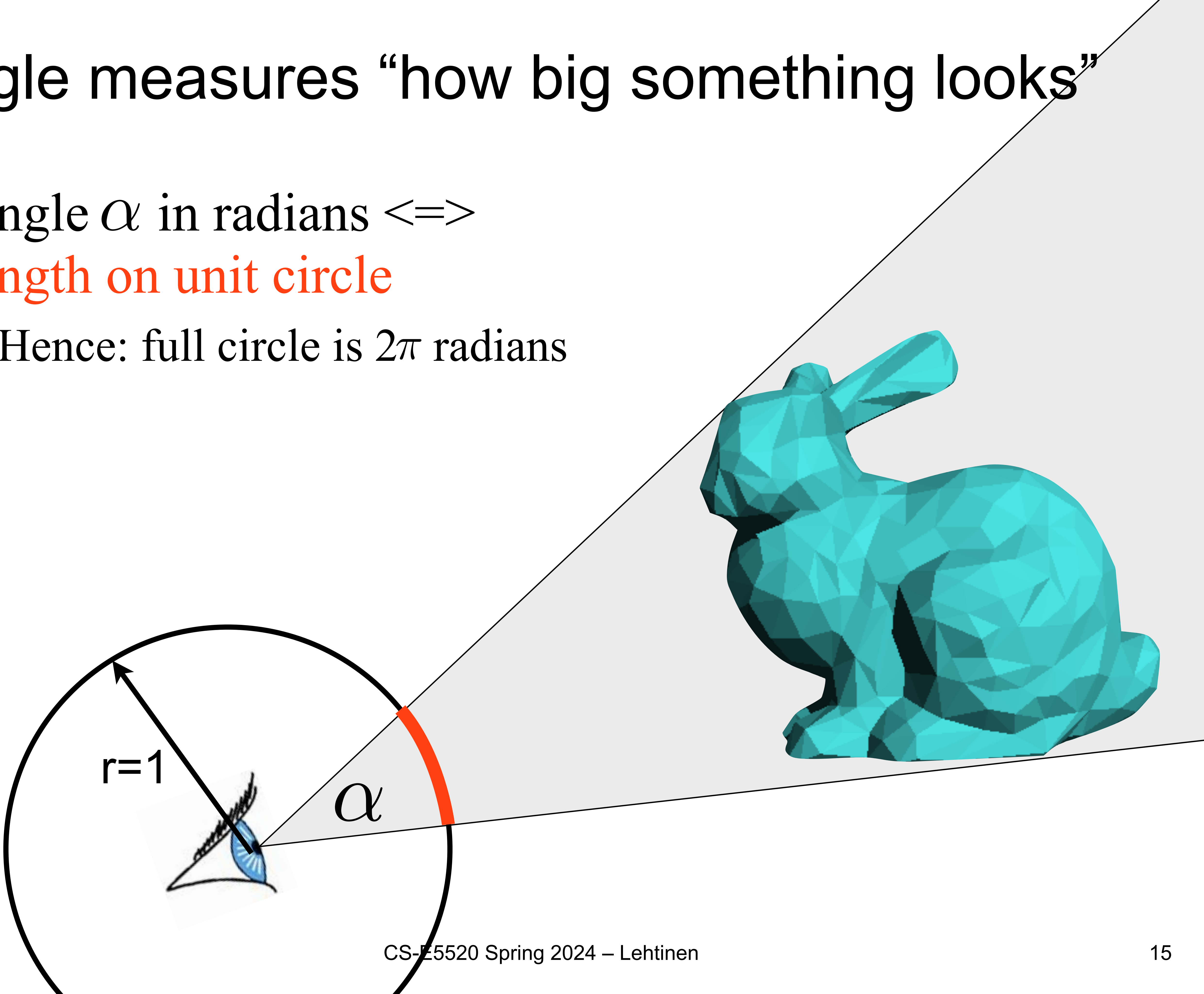


# Angle measures “how big something looks”



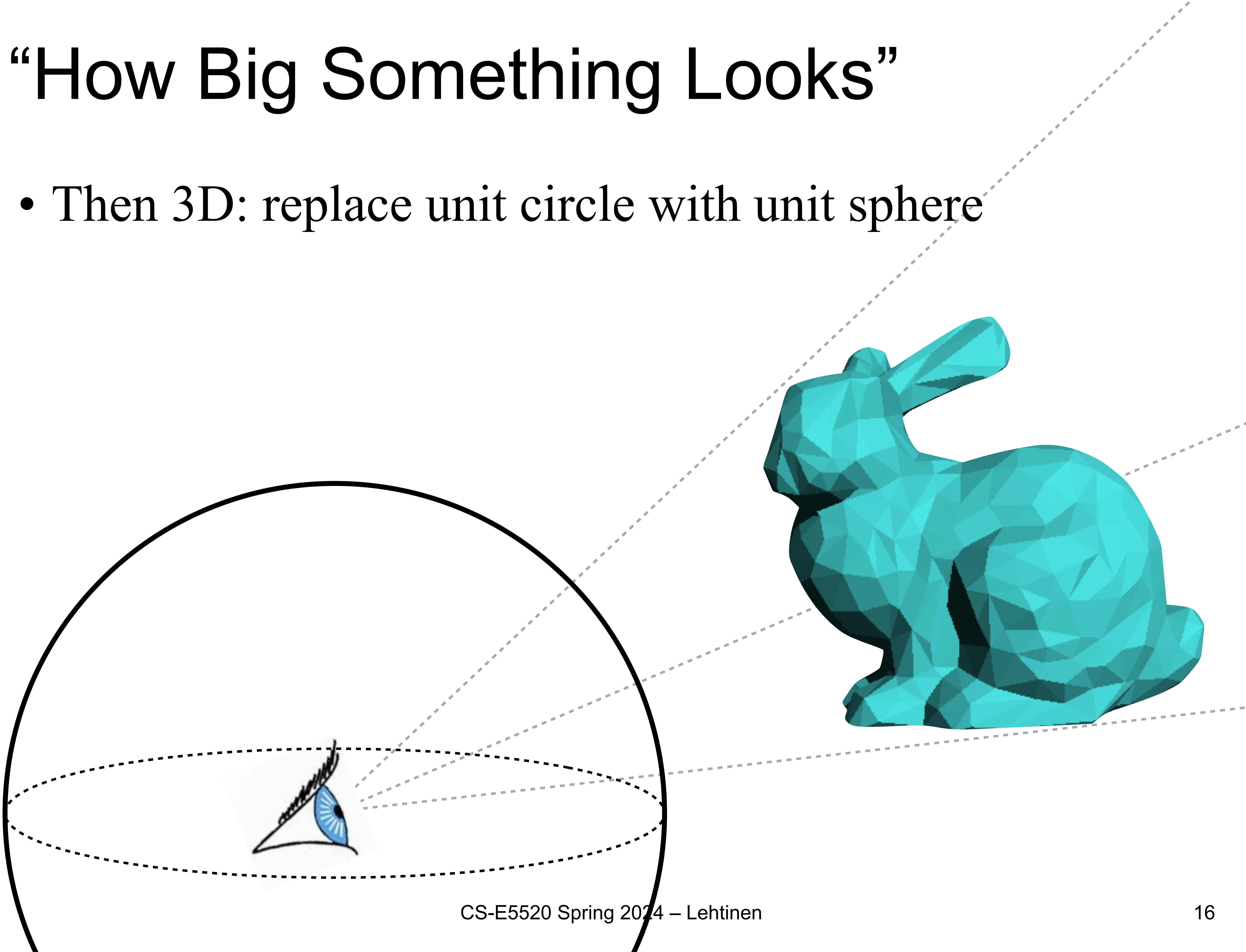
# Angle measures "how big something looks"

- Angle $\alpha$ in radians <=> length on unit circle
  - Hence: full circle is $2\pi$ radians



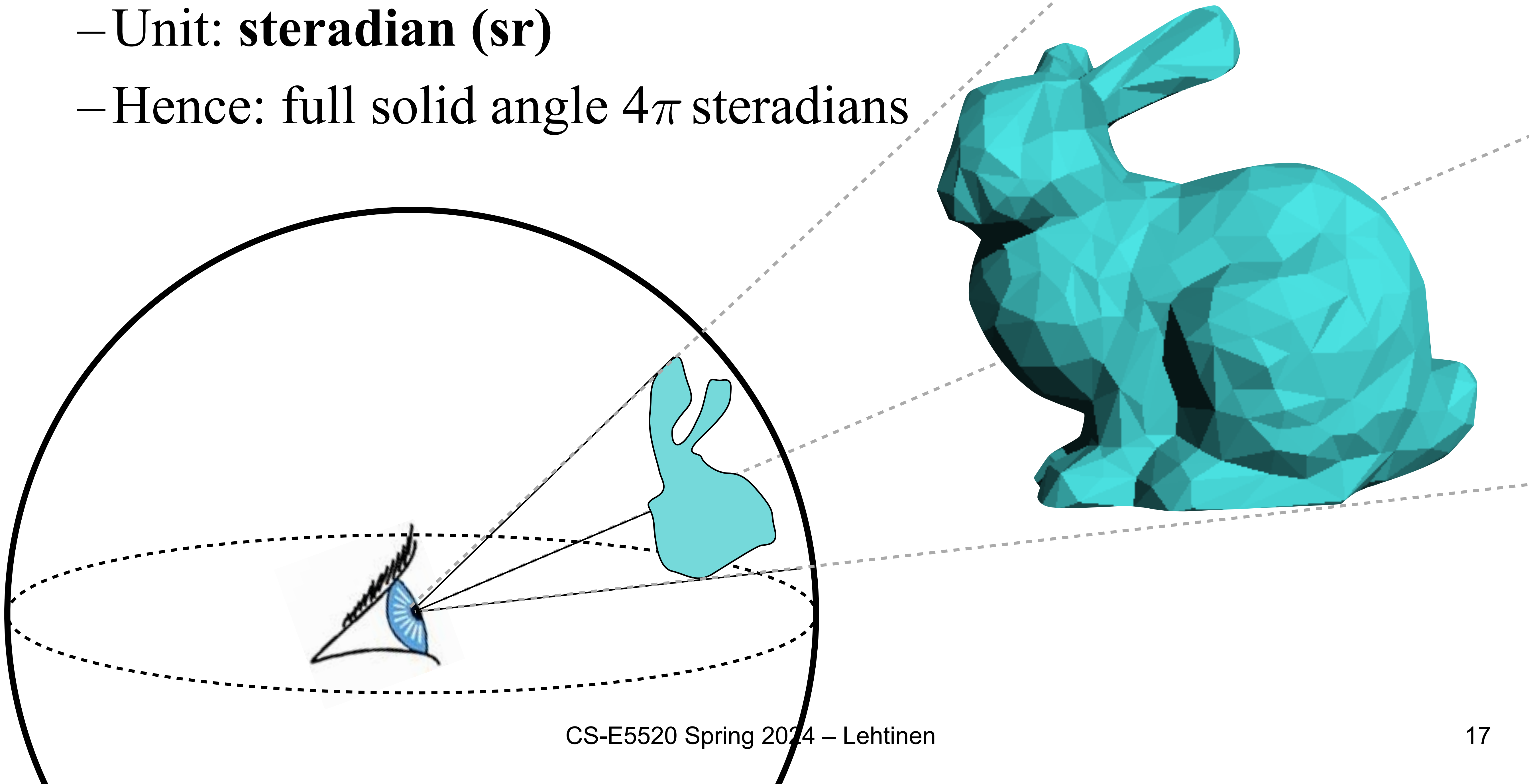
# "How Big Something Looks"

- Then 3D: replace unit circle with unit sphere

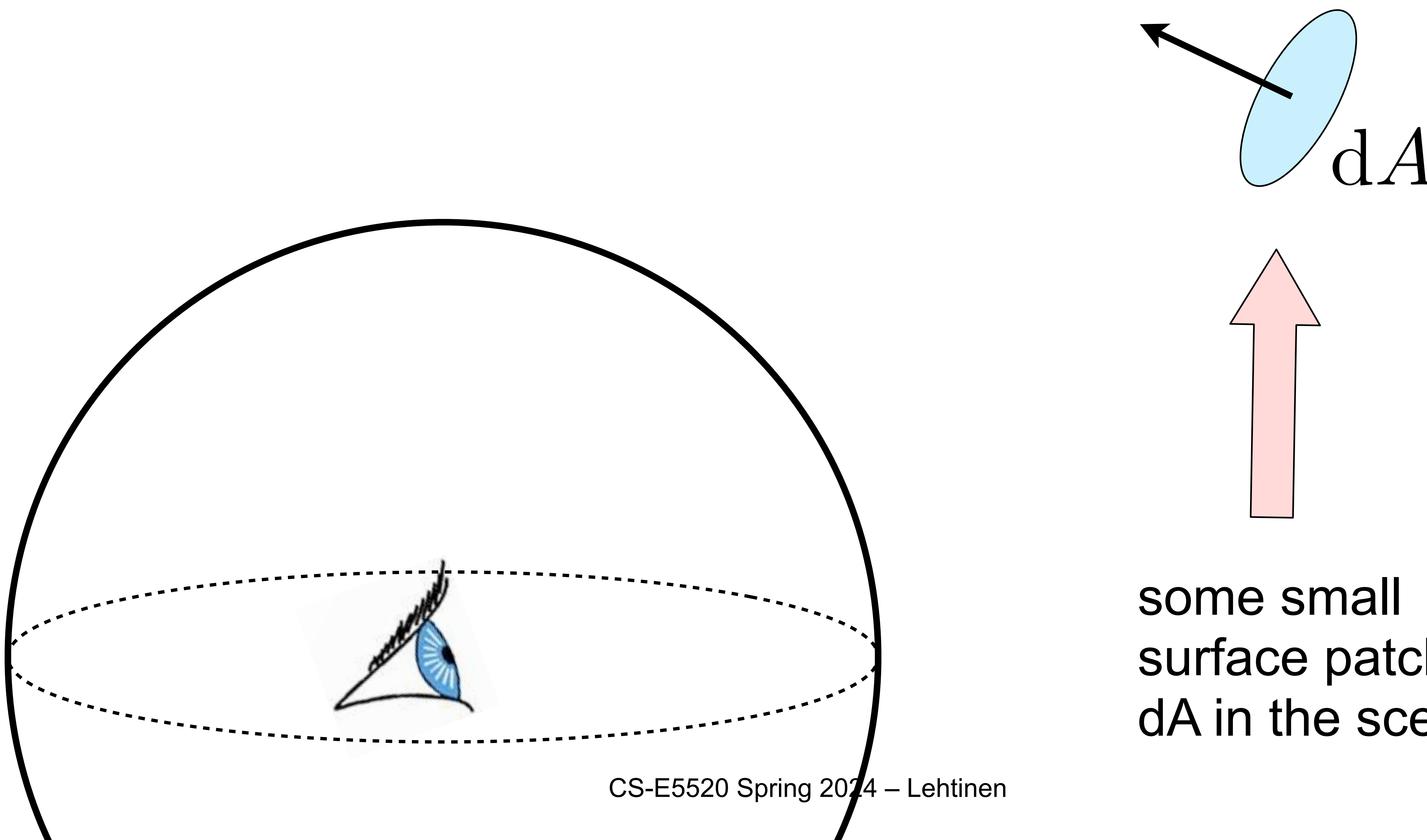


# “How Big Something Looks”

- Then 3D: replace unit circle with unit sphere
  - Same thing: solid angle <=> projected area on unit sphere
  - Unit: **steradian (sr)**
  - Hence: full solid angle $4\pi$ steradians



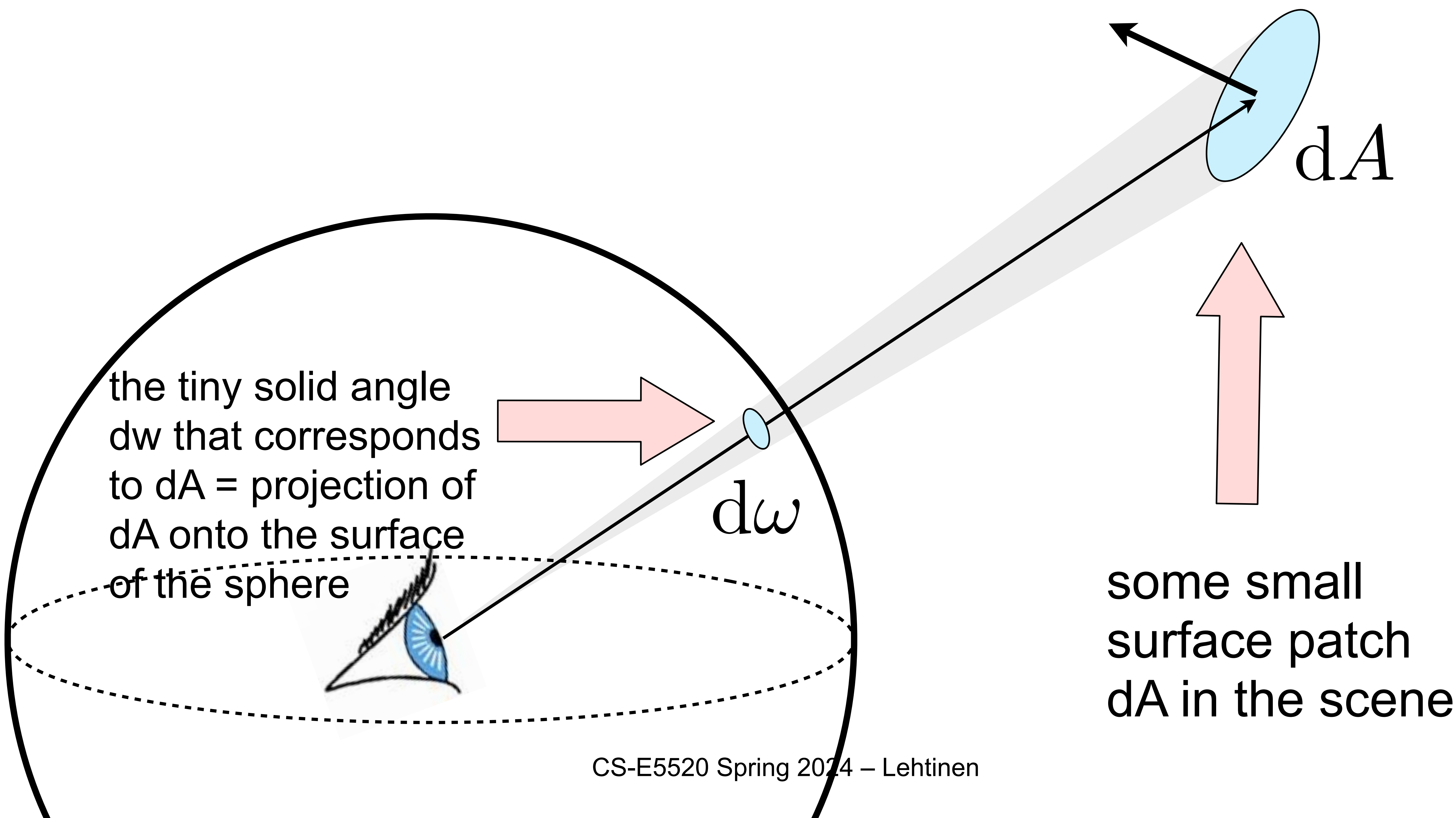
# Relationship of Area and Solid Angle




some small
surface patch
dA in the scene

# Relationship of Area and Solid Angle

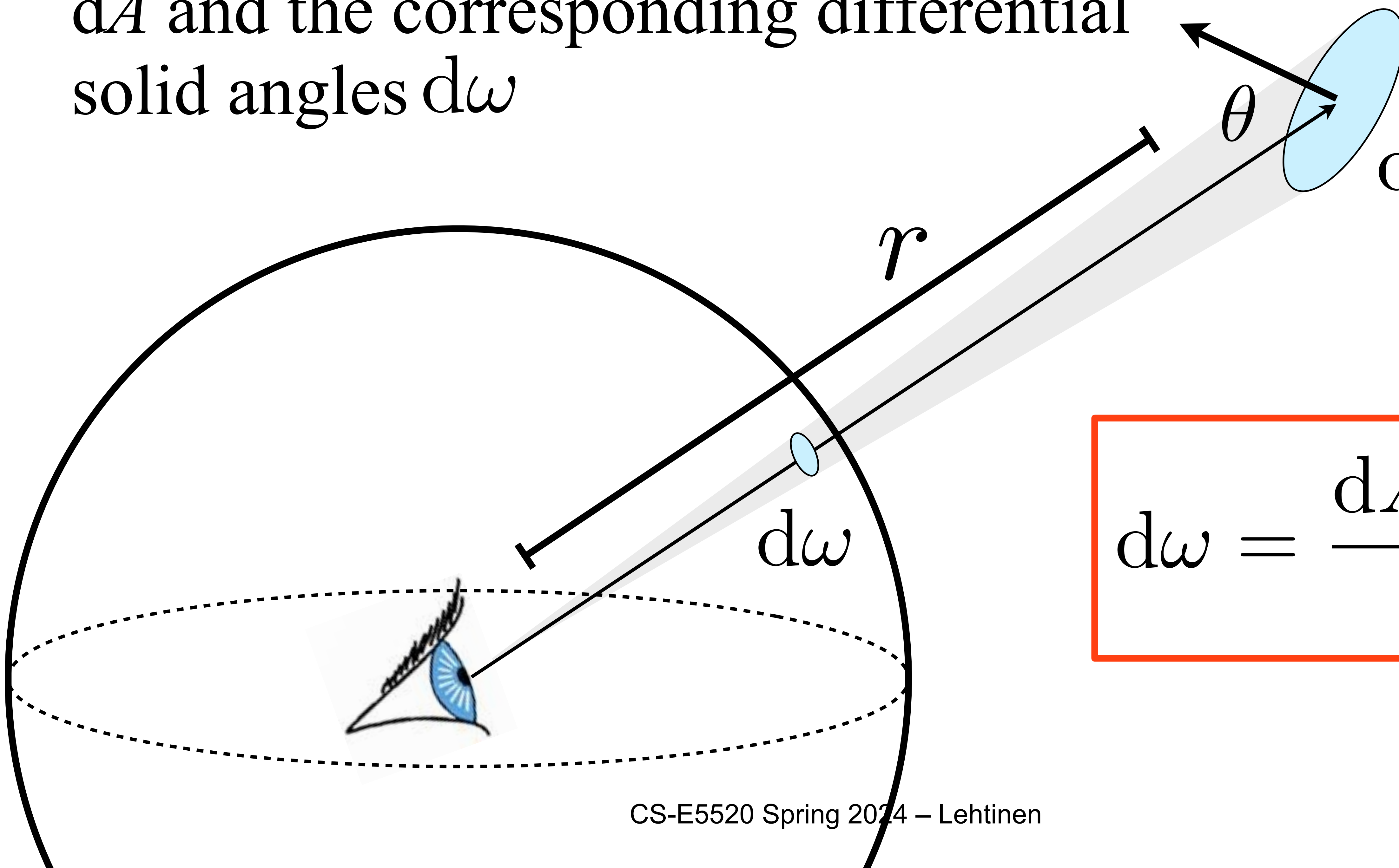
- What determines the area of the projected patch $\mathrm{d}\omega$ ?



# Relationship of Area and Solid Angle

- This simple relationship holds for infinitesimally small surface patches d$A$ and the corresponding differential solid angles $\mathrm{d}\omega$

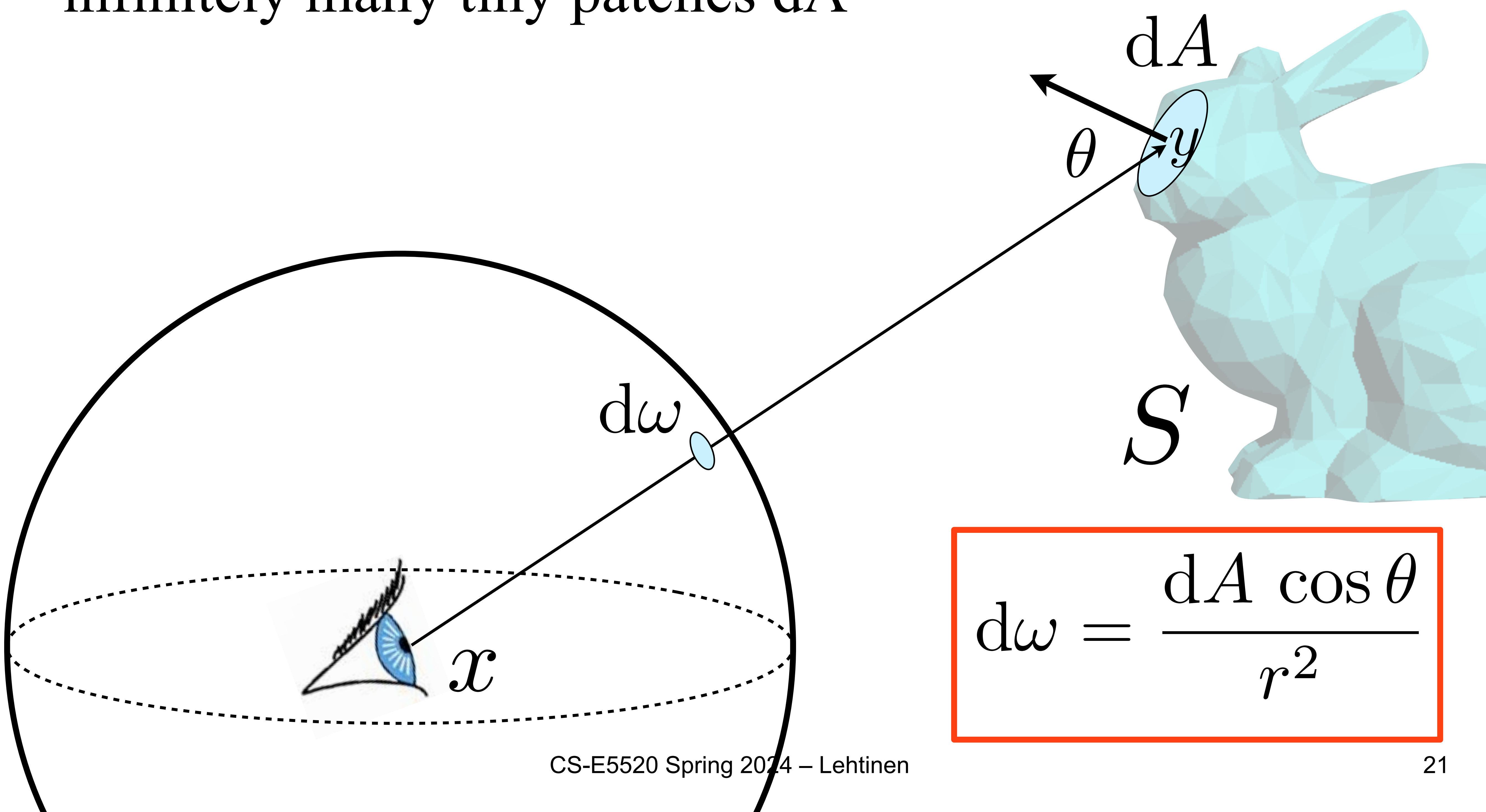
Distance *r*
Angle theta




$$\mathrm{d}\omega = \frac{\mathrm{d}A \cos\theta}{r^2}$$

# Larger Surfaces

- Actual surfaces consist of infinitely many tiny patches dA



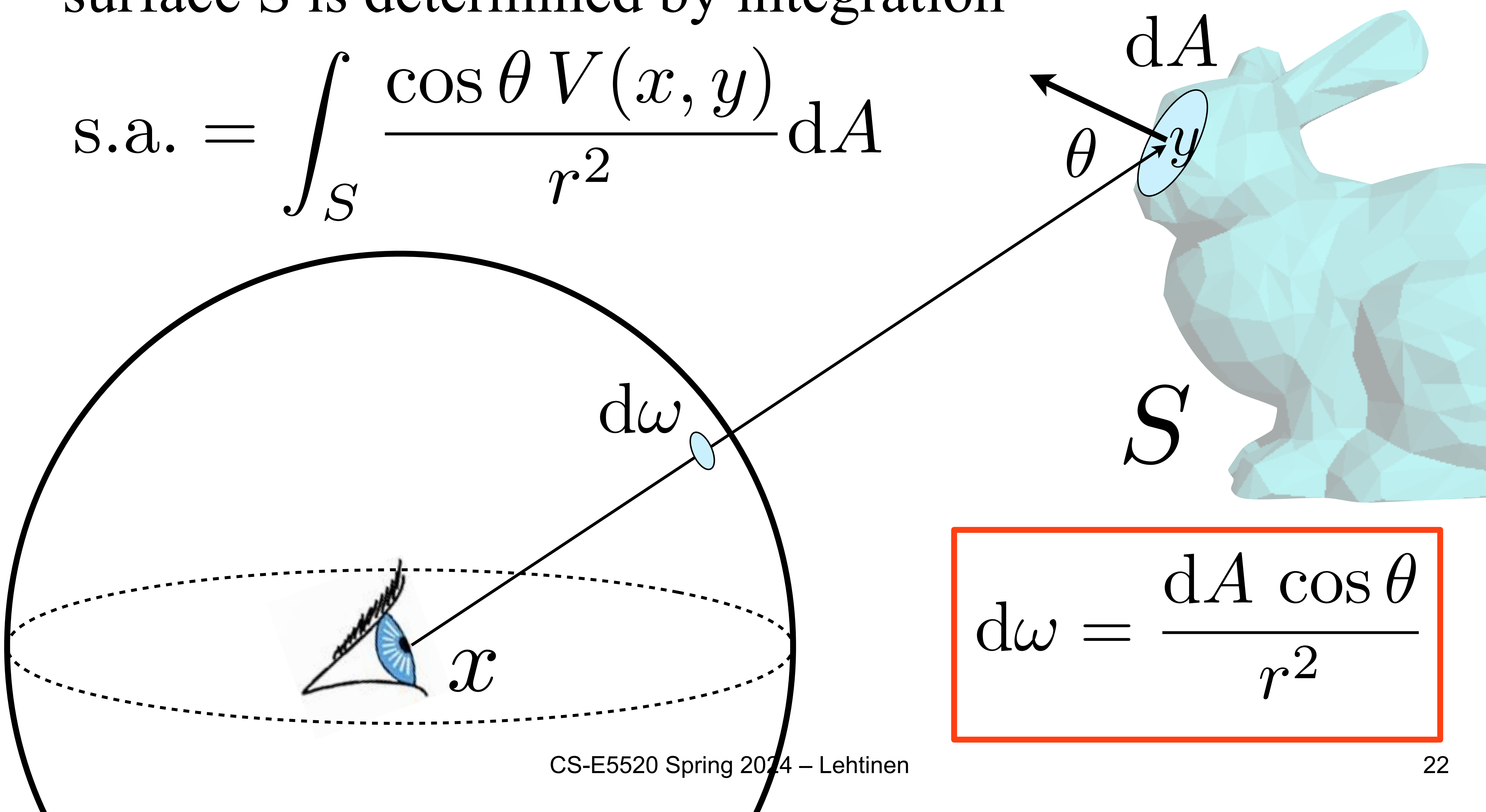

$$d\omega = \frac{dA \cos\theta}{r^2}$$

# Larger Surfaces

V(x,y) = (are x and y visible to each other? 1 : 0)

- Solid angle subtended by actual, non-infinitesimal surface S is determined by integration

$$\text{s.a.} = \int_S \frac{\cos\theta\, V(x,y)}{r^2} \mathrm{d}A$$




$$\mathrm{d}\omega = \frac{\mathrm{d}A \cos\theta}{r^2}$$

# A note on Cosine Terms

$$\text{s.a.} = \int_S \frac{\boxed{\cos\theta}\, V(x,y)}{r^2}\, \mathrm{d}A$$

- We write this, but we mean $\lfloor \cos\theta \rfloor = \max(0, \cos\theta)$
- Why? (When is the cosine negative?)

$$\mathrm{d}\omega = \frac{\mathrm{d}A\, \boxed{\cos\theta}}{r^2}$$

# A note on Cosine Terms

$$\text{s.a.} = \int_S \frac{\boxed{\cos\theta} V(x,y)}{r^2} \mathrm{d}A$$

- We write this, but we mean $\lfloor \cos\theta \rfloor = \max(0, \cos\theta)$
- Why? In order not to subtract light from below horizon

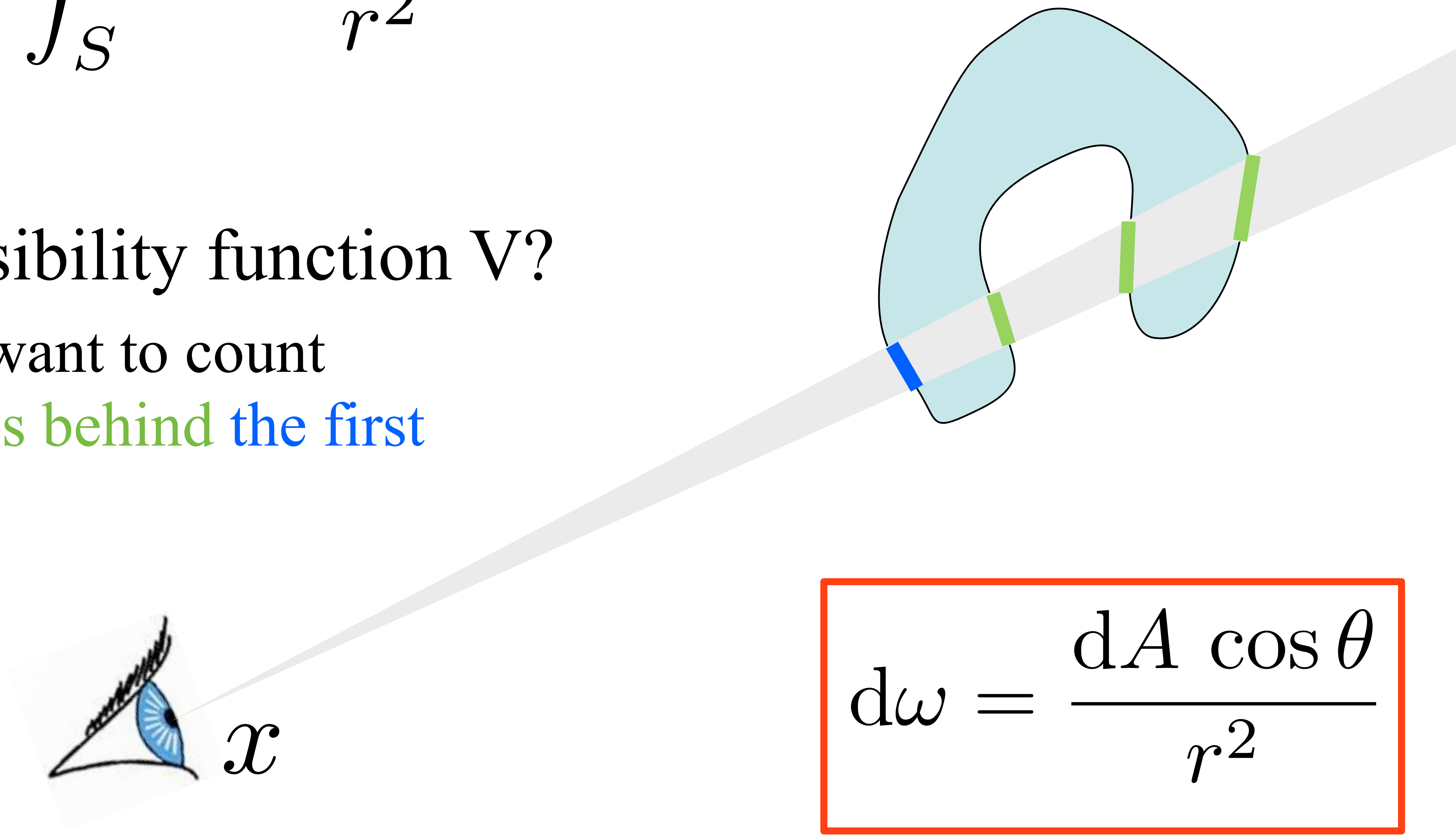
$$\mathrm{d}\omega = \frac{\mathrm{d}A \boxed{\cos\theta}}{r^2}$$

# Larger Surfaces

V(x,y) = (are x and y visible to each other? 1 : 0)

$$\text{s.a.} = \int_S \frac{\cos\theta \, V(x,y)}{r^2} \mathrm{d}A$$

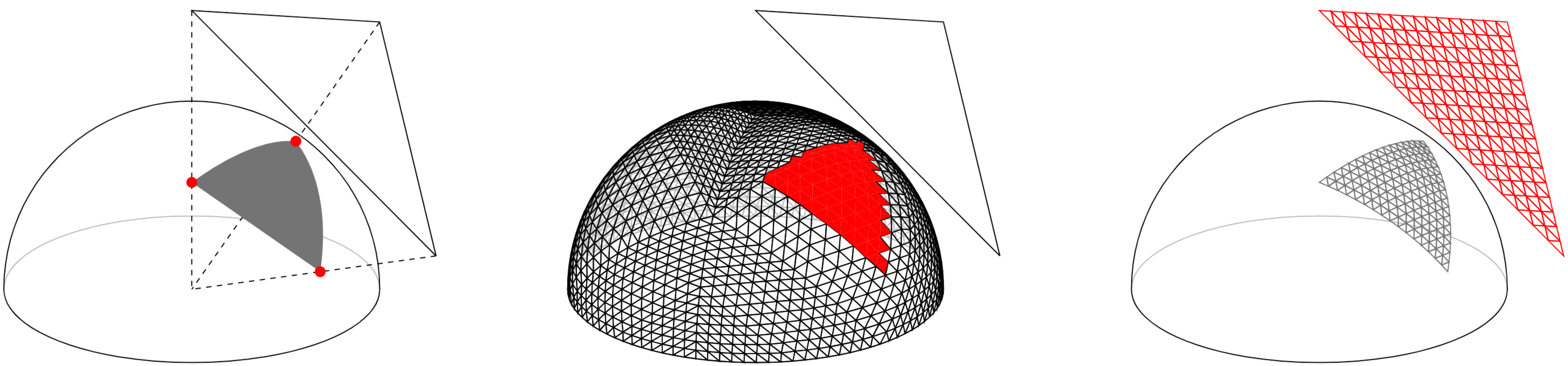
- Why visibility function V?
  - Don't want to count surfaces behind the first




$$\mathrm{d}\omega = \frac{\mathrm{d}A \cos\theta}{r^2}$$

# Visualisation by Pauli Kemppinen

- Compares different ways of integrating same thing



s.a.(Triangle)
(from the spherical excess of angles)

Direct evaluation: 0.3480304171399027sr

∫V(w) dw
(integral over hemisphere area)

Hemisphere discretization: 0.3585889439183479sr

Subdivide

Reset subdivision

∫V(A) cosθ/r² dA
(integral over triangle area)

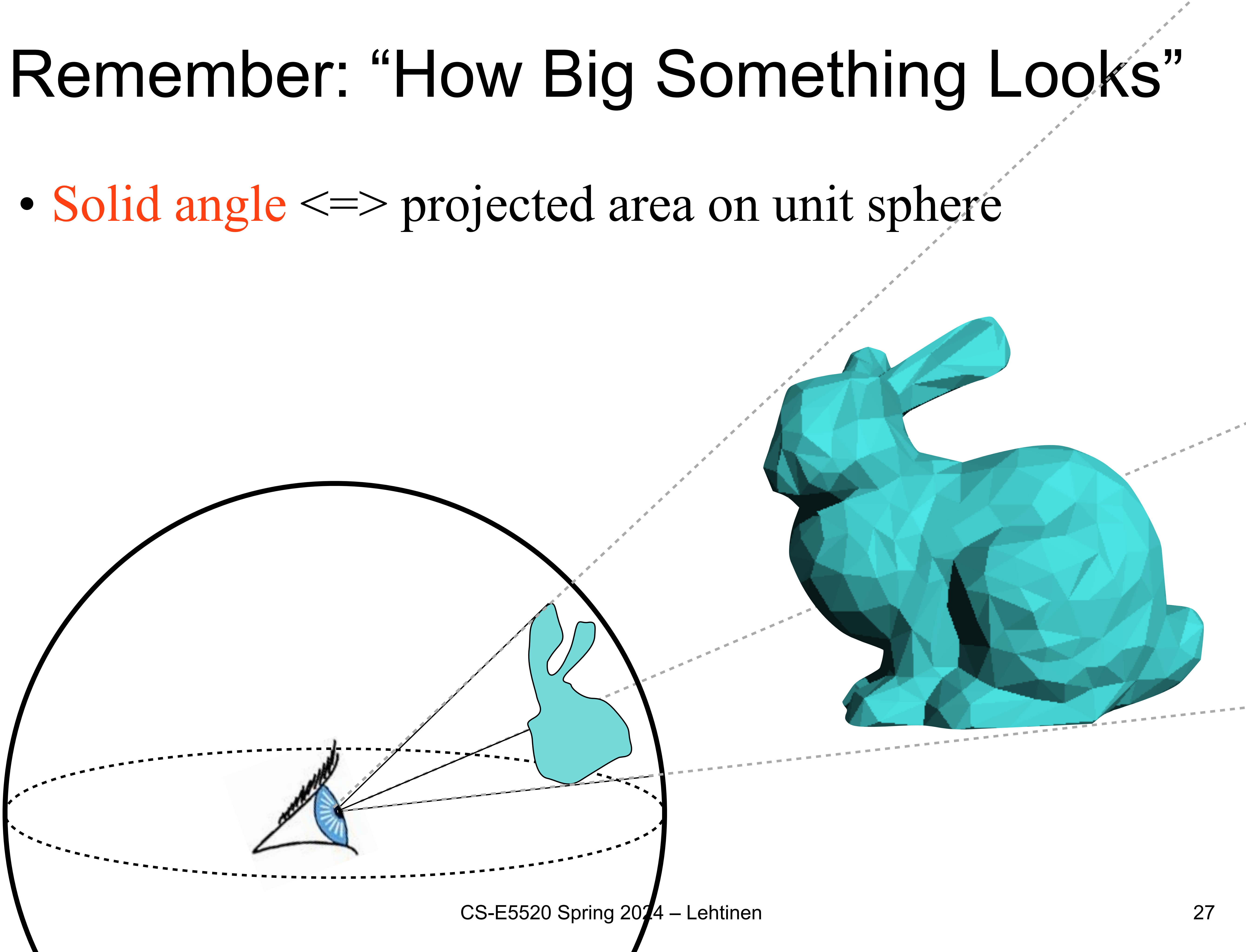
Area discretization: 0.34811299516831434sr

Subdivide

Reset subdivision

# Remember: “How Big Something Looks”

- Solid angle <=> projected area on unit sphere



# Don't be Scared of Integrals

- Think of Riemann sums from high school. Intuition:
  1. break the surface down into many, many tiny patches $A_i$
  2. evaluate integrand $f$ at a point $\boldsymbol{x}_i$ within each patch: $f(\boldsymbol{x}_i)$
  3. multiply by the area $\Delta A_i$ and then sum over all patches:

$$\sum_i f(\boldsymbol{x}_i)\,\Delta A_i$$

- Same holds for integrals over solid angle: they are just integrals over the surface of the sphere, that's all
  - Same logic applies: break sphere surface down to many tiny patches, sum them up

# Area Integrals as Riemann Sums

- break the surface down into many, many tiny patches, evaluate the integrand, multiply by the area $\Delta A$, and then sum over all patches

$$\text{s.a.} = \int_S \frac{\lfloor \cos\theta \rfloor \; V(x,y)}{r^2} \mathrm{d}A$$

$$= \lim_{N \to \infty} \sum_{i=1}^{N} \frac{\lfloor \cos\theta \rfloor \; V(x,y)}{r^2} \Delta A$$

# OK, Let's Explain the Intuition

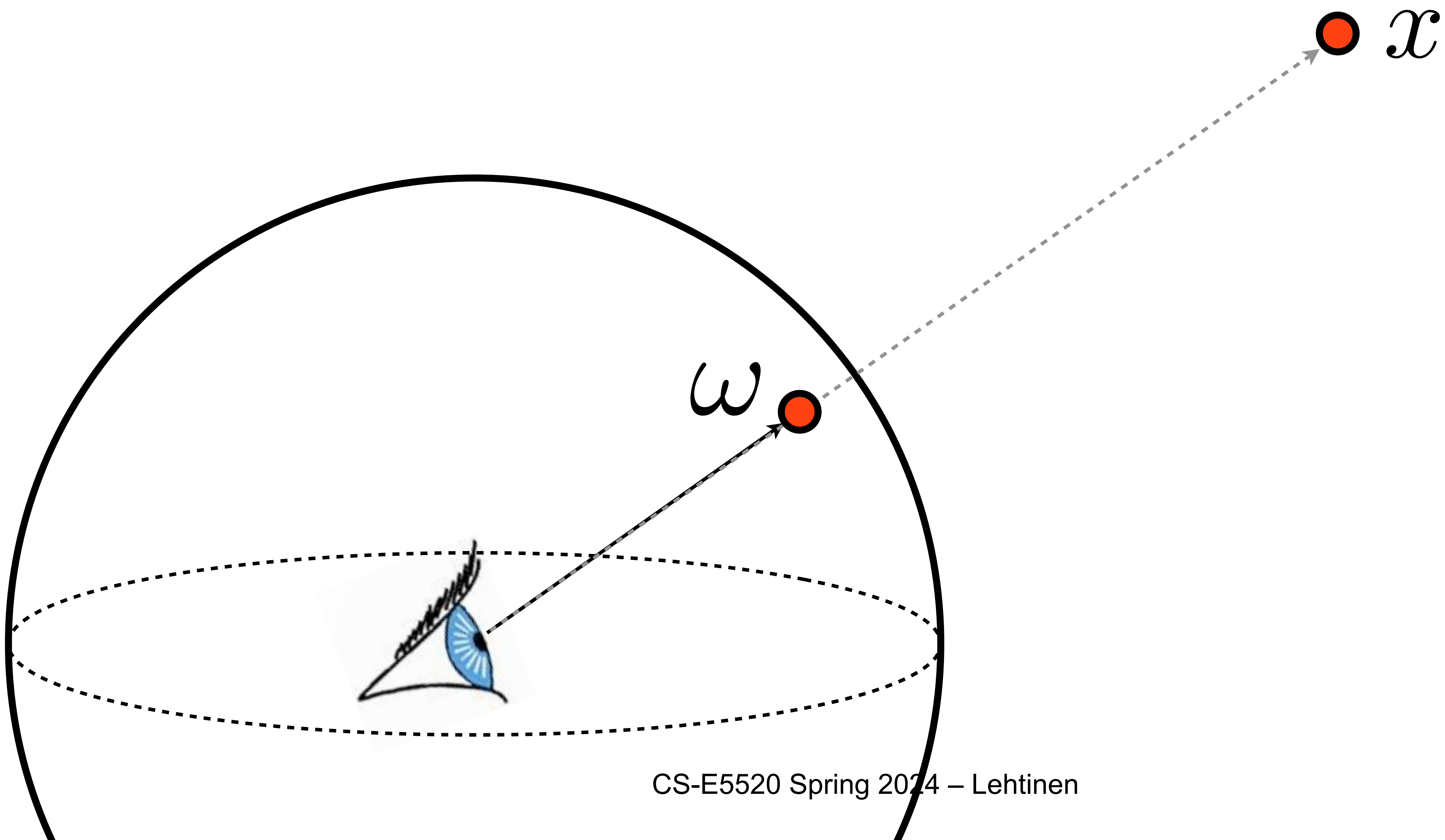

- Take the lamp further away
  => **solid angle decreases**
  => illumination less powerful

- Tilt the lamp away from yourself
  => **solid angle decreases**
  => illumination less powerful

- But all the time, the points on the lamp are ~constant "brightness"!

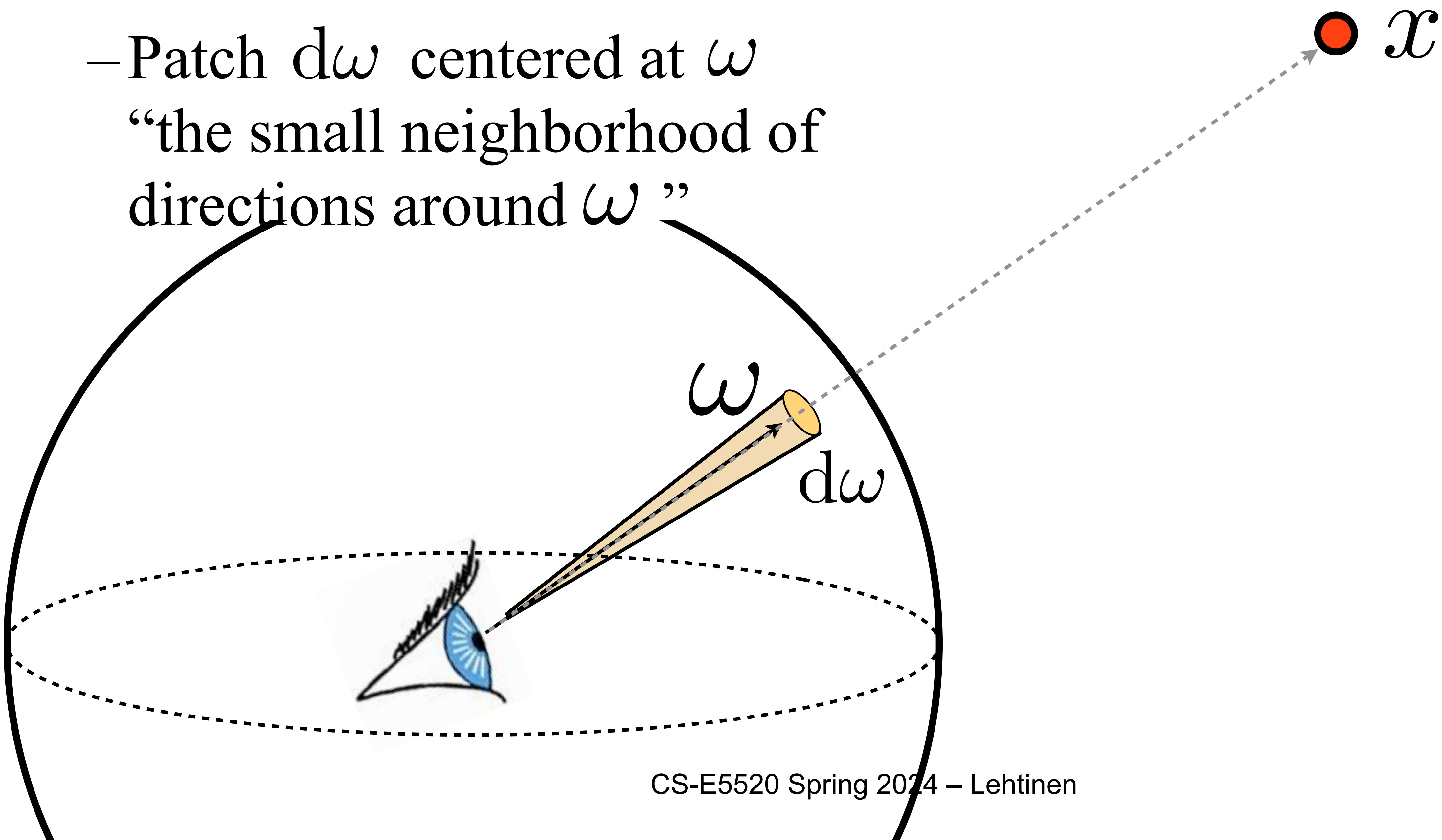
# Points on Sphere also Encode Direction

- Point on unit sphere <=> direction
  - Just as with usual angles in the plane
  - "Point x is in direction $\omega$"



# Points on Sphere also Encode Direction

- Point on unit sphere <=> direction
  - Just as with usual angles in the plane
  - "Point x is in direction $\omega$"
  - Patch $\mathrm{d}\omega$ centered at $\omega$ "the small neighborhood of directions around $\omega$"



# Questions?

J. Lehtinen

# How to Measure Light?

- Geometric optics assumes light energy is a continuum defined over continuous area and angle measurements
  - Basically: how much "stuff" flows in a certain area and direction

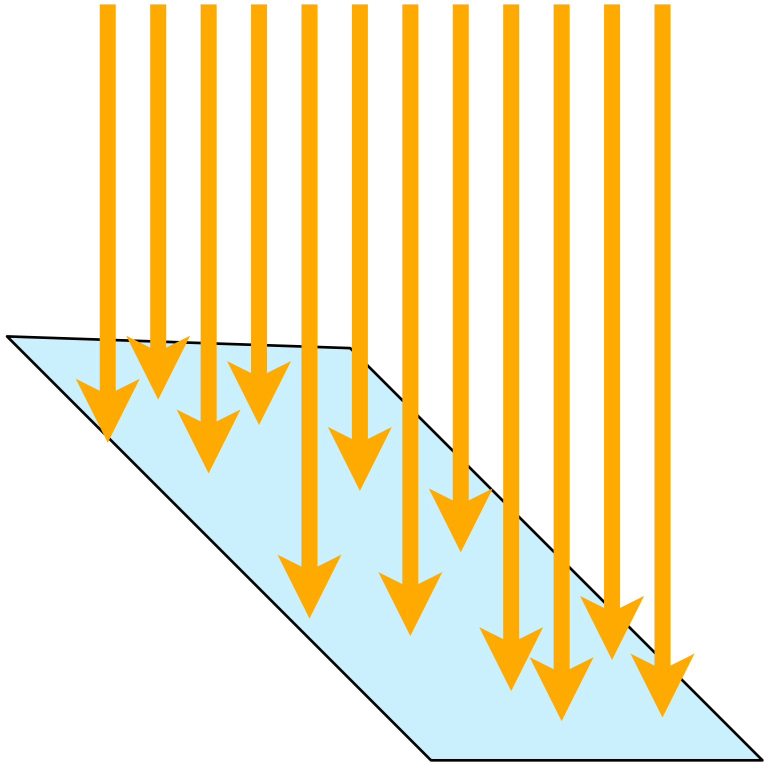
- Not incompatible with photons
  - We can think of measuring how many photons land on a small surface from a tiny set of directions in a second
  - Each photon carries some constant energy (dependending on its wavelength), so [photons/second] <=> [J/s] = [W]
  - Power carried by light is called **flux**, denoted $\Phi$

# A Little More Formally: Irradiance

- **Irradiance** E is the flux Φ [W] per unit area [1/m²] landing on a surface

$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A} \qquad \left[\frac{W}{m^2}\right]$$

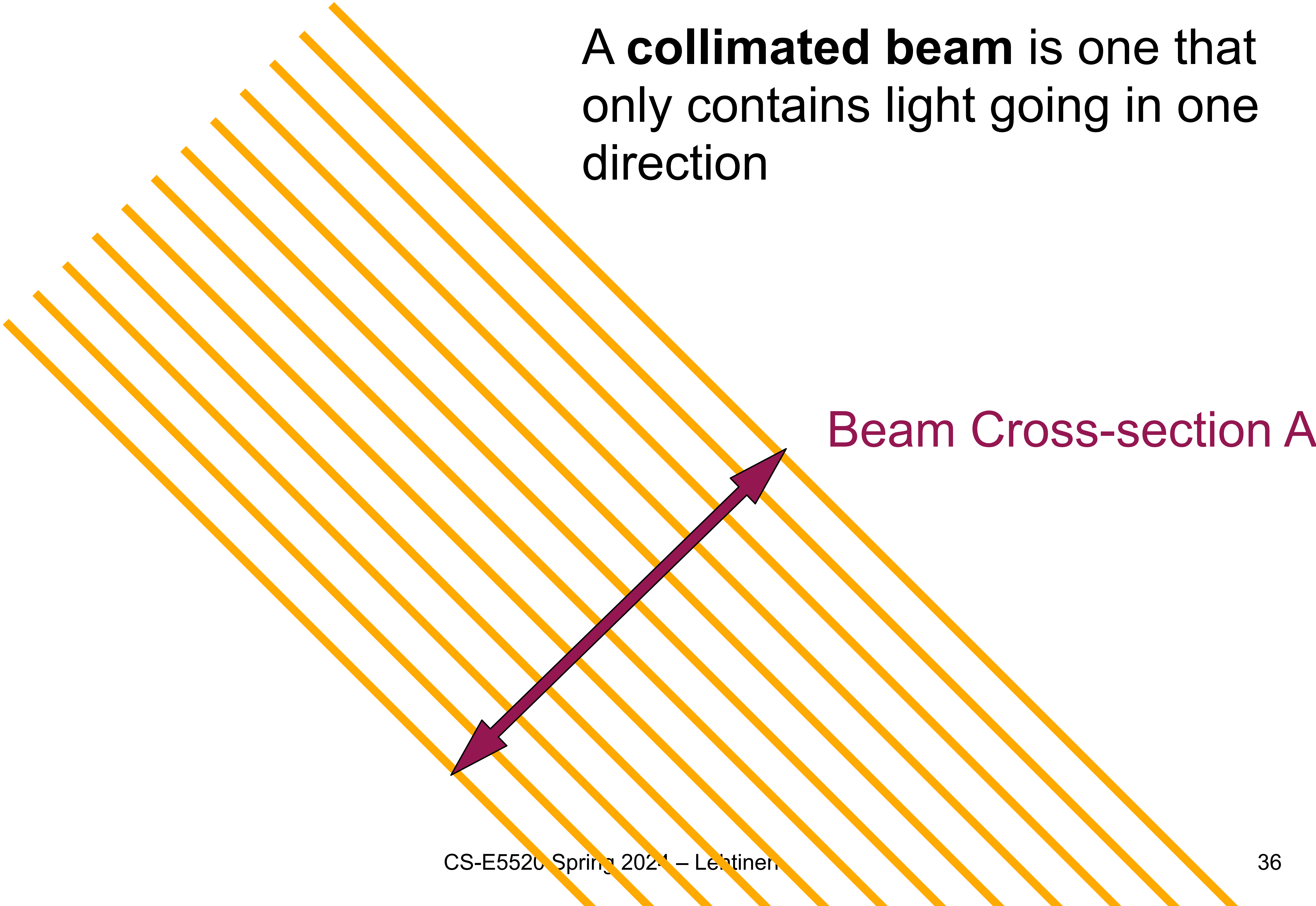
  – You can really think of counting photons



- (Brightness of diffuse surface determined directly by irradiance)
  – (We'll come to this in a bit)

# Beam Power

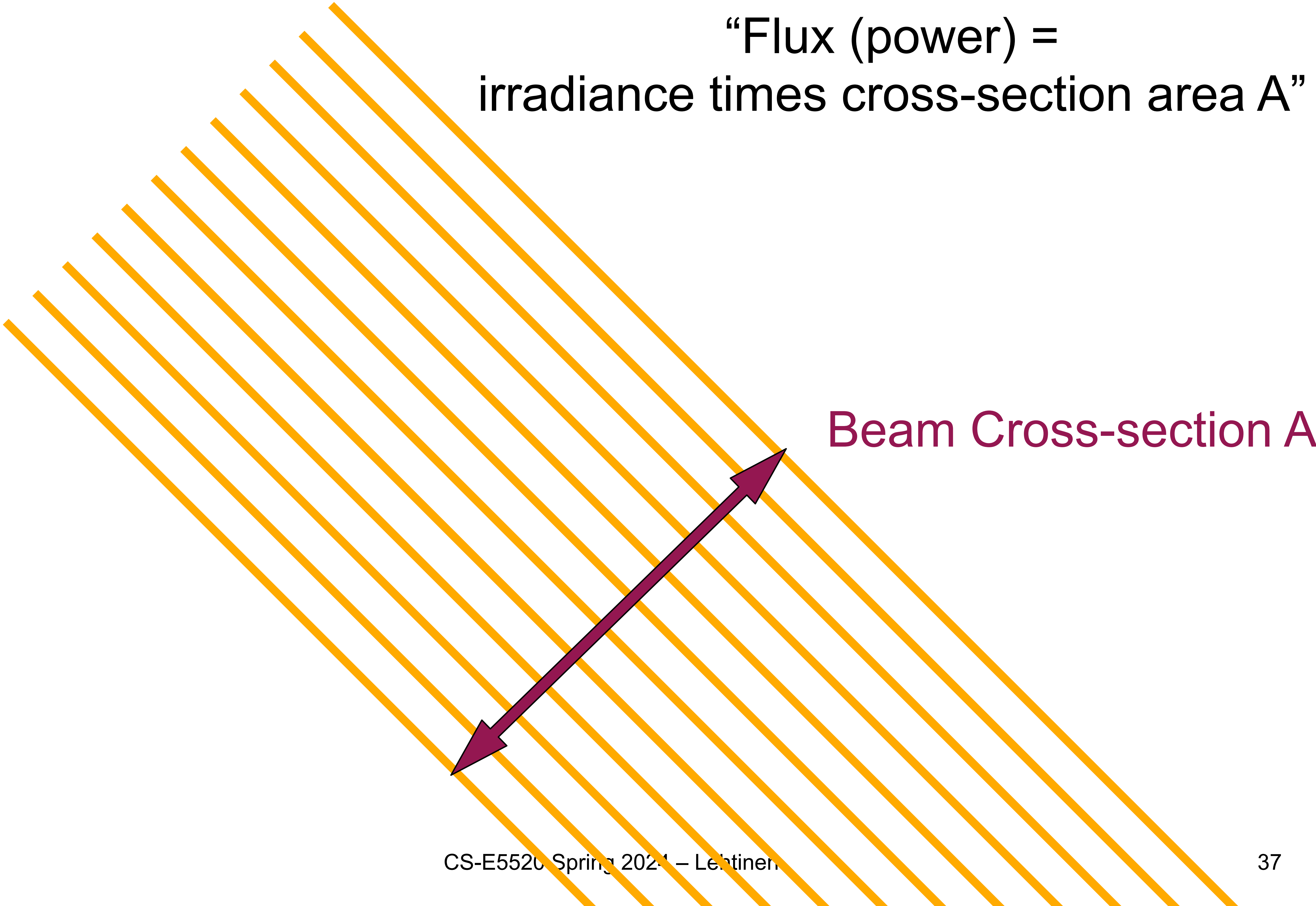
A **collimated beam** is one that only contains light going in one direction



# Beam Power

$$\Phi = EA$$

"Flux (power) =
irradiance times cross-section area A"

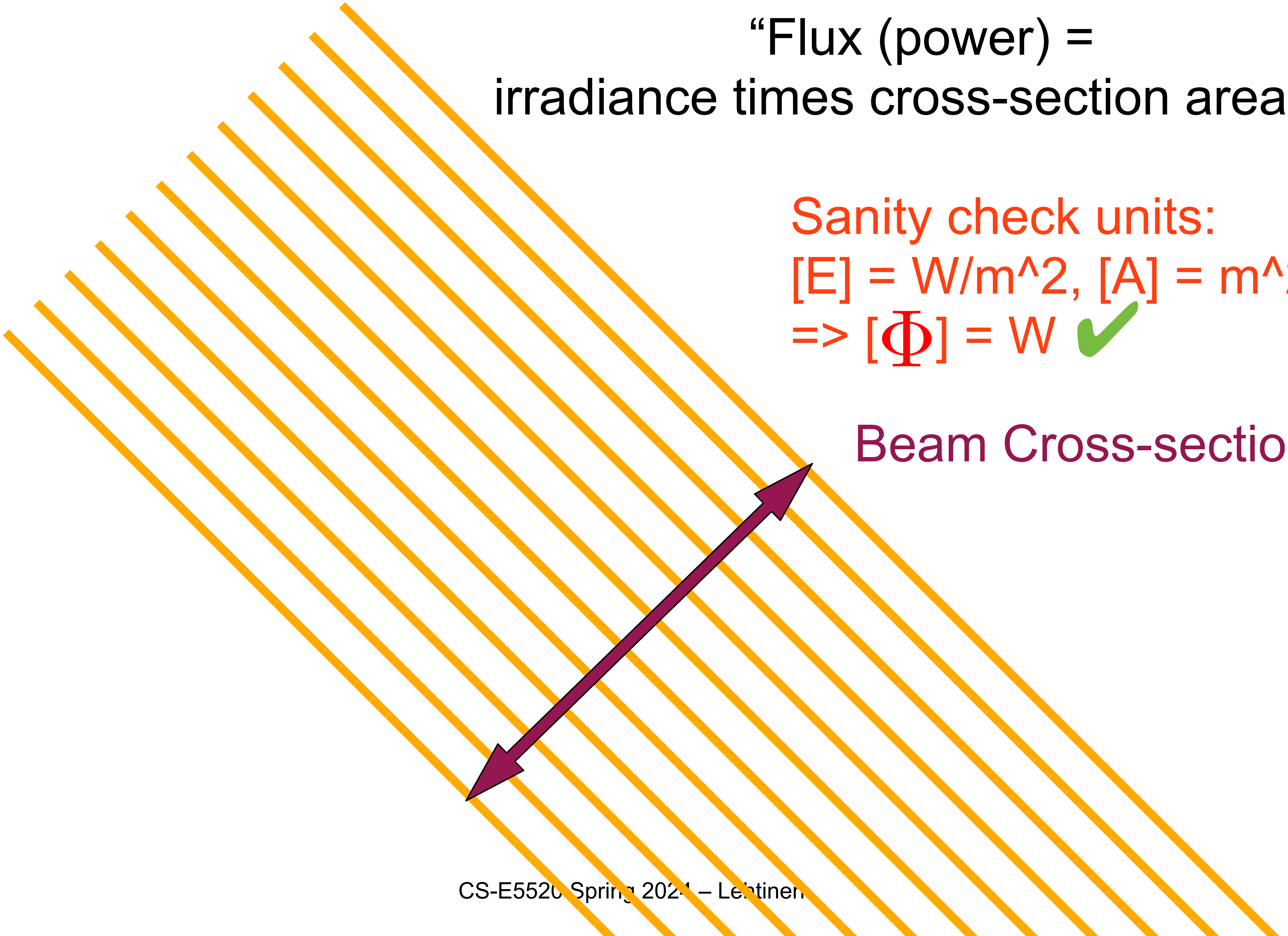


# Beam Power

$$\Phi = EA$$

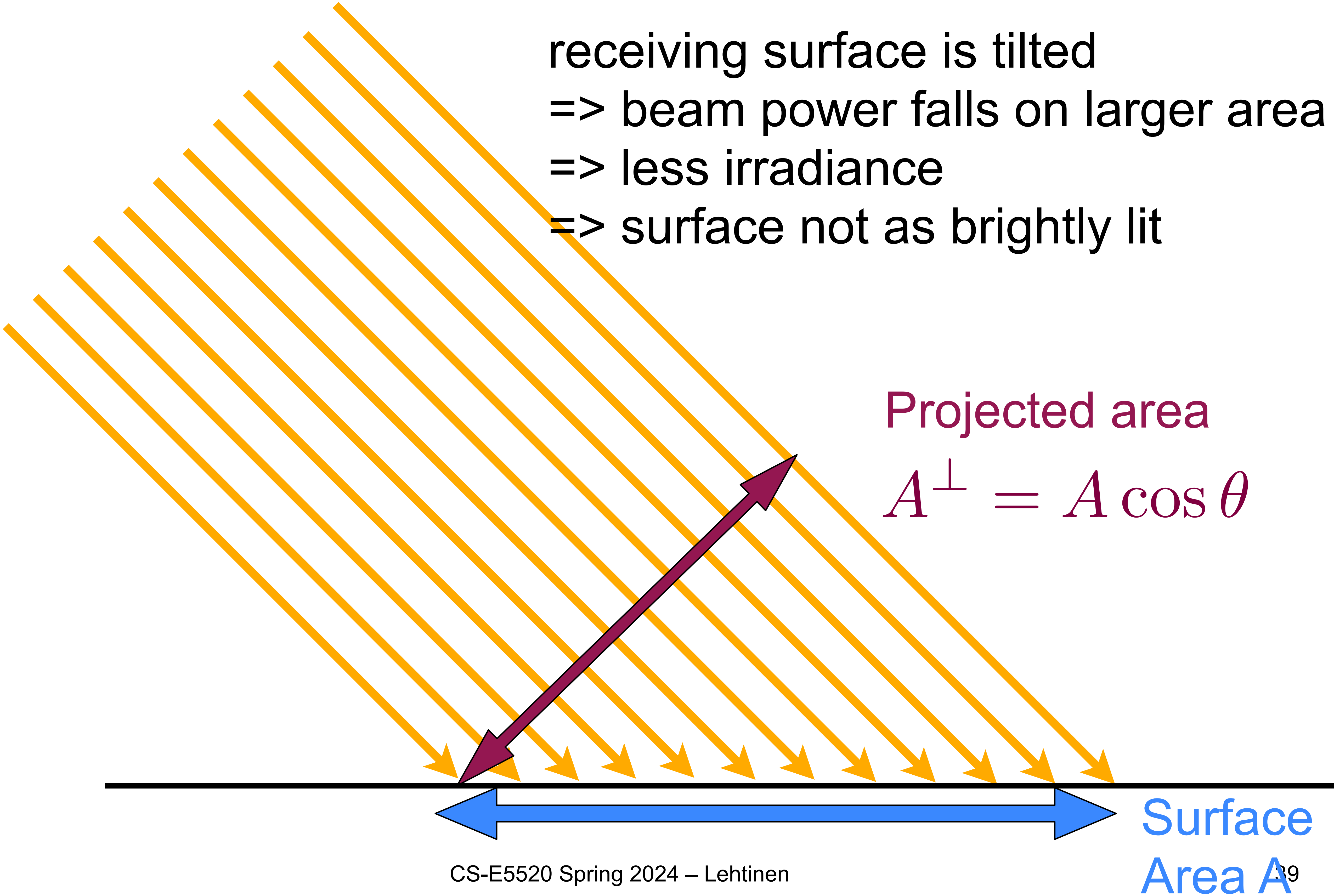
"Flux (power) =
irradiance times cross-section area A"

Sanity check units:
[E] = W/m^2, [A] = m^2
=> [$\Phi$] = W ✔

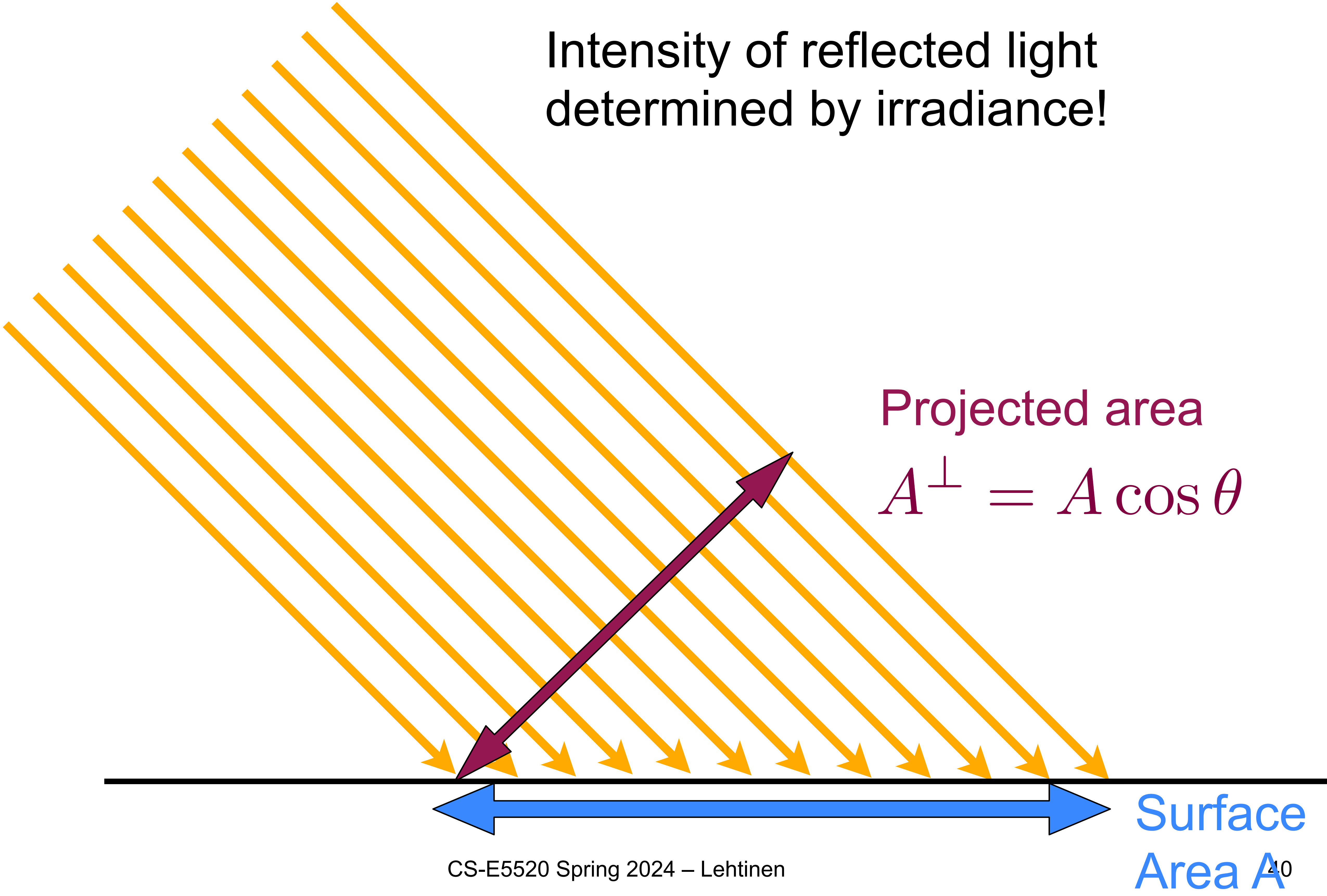


# Projected Area and Irradiance

receiving surface is tilted
=> beam power falls on larger area
=> less irradiance
=> surface not as brightly lit



# Projected Area and Irradiance



# That's Not the Whole Story

- Clearly, light is rarely collimated
- Clearly, there is light everywhere going to every direction

# Radiance

- **Radiance** is the fundamental quantity that simultaneously explains effects of both light source size and receiver orientation

# Radiance

- Let's consider a tiny almost-collimated beam of cross-section $\mathrm{d}A^{\perp} = \mathrm{d}A\cos\theta$ where the directions are all within a differential angle $\mathrm{d}\omega$ of each other

# Radiance

- Radiance L =
**flux per unit projected area per unit solid angle**

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

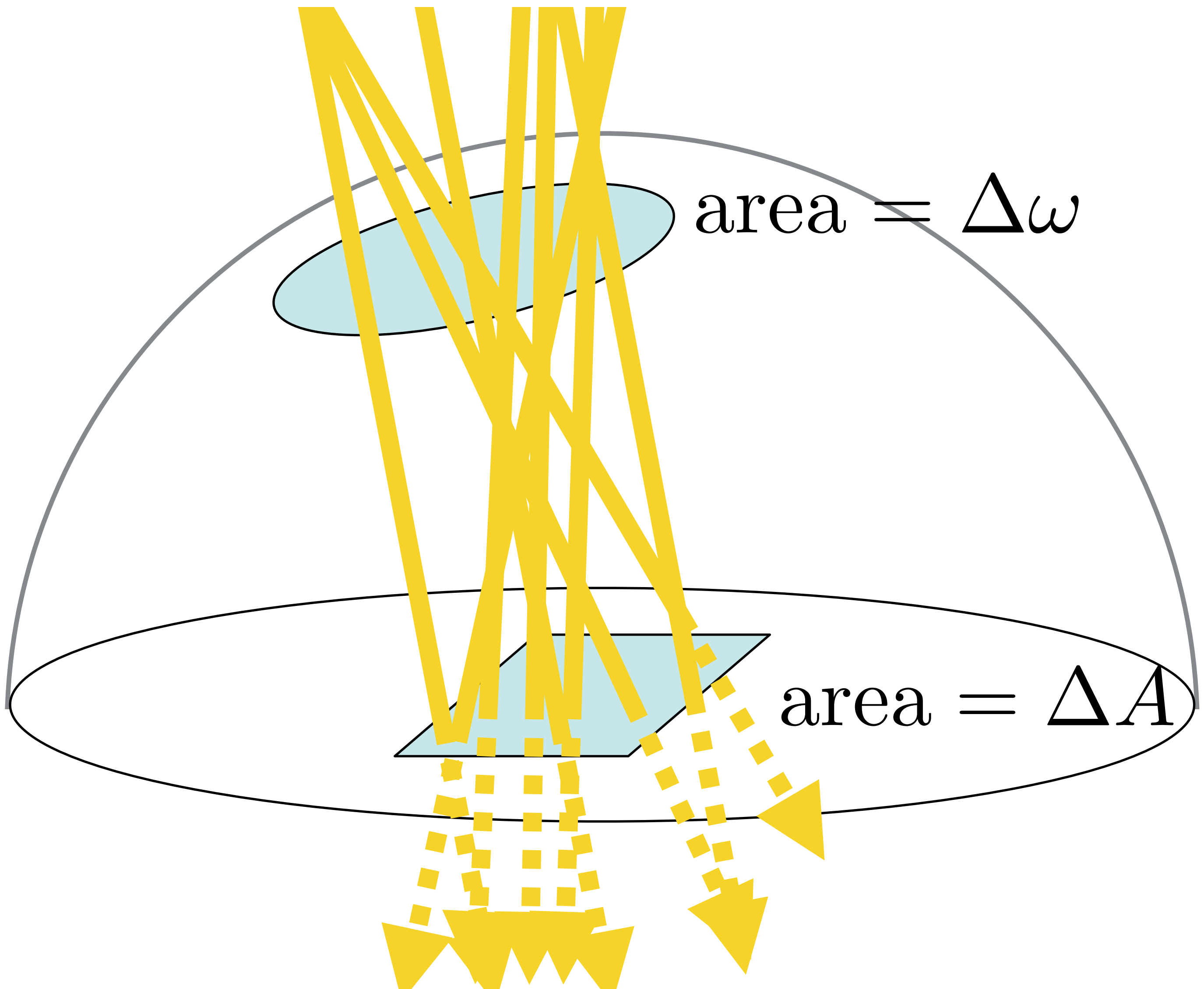
$$[L] = \left[\frac{W}{m^2\ sr}\right]$$

# Radiance, intuitively

- Let's count energy packets, each ray carries the same $\Delta\Phi$

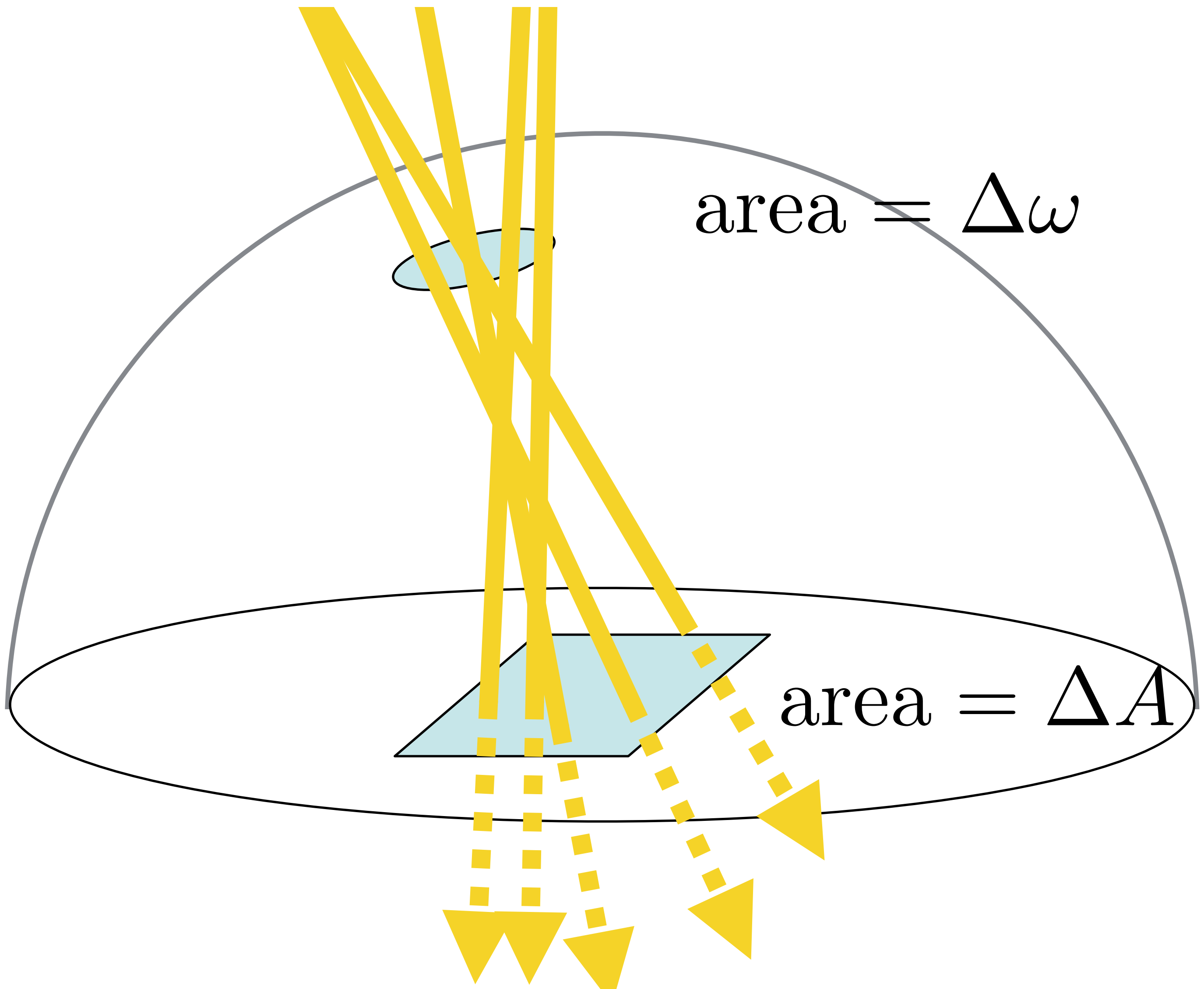
$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

$$[L] = \left[\frac{W}{m^2\ sr}\right]$$



# Radiance, intuitively

- Smaller solid angle => fewer rays => less energy




$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

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# Radiance, intuitively

- Smaller projected surface area => fewer rays => less energy

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

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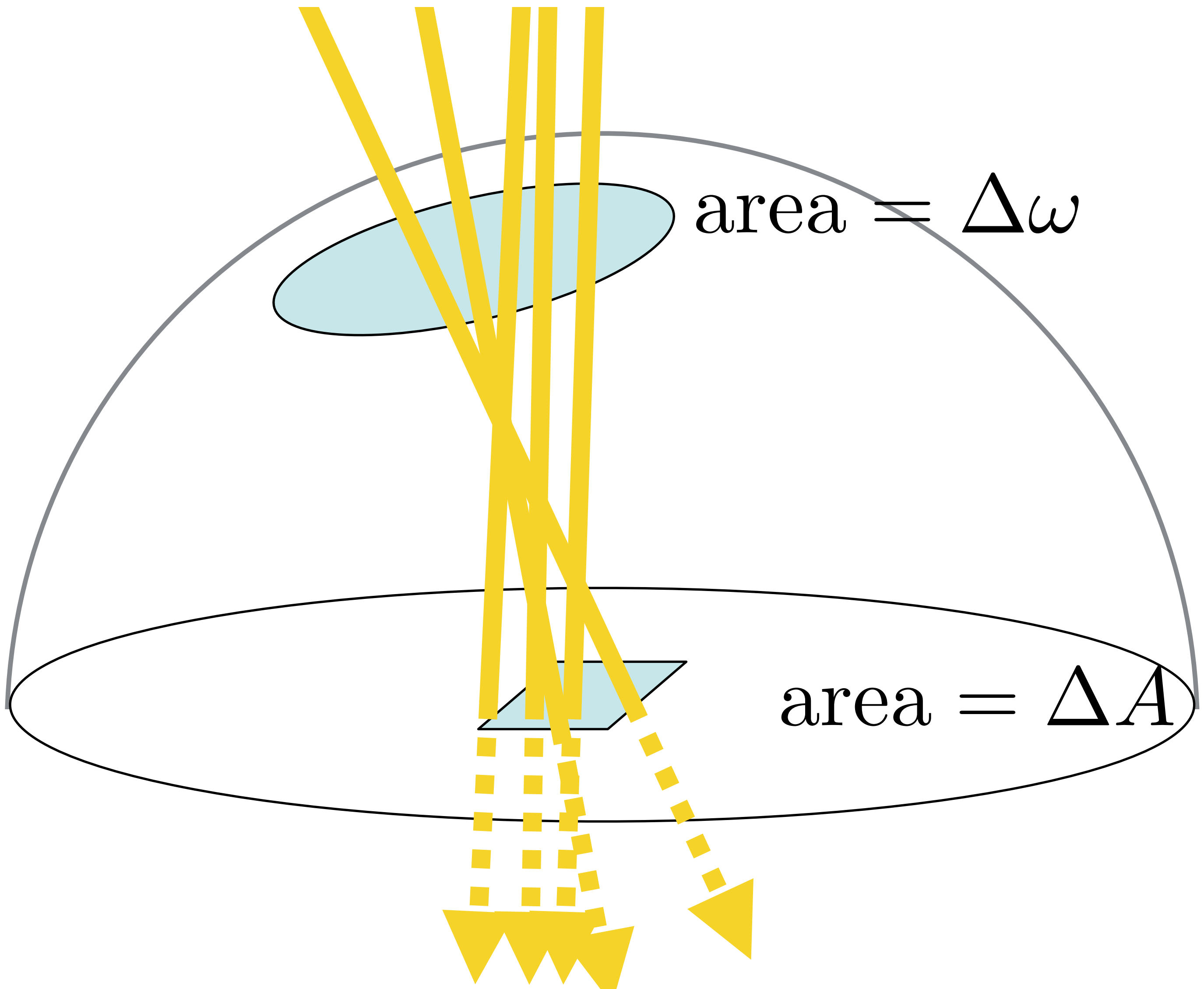
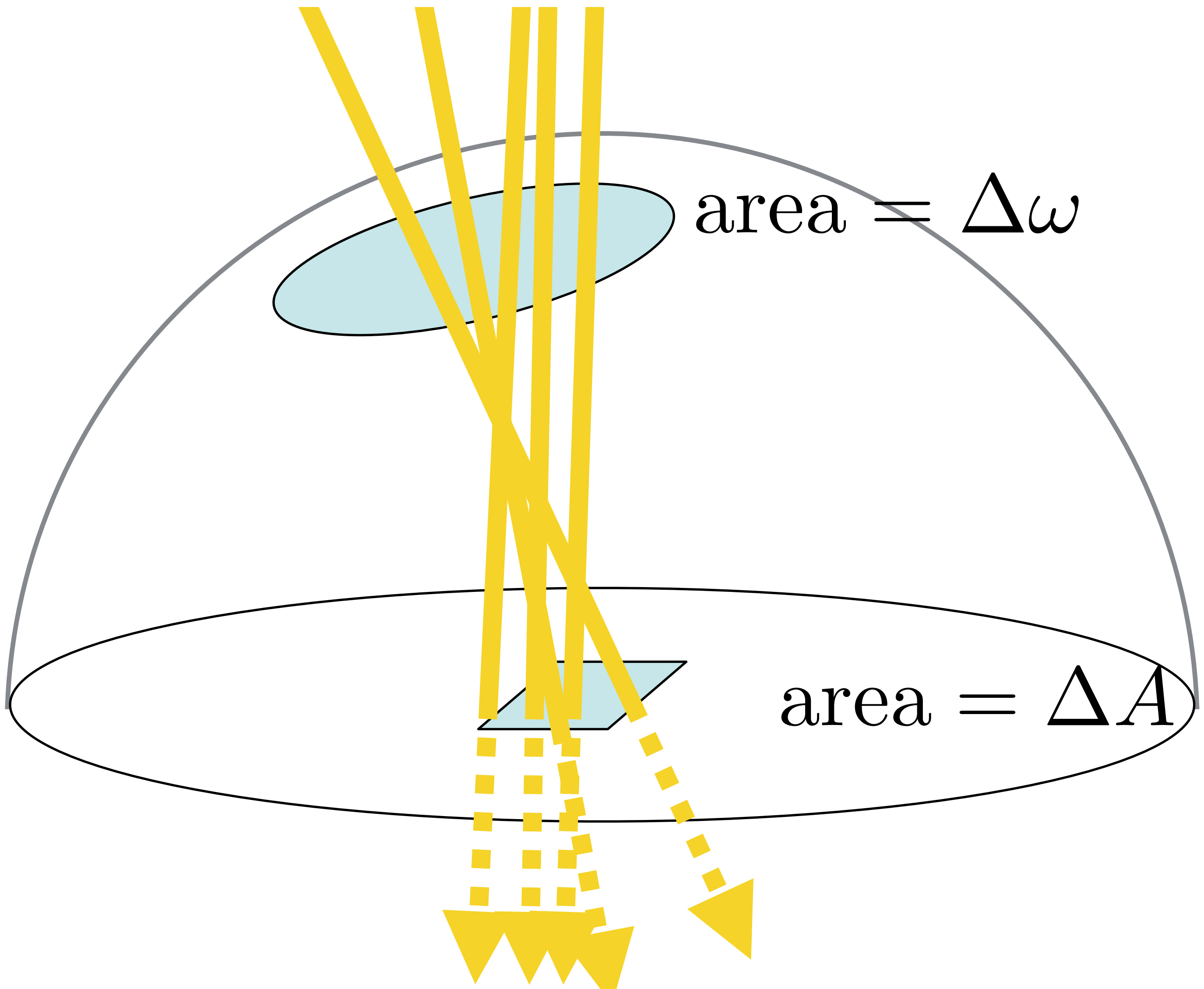


# Radiance, intuitively

- I.e., *radiance is a density over both space and angle*

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$



# Radiance

- **Sensors are sensitive to radiance**
  - It's what you assign to pixels
  - The fundamental quantity in image synthesis

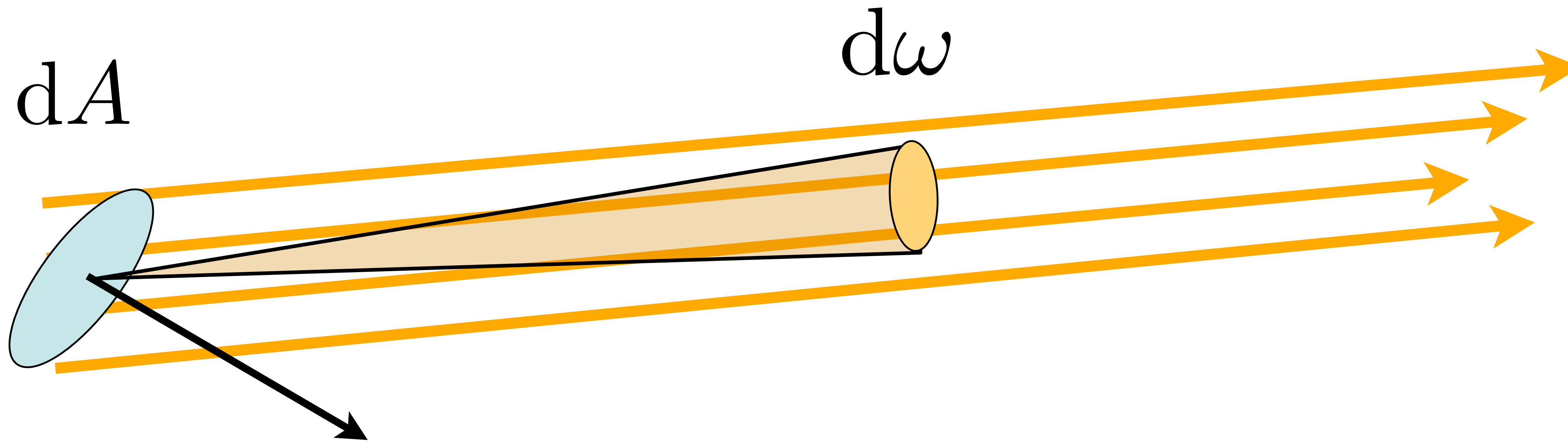
- "Intensity does not attenuate with distance"
  <=> **radiance stays constant along straight lines****

- **All relevant quantities (irradiance, etc.) can be derived from radiance**

**unless the medium is participating, e.g., smoke, fog

# Radiance

- Characterizes
  - Lighting that leaves a surface patch d$A$ to a given direction
  - Lighting that impinges d$A$ from a given direction
    - Just flip direction



# Radiance

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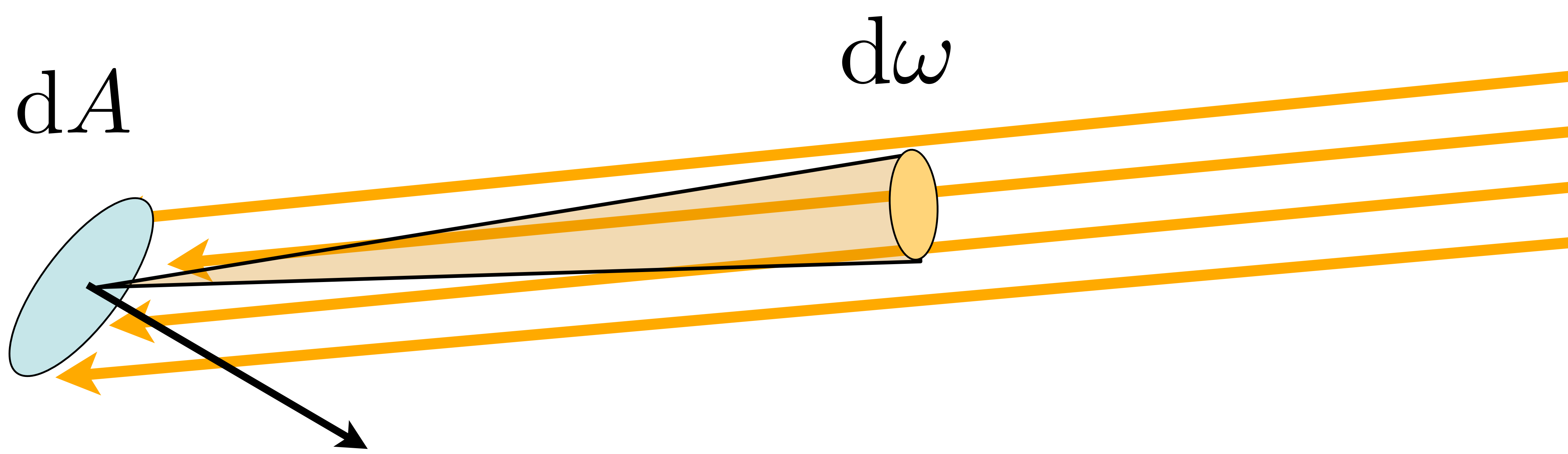
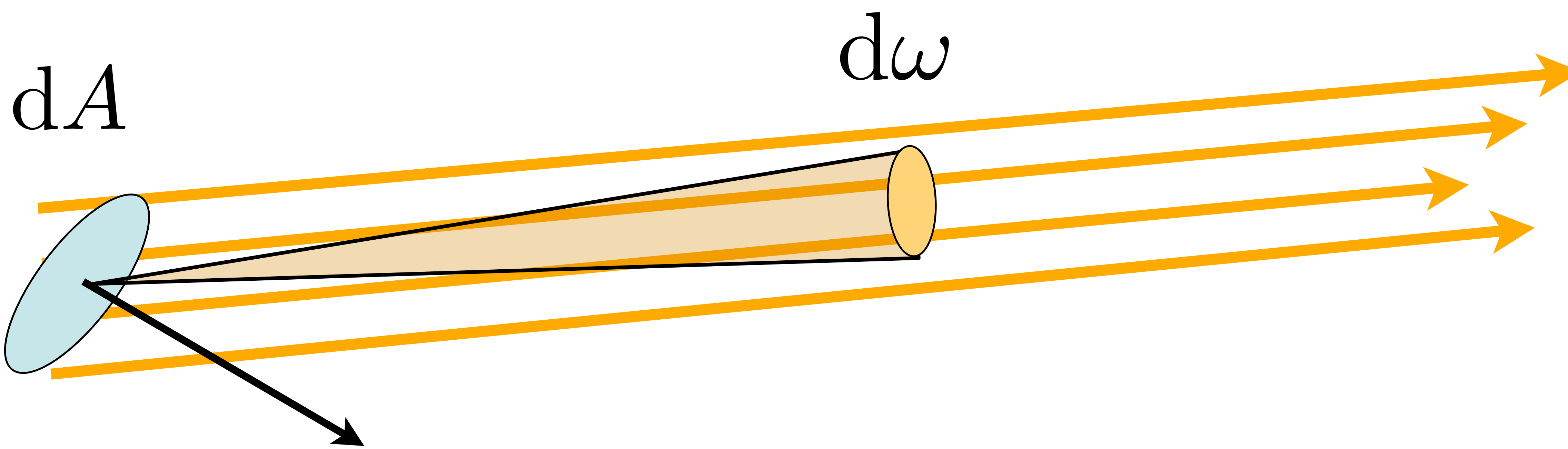


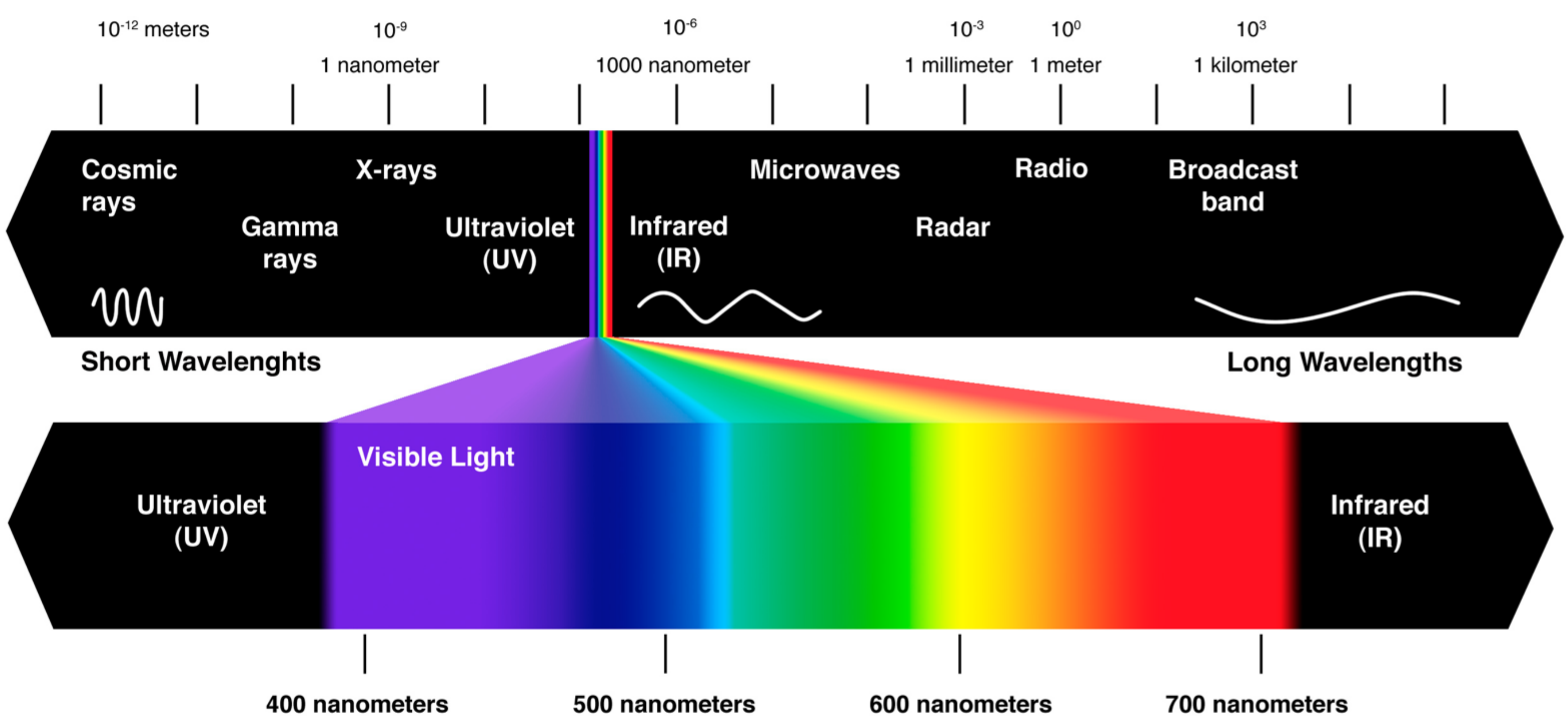
# Radiance

- Also empty space, away from surfaces
  - Radiance $L(x, \omega)$ , when taken as a 5D function of position (3D) and direction (2D) completely nails down the light flow in a scene
  - Sometimes called the "plenoptic function"



# A Word on Color

- Spectral radiance $L(x, \omega, \lambda)$ is the radiance in a small band $\mathrm{d}\lambda$ of wavelengths
- You get the total energy by integrating over the visible range

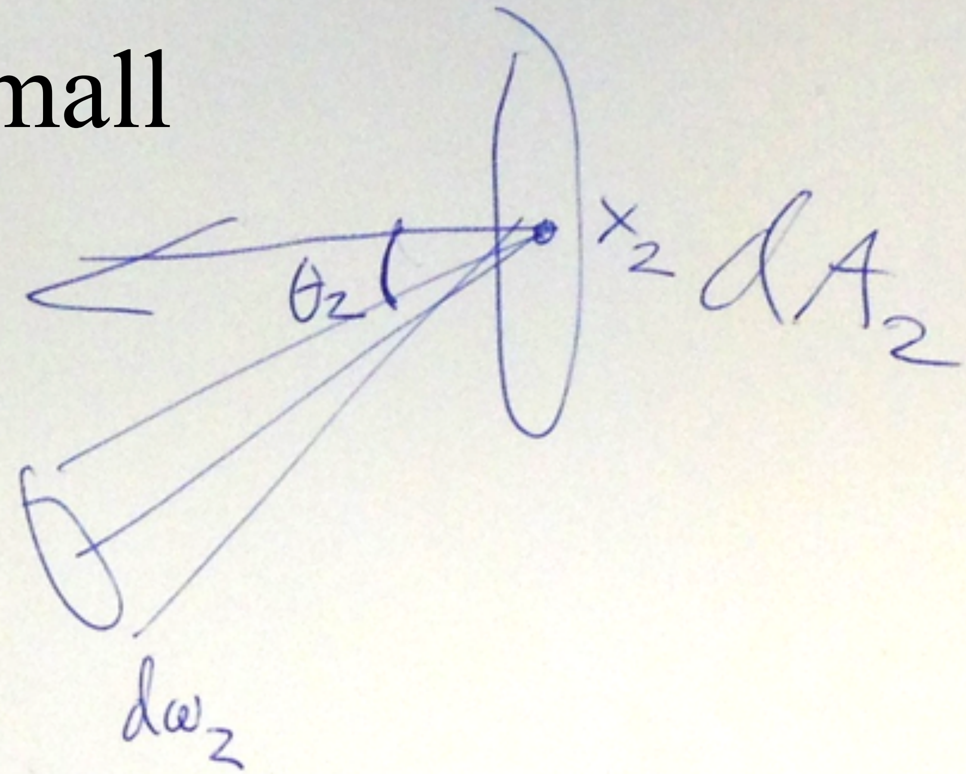


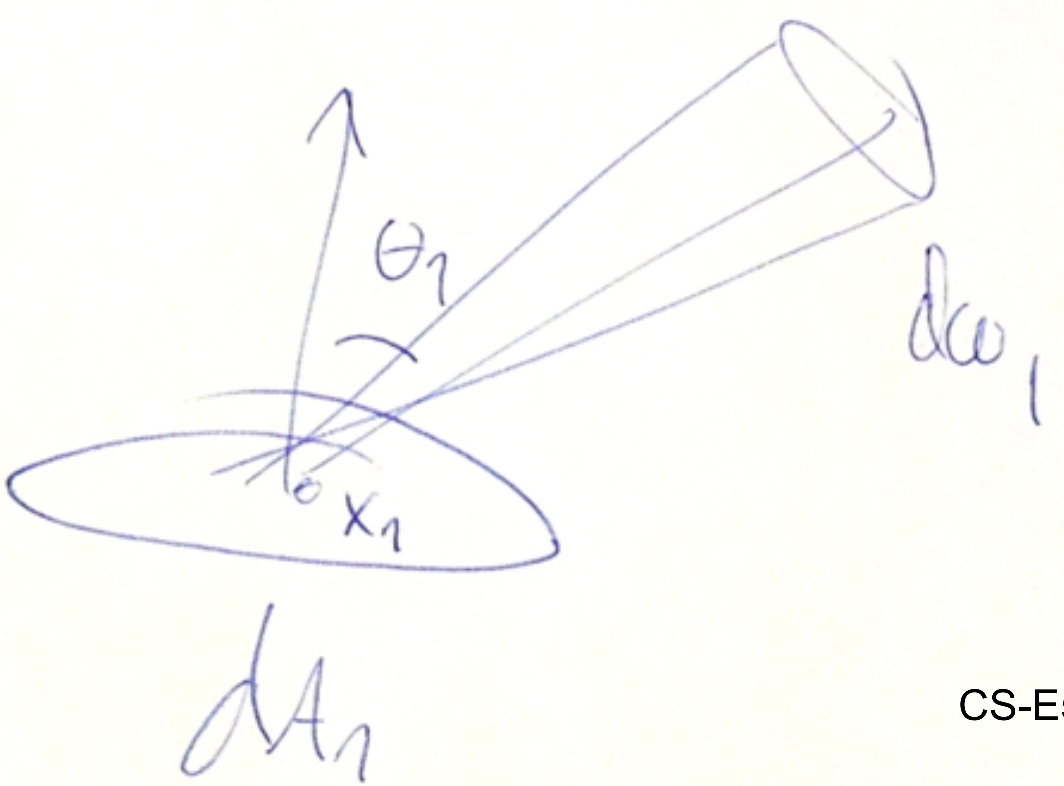
# About Color

- We'll mostly not talk about it in this class
- But not difficult to do "right"
- See e.g. Chapter 5 in the excellent Physically Based Rendering: From Theory to Implementation, 3rd ed.

# Constancy Along Straight Lines

- Let's look at the flux sent by a small patch onto another small patch

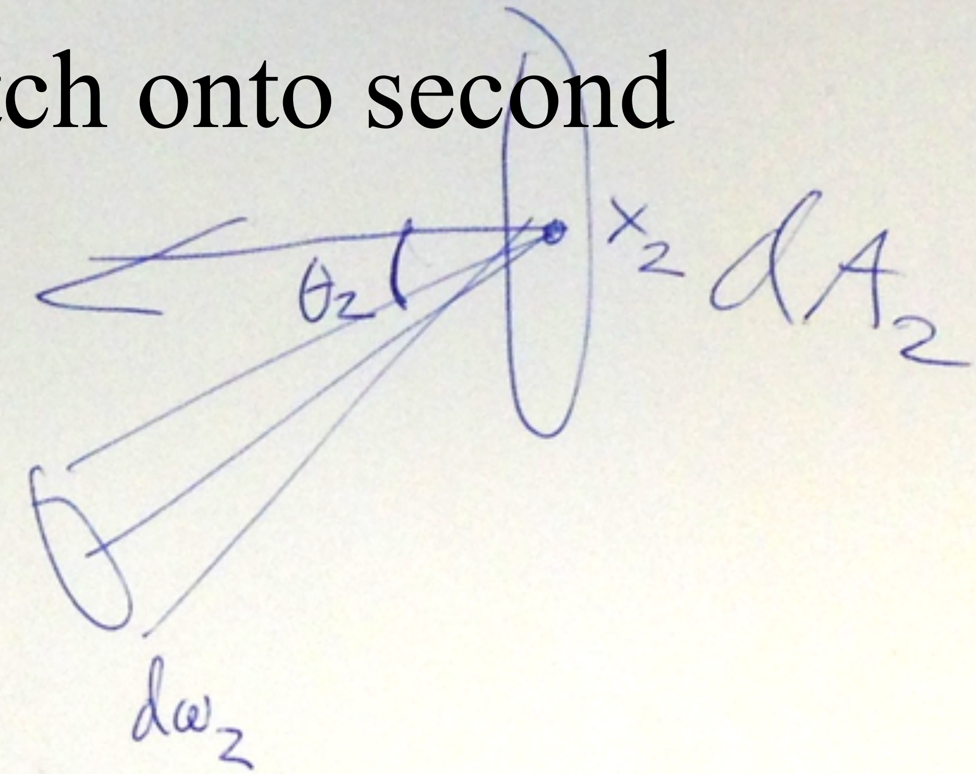


# Constancy Along Straight Lines

- Differential flux sent by first patch onto second

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$





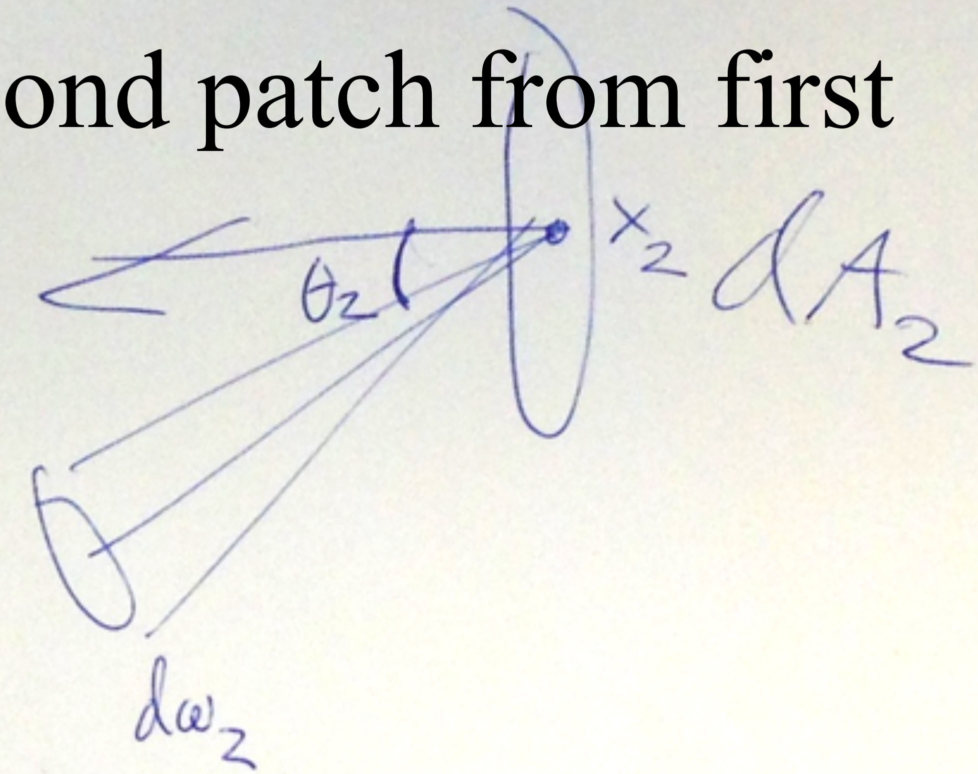
Solid angle $\mathrm{d}\omega_1$ subtended by dA2 as seen from dA1

$$\mathrm{d}\Phi = L(x_1 \rightarrow \omega_1)\,\overbrace{\cos\theta_1 \mathrm{d}A_1}^{\mathrm{d}A_1^{\perp}}\,\overbrace{\frac{\mathrm{d}A_2\,\cos\theta_2}{r^2}}$$

# Constancy Along Straight Lines

- Differential flux received by second patch from first

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$





Solid angle $\mathrm{d}\omega_2$ subtended by dA1 as seen from dA2

$$\mathrm{d}\Phi = L(x_2 \leftarrow \omega_2)\, \overbrace{\cos\theta_2 \mathrm{d}A_2}^{\mathrm{d}A_2^{\perp}}\, \overbrace{\frac{\mathrm{d}A_1\, \cos\theta_1}{r^2}}$$

# Eureka

$$\mathrm{d}\Phi = L(x_2 \leftarrow \omega_2)\ \cos\theta_2 \mathrm{d}A_2\ \frac{\mathrm{d}A_1\ \cos\theta_1}{r^2}$$

$$\mathrm{d}\Phi = L(x_1 \rightarrow \omega_1)\ \cos\theta_1 \mathrm{d}A_1\ \frac{\mathrm{d}A_2\ \cos\theta_2}{r^2}$$

# Eureka

$$\mathrm{d}\Phi = L(x_2 \leftarrow \omega_2)\ \cos\theta_2 \mathrm{d}A_2\ \frac{\mathrm{d}A_1\ \cos\theta_1}{r^2}$$

$$\mathrm{d}\Phi = L(x_1 \rightarrow \omega_1)\ \cos\theta_1 \mathrm{d}A_1\ \frac{\mathrm{d}A_2\ \cos\theta_2}{r^2}$$

$$\Rightarrow L(x_1 \rightarrow \omega_1) = L(x_2 \leftarrow \omega_2)$$

# Eureka

- Radiance is constant along straight lines
  - I.e. radiance sent by $dA_1$ into the direction of $dA_2$ is the same as radiance received by $dA_2$ from the direction of $dA_1$.
- This is why the lamp appears "as bright" no matter how far you look at it from

$$\Rightarrow L(x_1 \to \omega_1) = L(x_2 \leftarrow \omega_2)$$

**Rendering <=>**
**what is the radiance hitting my sensor?**

# Let's Look at Irradiance Again

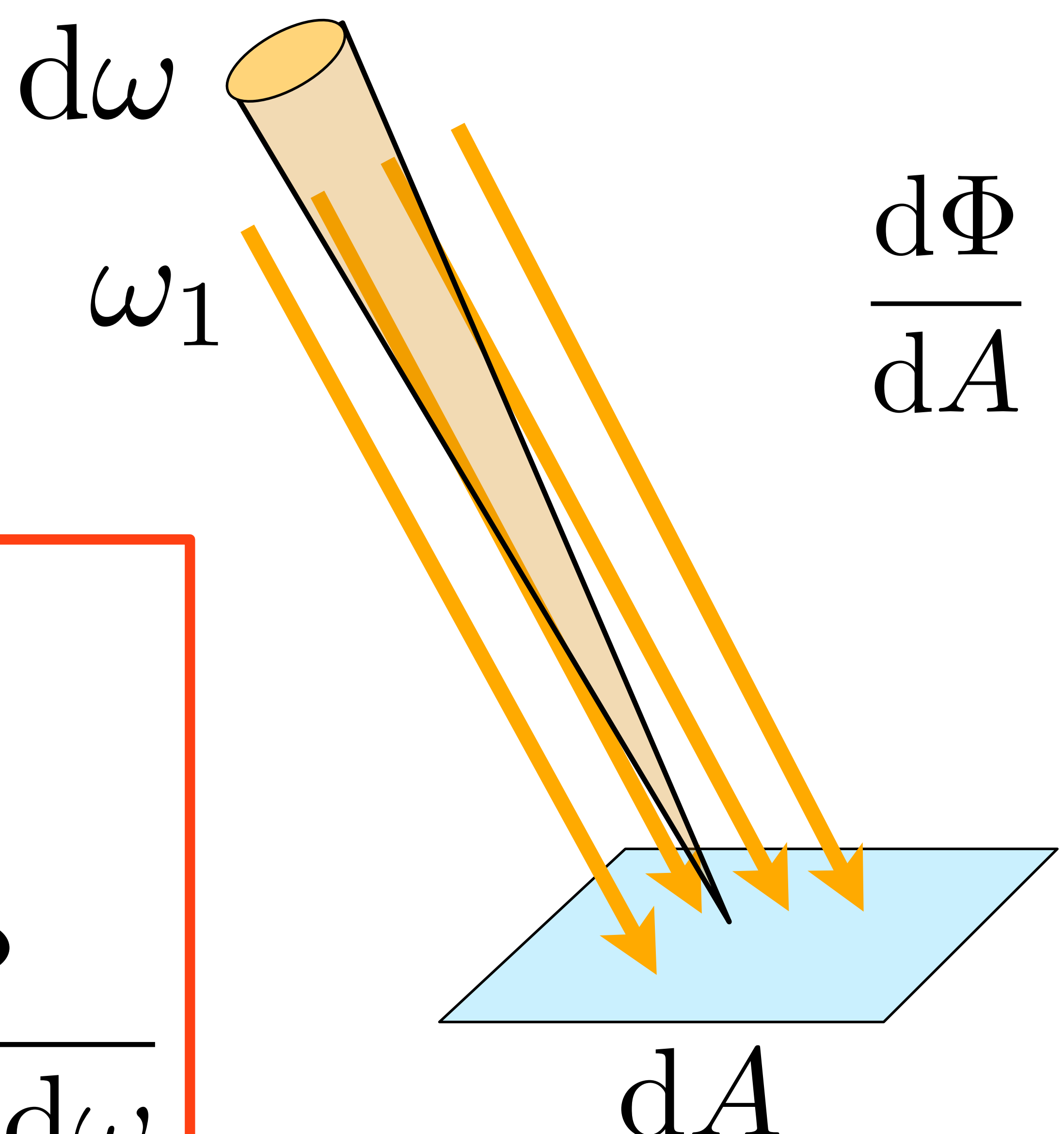
- Remember, irradiance is radiant power landing on a surface per unit area (from all directions)
  - So far we only looked at tiny collimated beams

$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A} \qquad \left[\frac{W}{m^2}\right]$$

$\mathrm{d}A$

# Let's Look at Irradiance Again

- Let's count irradiance, add up the radiance from all the differential beams from all directions




$$\frac{\mathrm{d}\Phi}{\mathrm{d}A} = L(\omega_1)\cos\theta\,\mathrm{d}\omega$$

$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A}$$

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

**Remember: omega is a single direction, dw is the small solid angle around it**

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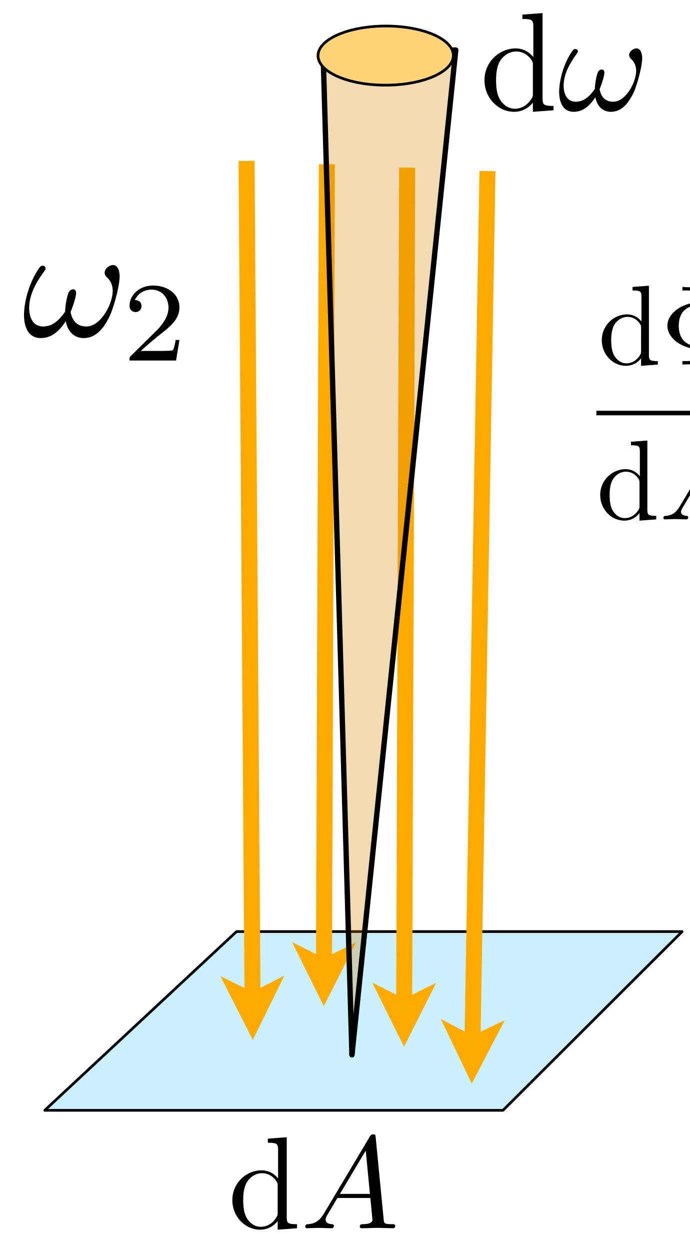



$$\frac{\mathrm{d}\Phi}{\mathrm{d}A} = L(\omega_2)\cos\theta\,\mathrm{d}\omega$$

$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A}$$

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

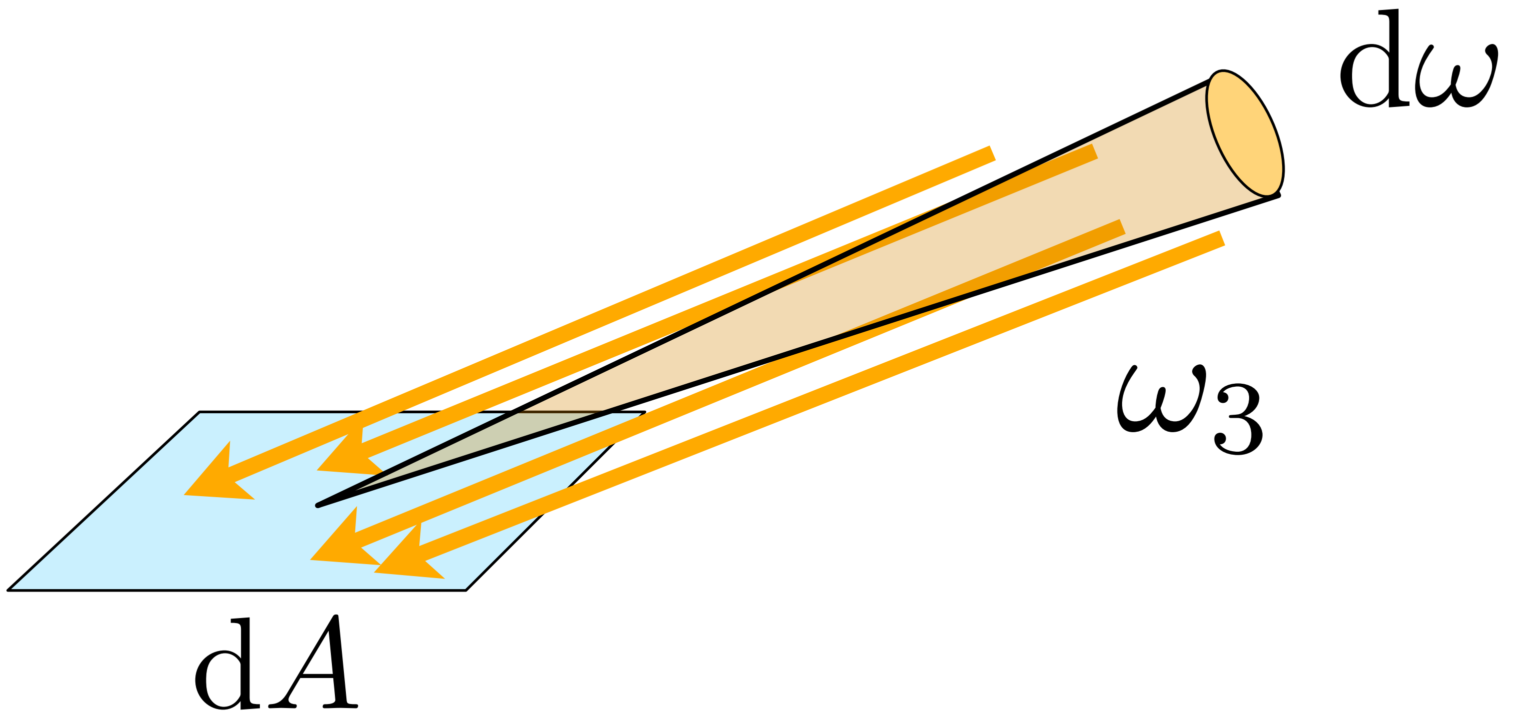
# This Happens for All Directions

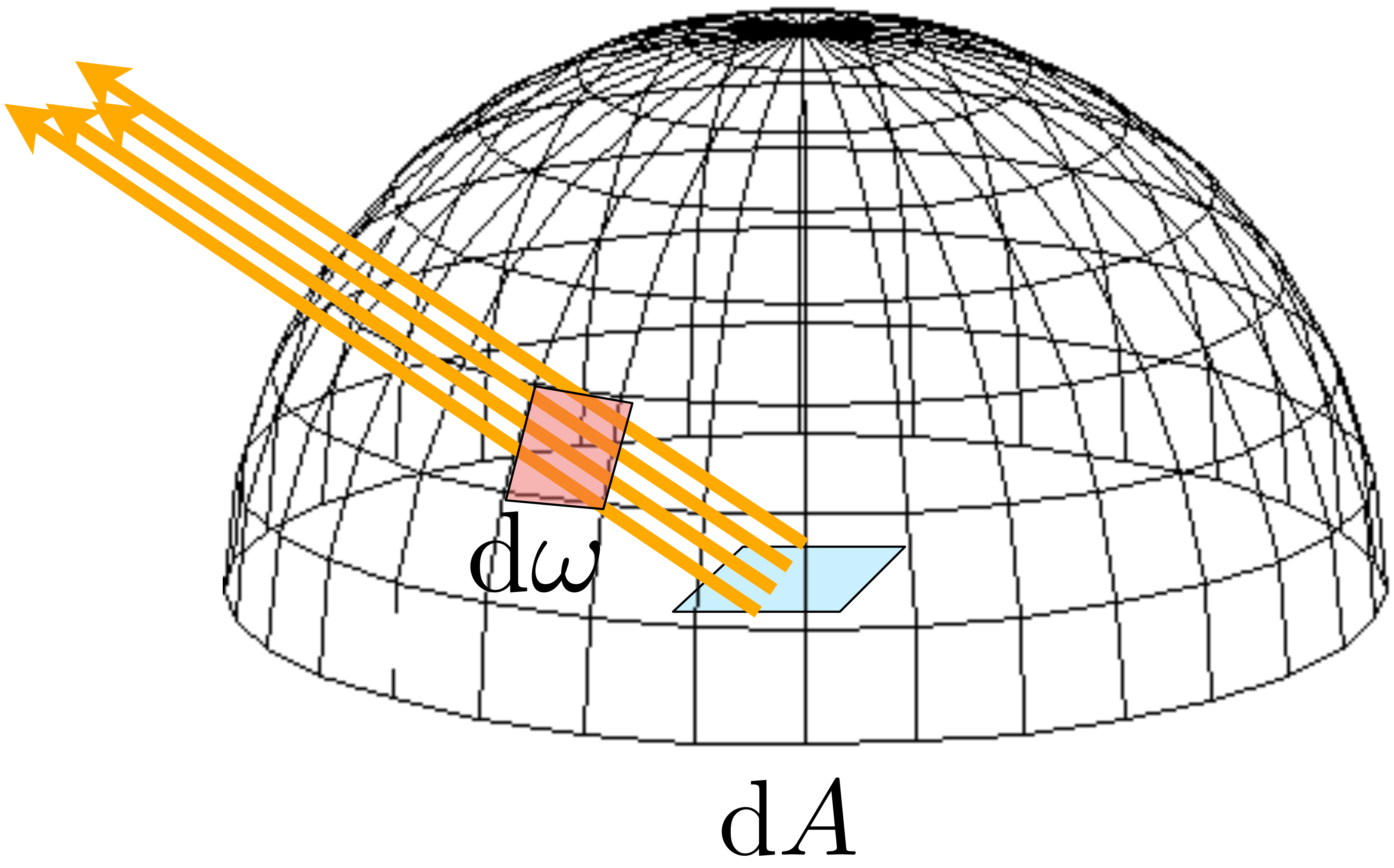
- Infinitely many of incident directions
  - Yes, you guessed it: integral over solid angle

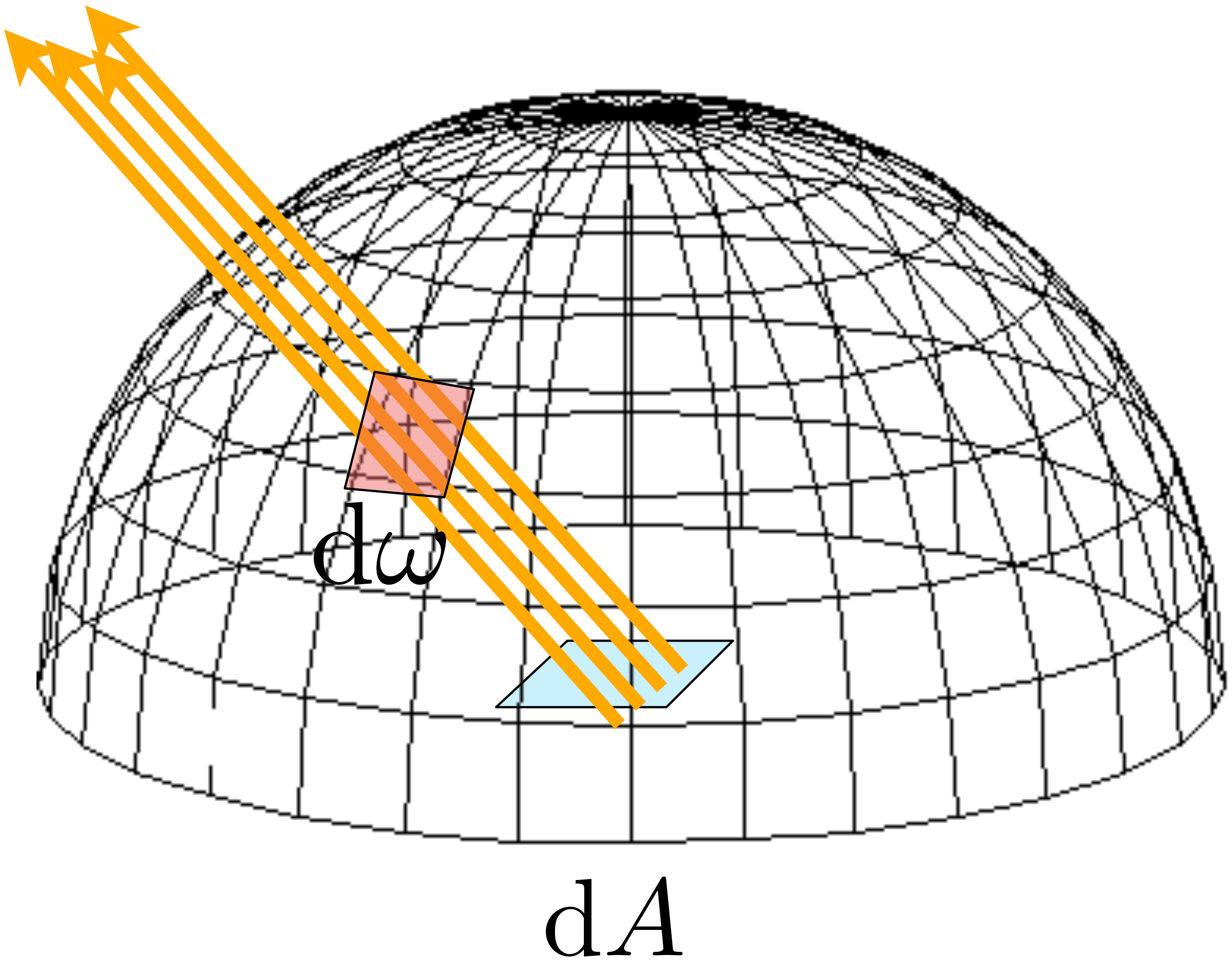
$$\frac{\mathrm{d}\Phi}{\mathrm{d}A} = L(\omega_3)\cos\theta\ \mathrm{d}\omega$$

$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A}$$

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\ \mathrm{d}\omega}$$

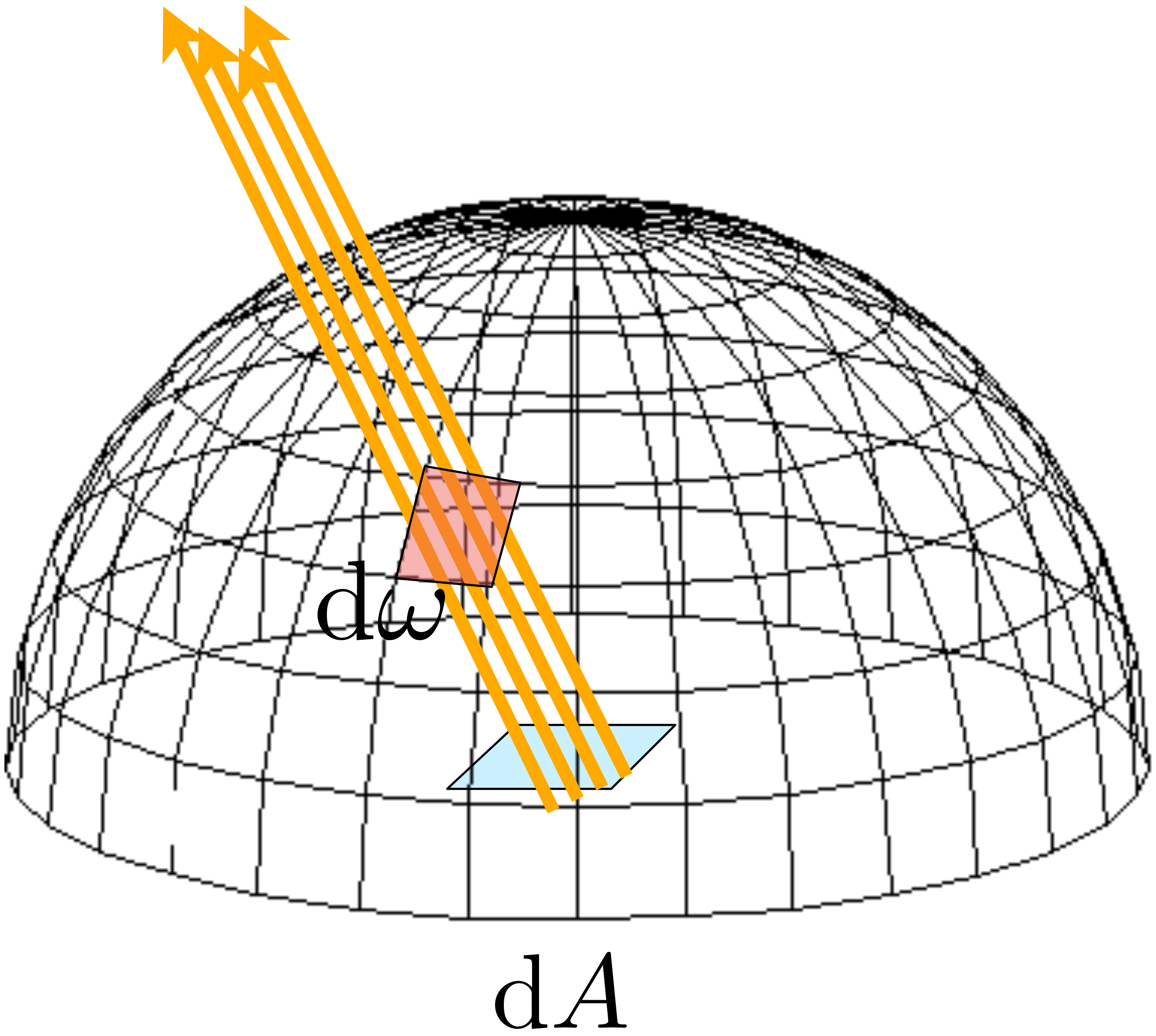



$\mathrm{d}\omega$
$\mathrm{d}A$



$\mathrm{d}\omega$
$\mathrm{d}A$

- ...



$\mathrm{d}\omega$
$\mathrm{d}A$

# Irradiance

- Integrate incident radiance times cosine over the hemisphere $\Omega$

$$E = \int_{\Omega} L(\omega)\ \cos\theta\ \mathrm{d}\omega$$

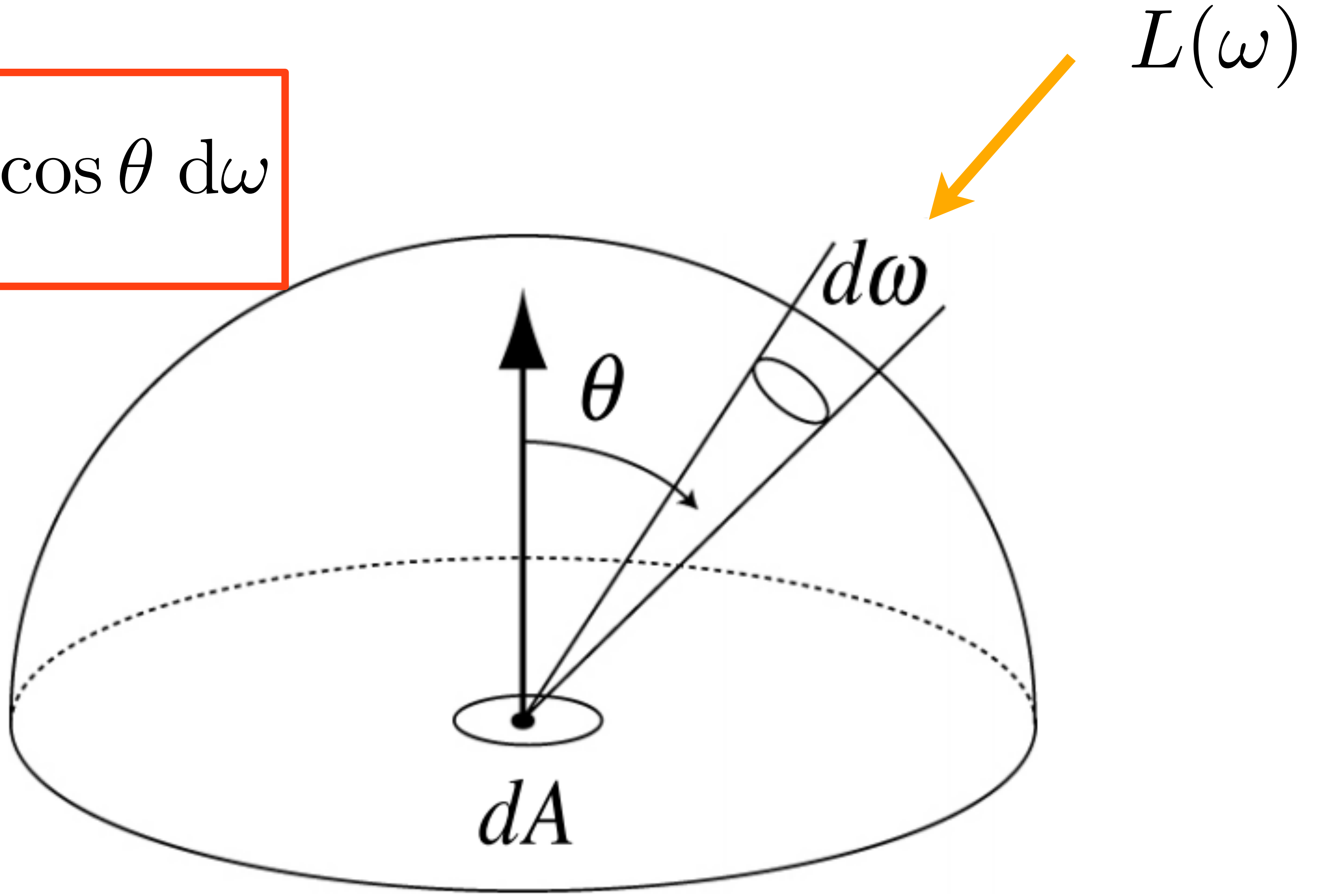



Figure adapted from Pat Hanrahan

# Eureka, Part Deux

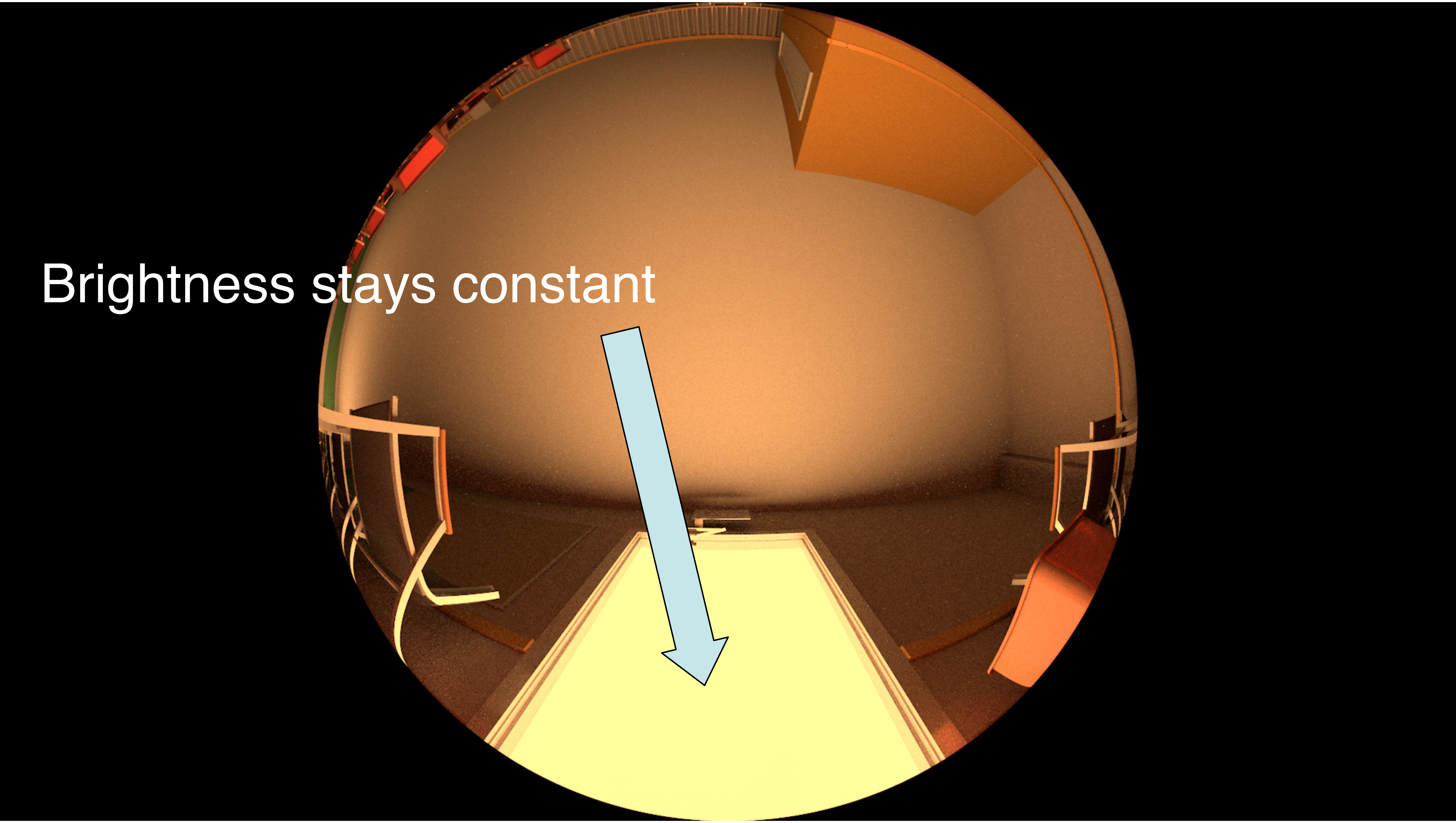
- Radiance is constant along straight lines ✔
  - I.e. radiance sent by $dA_1$ into the direction of $dA_2$ is the same as radiance received by $dA_2$ from the direction of $dA_1$.

- This is why the lamp appears "as bright" no matter how far you look at it from ✔
  - BUT: The solid angle subtended by the lamp decreases with distance, so irradiance, which is the integral of radiance over solid angle, decreases => less light is reflected

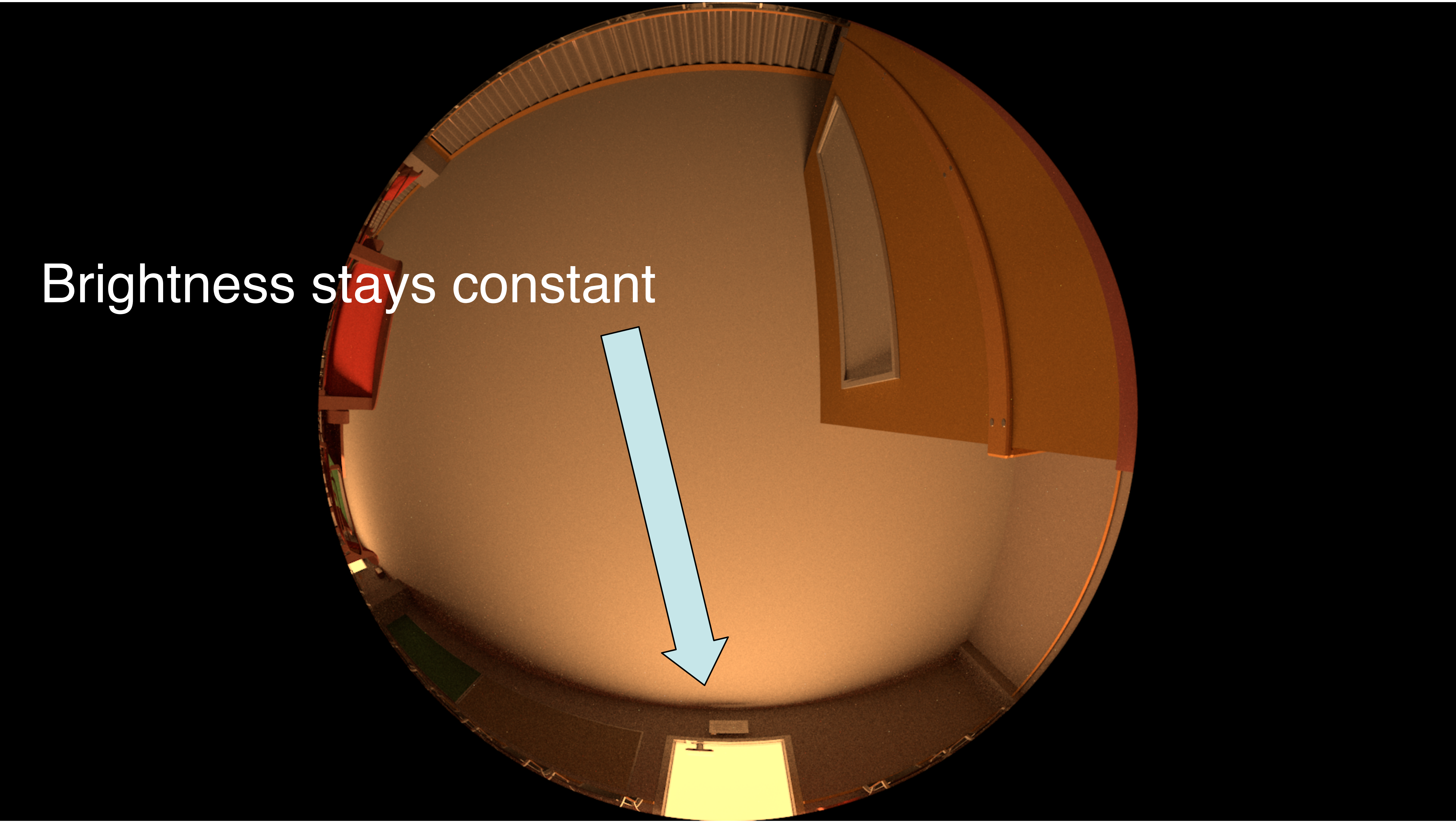
$$E = \int_\Omega L(\omega)\ \cos\theta\ \mathrm{d}\omega$$


A
B
C

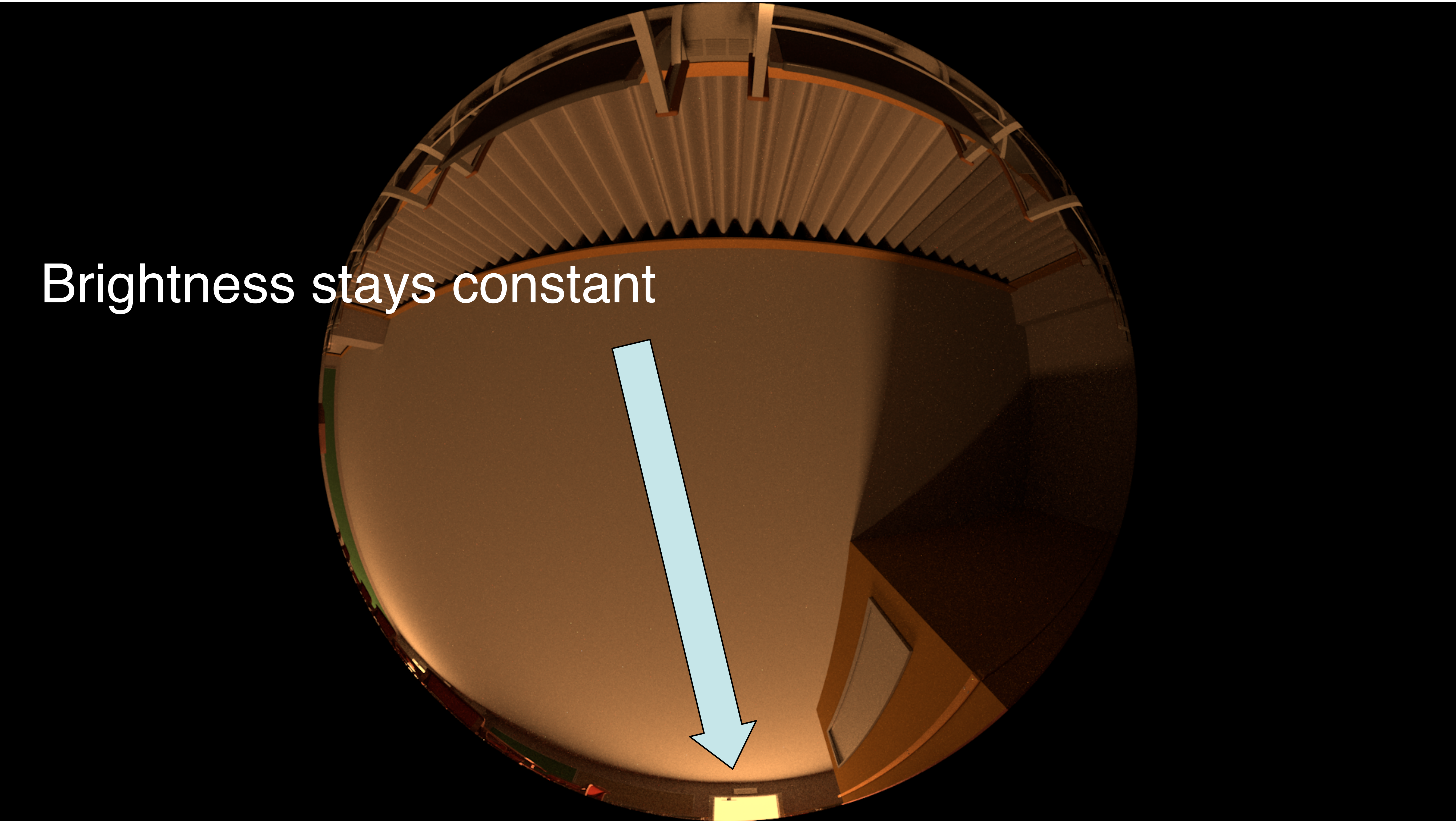
# View from A

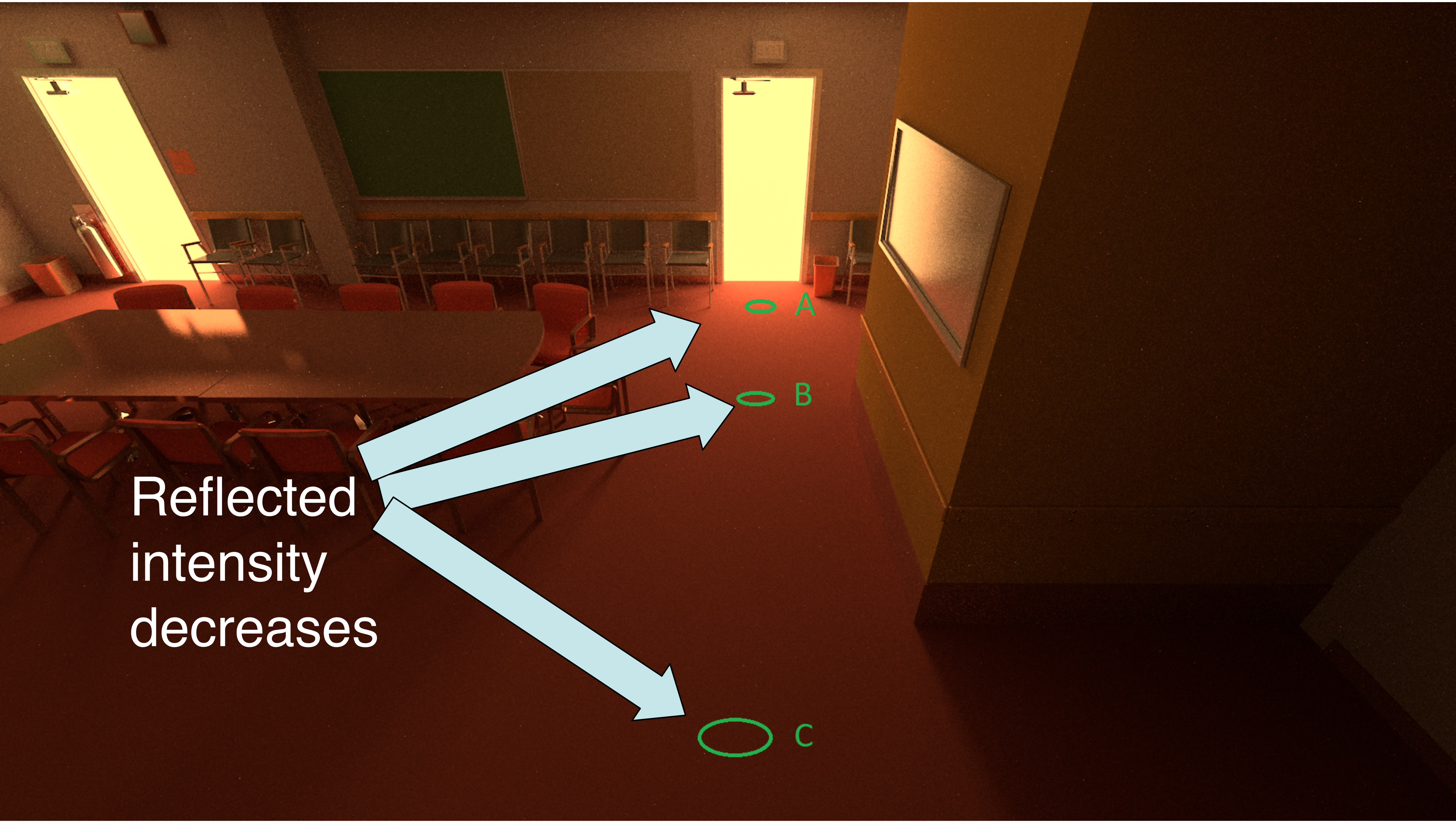


# View from B



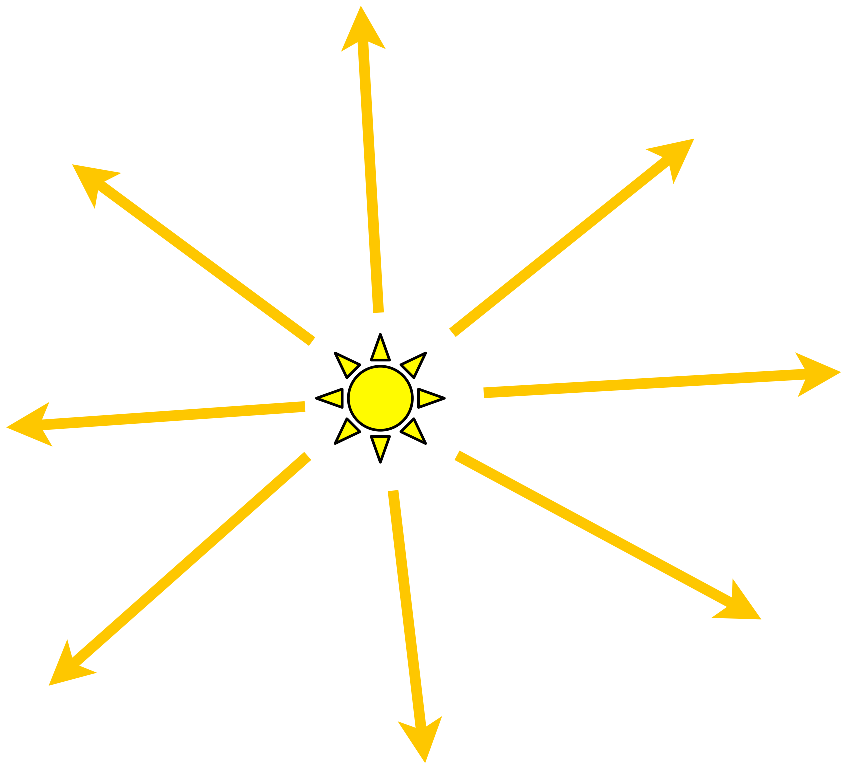
# View from C




A
B
C
Reflected intensity decreases

# What About Pointlights?

- A pointlight has no area
  - Hence we can't define radiance easily
  - However, differential irradiance is easy



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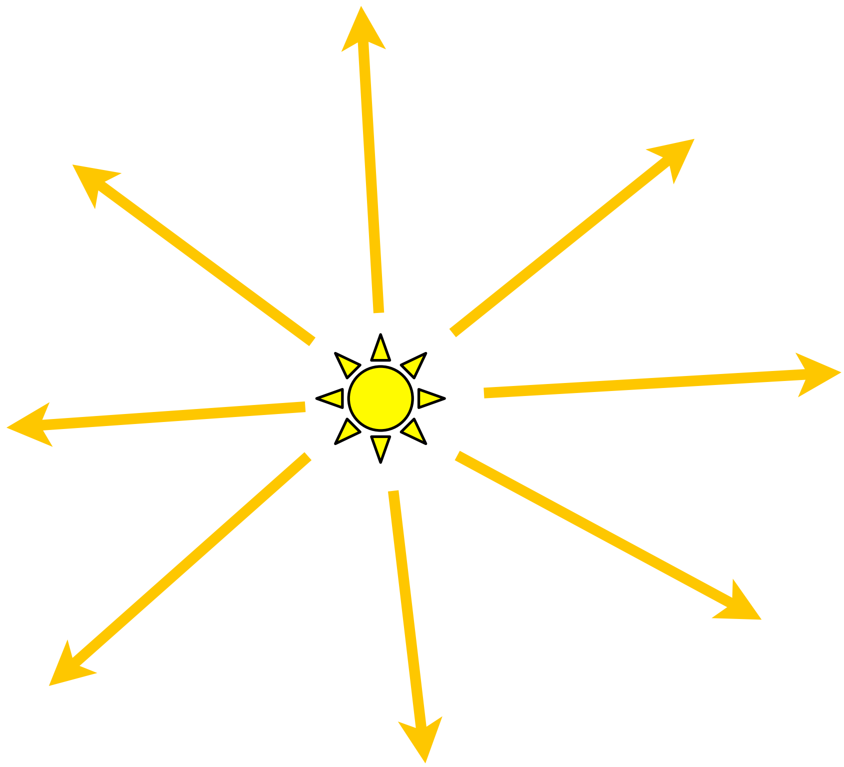


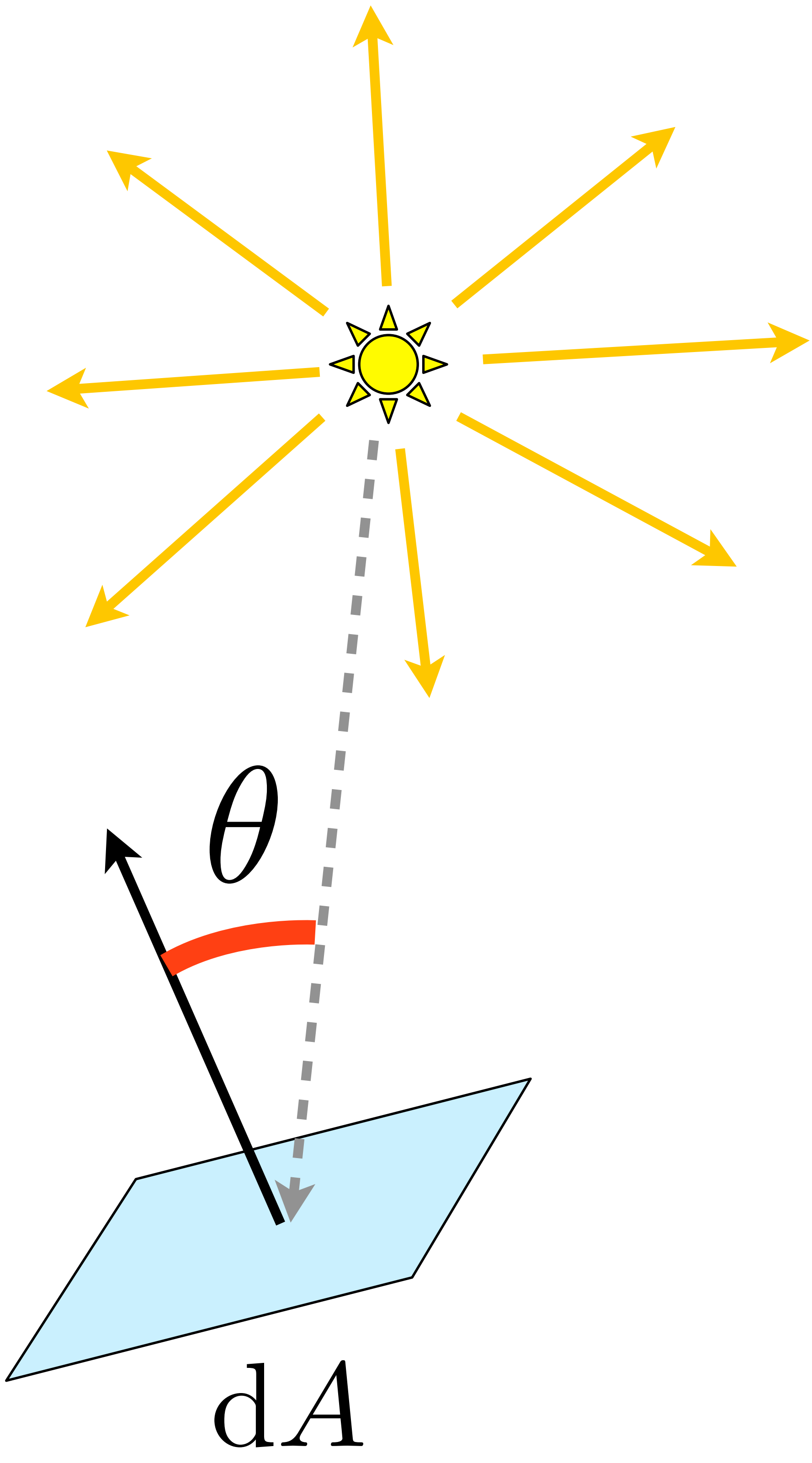
- The emission of a pointlight measured by *intensity* I, flux per solid angle

$$I = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega} \qquad [I] = \left[\frac{W}{sr}\right]$$

# Irradiance due to a Pointlight

- What's the irradiance received by d$A$ from the light <=> what's the solid angle subtended by d$A$ as seen from the light?
  – We know the answer...

$$I = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega} \qquad [I] = \left[\frac{W}{sr}\right]$$



# Irradiance due to a Pointlight

- What's the irradiance received by d$A$ from the light <=> what's the solid angle subtended by d$A$ as seen from the light?
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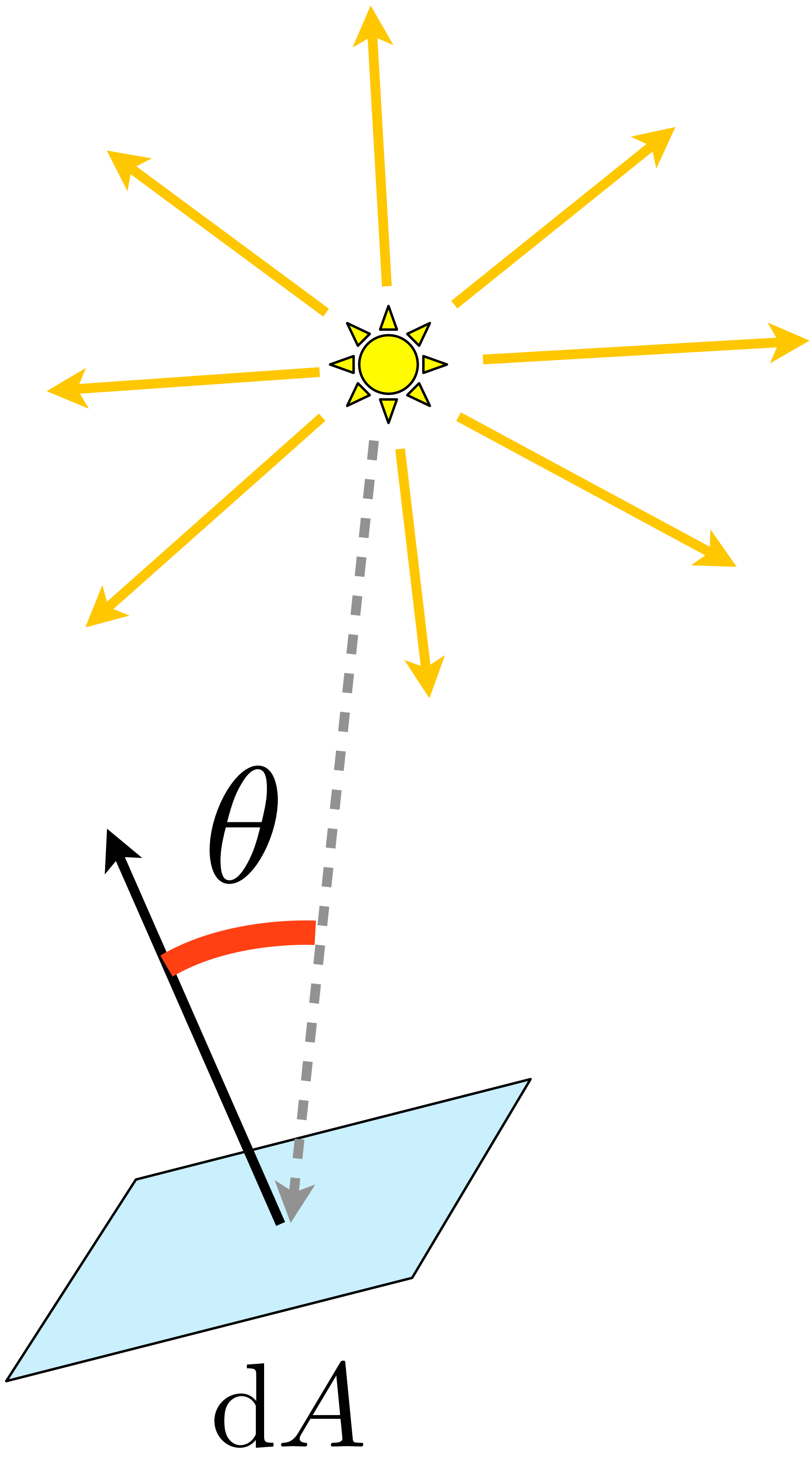
$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A} = I\frac{\mathrm{d}\omega}{\mathrm{d}A} = I\frac{\cos\theta}{r^2}$$

$$I = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega} \qquad [I] = \left[\frac{W}{sr}\right]$$



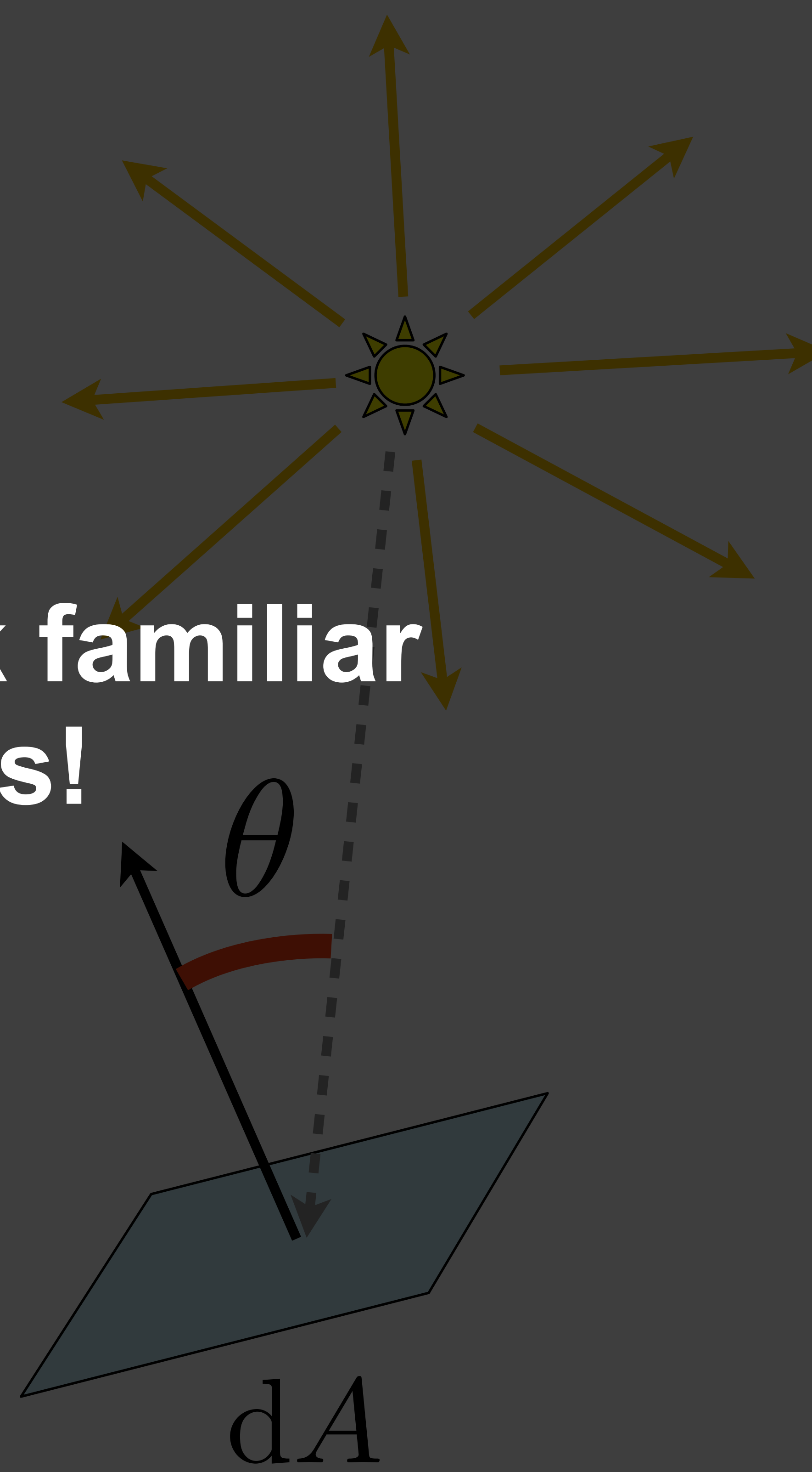
# Irradiance due to a Pointlight

- What's the irradiance received by d$A$ from the light <=> what's the solid angle subtended by d$A$ as seen from the light?
  - We know the answer...

**This formula should look familiar from the intro class!**

$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A} = I\frac{\mathrm{d}\omega}{\mathrm{d}A} = I\frac{\cos\theta}{r^2}$$

$$I = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega} \qquad [I] = \left[\frac{W}{sr}\right]$$



# "White Furnace Test"

- Integrate incident radiance times cosine over the hemisphere $\Omega$ above surface normal

$$E = \int_{\Omega} L(\omega) \, \cos\theta \, \mathrm{d}\omega$$

- Sanity check: What if we get unit intensity in, i.e., L=1 for all incident directions?
  - The so-called "white furnace test"
  - We'd expect the surface not to emit more than 1 unit of radiance.. Conservation of energy!
  - *Good idea to perform this in code for validation*

# Interlude

- Remember polar coordinates? $\mathrm{d}\omega = \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$

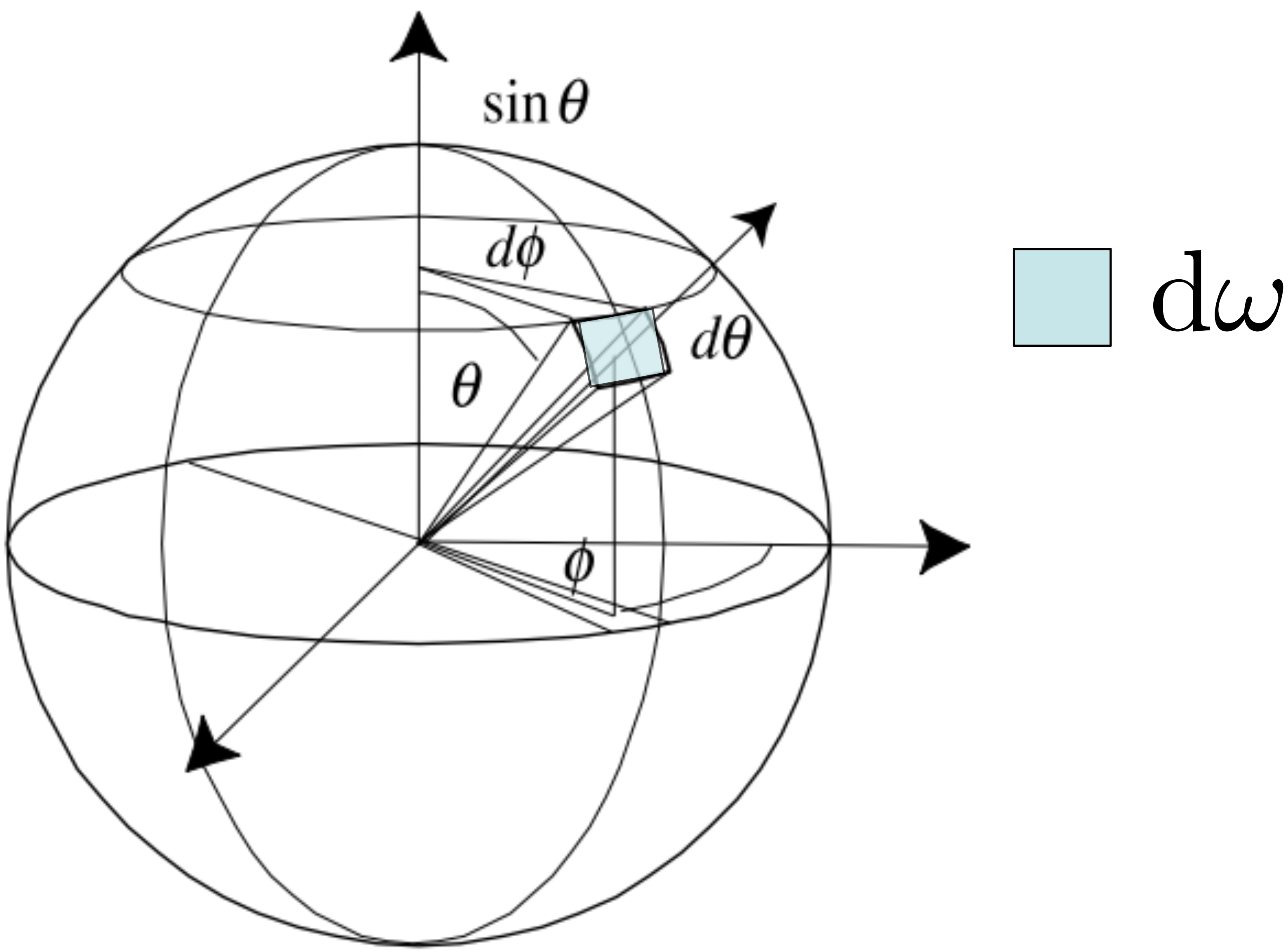



Figure adapted from Pat Hanrahan

# White Furnace, cont'd

- Sanity check: What if we get unit intensity in, i.e., L=1 for all incident directions?

$$E = \int_{\Omega} L(\omega)\ \cos\theta \, \mathrm{d}\omega$$

$$= \int 1 \cos\theta\ \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

← integral over hemisphere in polar coordinates

$$\phi \in \{0, 2\pi\}$$

$$\theta \in \{0, \pi/2\}$$

# White Furnace, cont'd

- Sanity check: What if we get unit intensity in, i.e., L=1 for all incident directions?

$$E = \int_\Omega L(\omega)\ \cos\theta\ \mathrm{d}\omega$$

See it for yourself in Wolfram Alpha (click here)

$$= \int 1 \cos\theta\ \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi \boxed{= \pi}$$

$$\phi \in \{0, 2\pi\}$$
$$\theta \in \{0, \pi/2\}$$

Hmm, intuition says: if you light a perfectly reflecting diffuse surface with uniform lighting, you should get the same "intensity" out

# From Irradiance to Radiosity

- The reflectivity of a diffuse surface is determined by its *albedo* $\rho$
  - This is the "diffuse color $k_d$" from your ray tracer in C3100

- The flux emitted by a diffuse surface per unit area is called *radiosity* B
  - Same units as irradiance, [B] = [W/m^2]
  - Hence

$$\boxed{B = \frac{\rho E}{\pi}}$$

(Danger spot! What if you forget to divide your albedo by pi?)

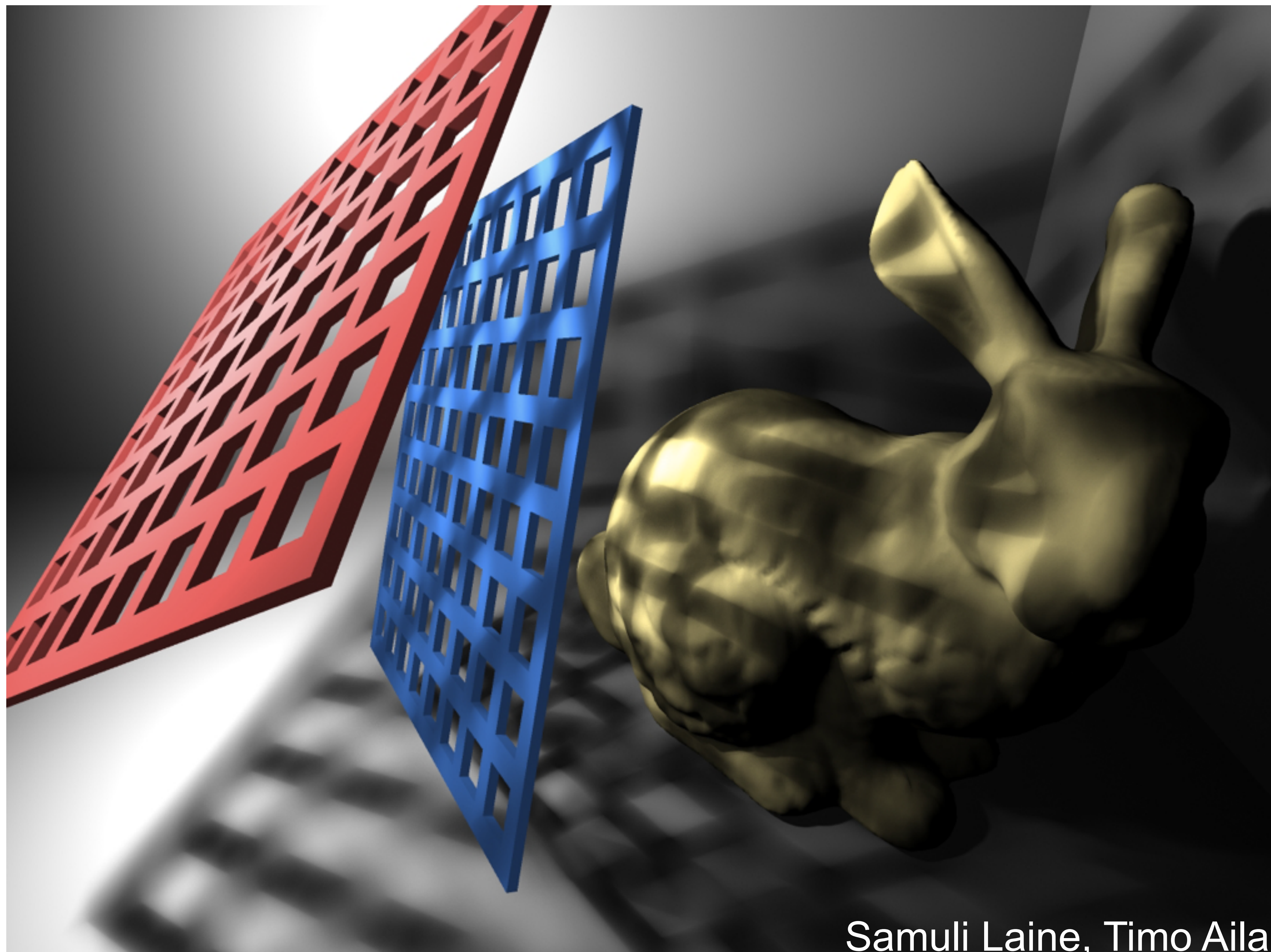
# Radiosity cont'd

- For a diffuse surface, the outgoing radiance is constant over all directions, and L = B
- Diffuseness is a pretty strict approximation (not many surfaces are really like that) but diffuse GI can look very good when done right
  - We did this for Max Payne 1 & 2
  - Easy to combine diffuse GI solution with "fake" glossy/ specular reflections computed on top of it


Jason Jacobs

# Enough Theory, Let's Apply This

- How to compute soft shadows from an area light source on a diffuse receiver?



# Lambertian Soft Shadows

differential solid angle

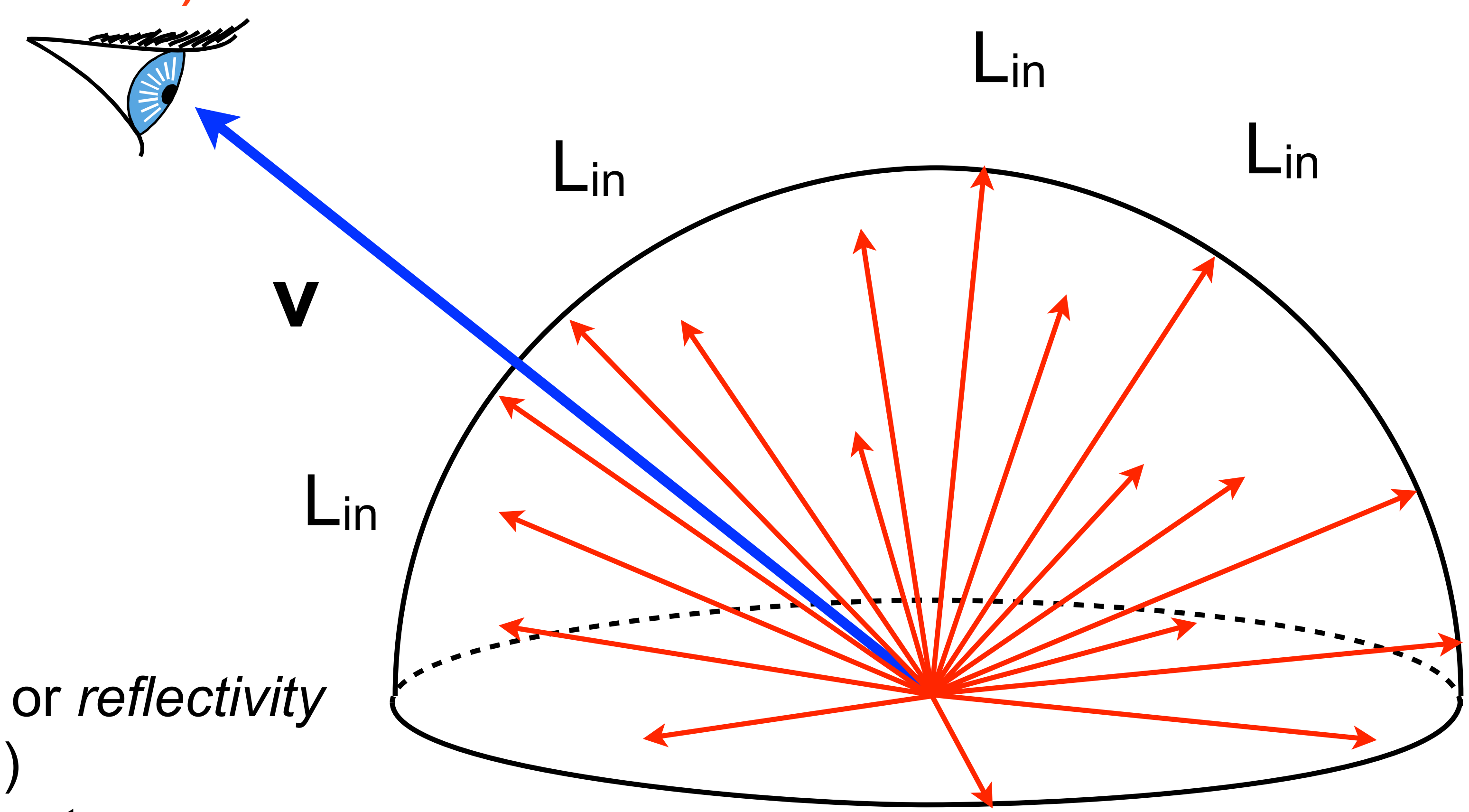
$$L_{\text{out}}(x) = \frac{\rho(x)}{\pi} \int_{\Omega} L_{\text{in}}(x, \omega)\ \cos\theta\, \mathrm{d}\omega$$

outgoing light (diffuse => independent of direction v)

albedo/pi

incident radiance

cosine term




Sum (integrate) over every direction on the hemisphere, modulate incident illumination by cosine, albedo/pi

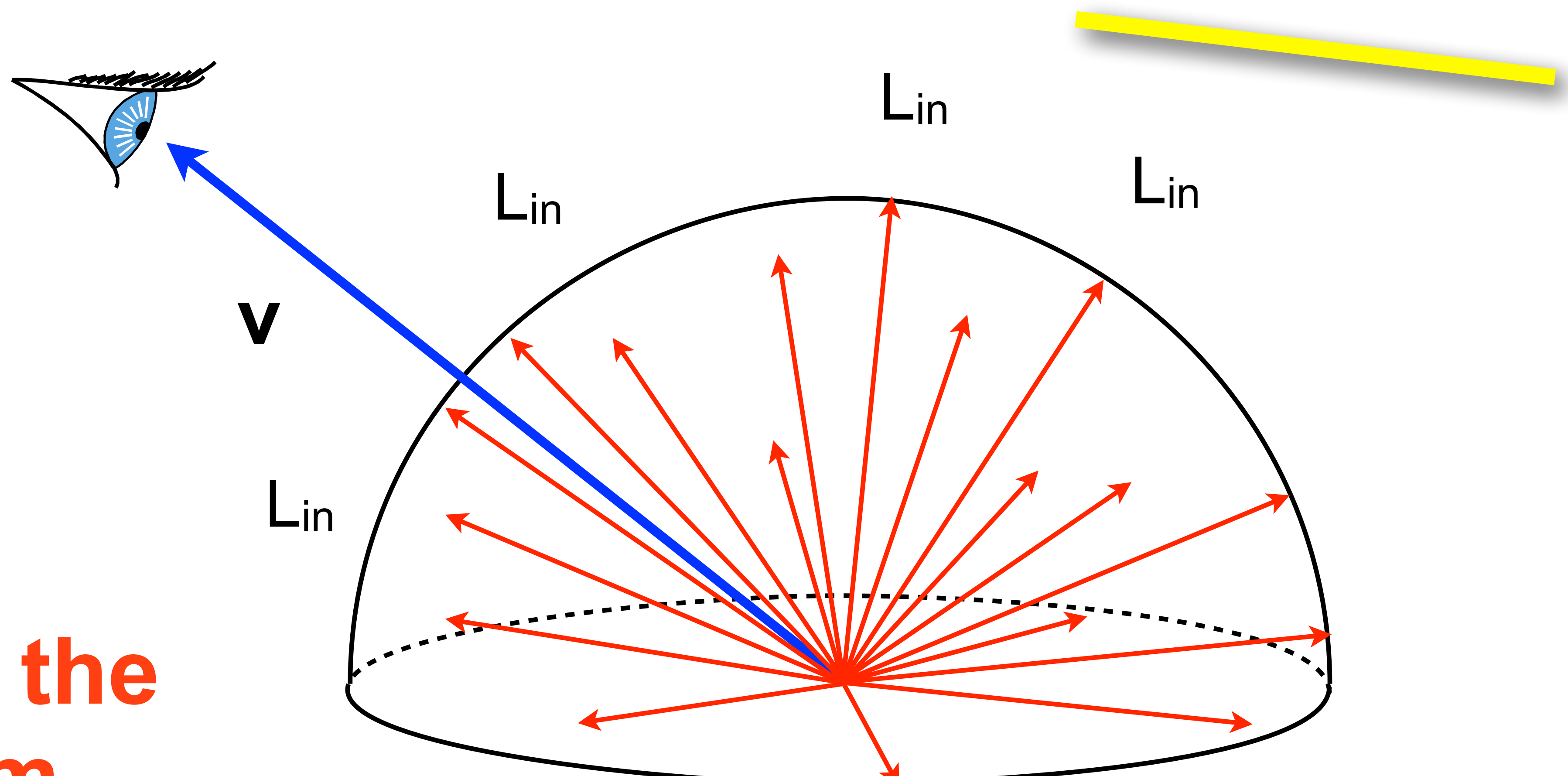
$$\rho(x)$$

is the albedo or *reflectivity* (between 0,1) of the surface at x

# Incident Light: Area Light Source

$$L_{\mathrm{out}}(x) = \frac{\rho(x)}{\pi} \int_{\Omega} L_{\mathrm{in}}(x, \omega)\, \cos\theta \, \mathrm{d}\omega$$

incident light
from direction $\omega$

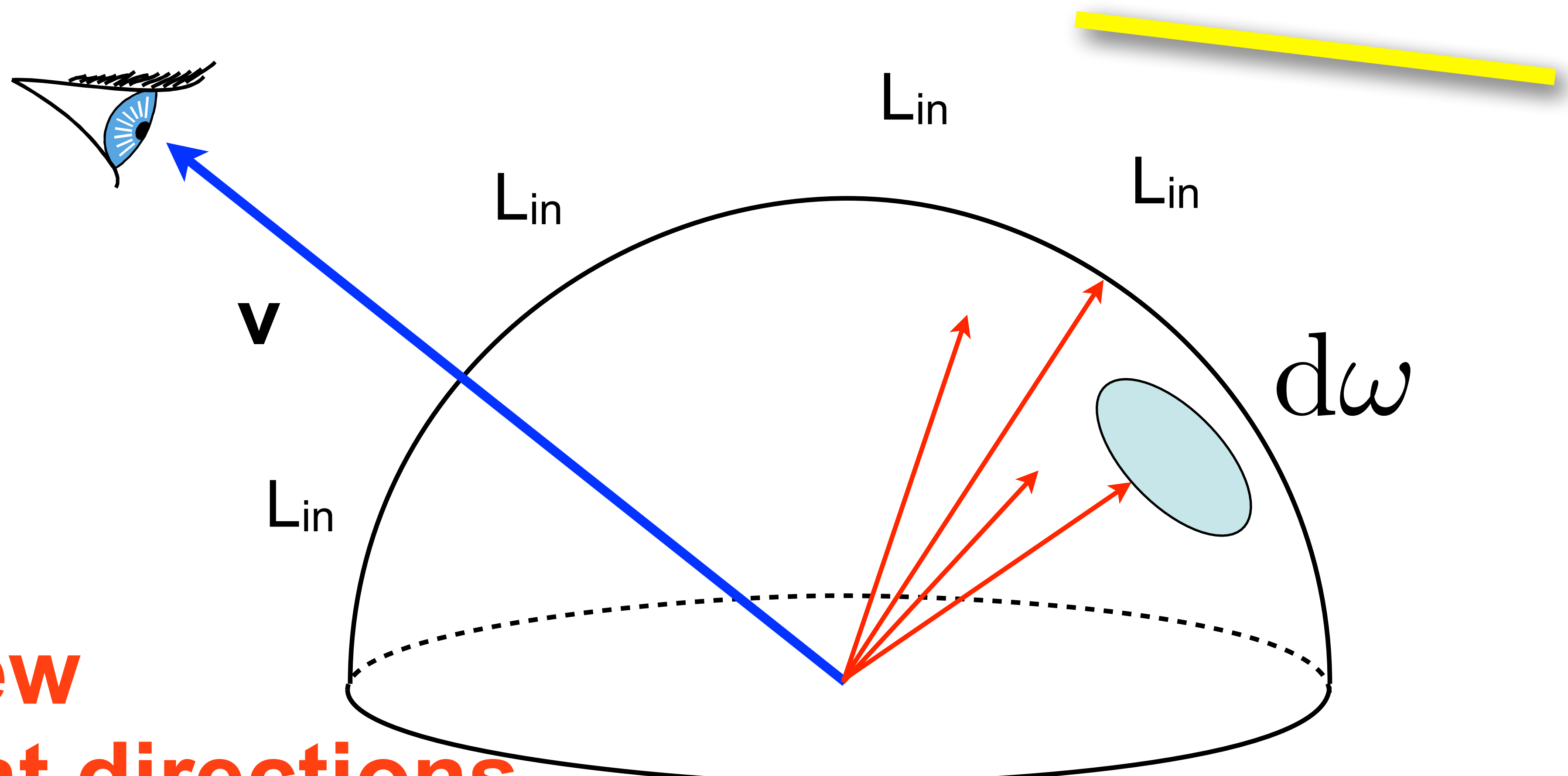



**What's the problem here?**

# Incident Light: Area Light Source

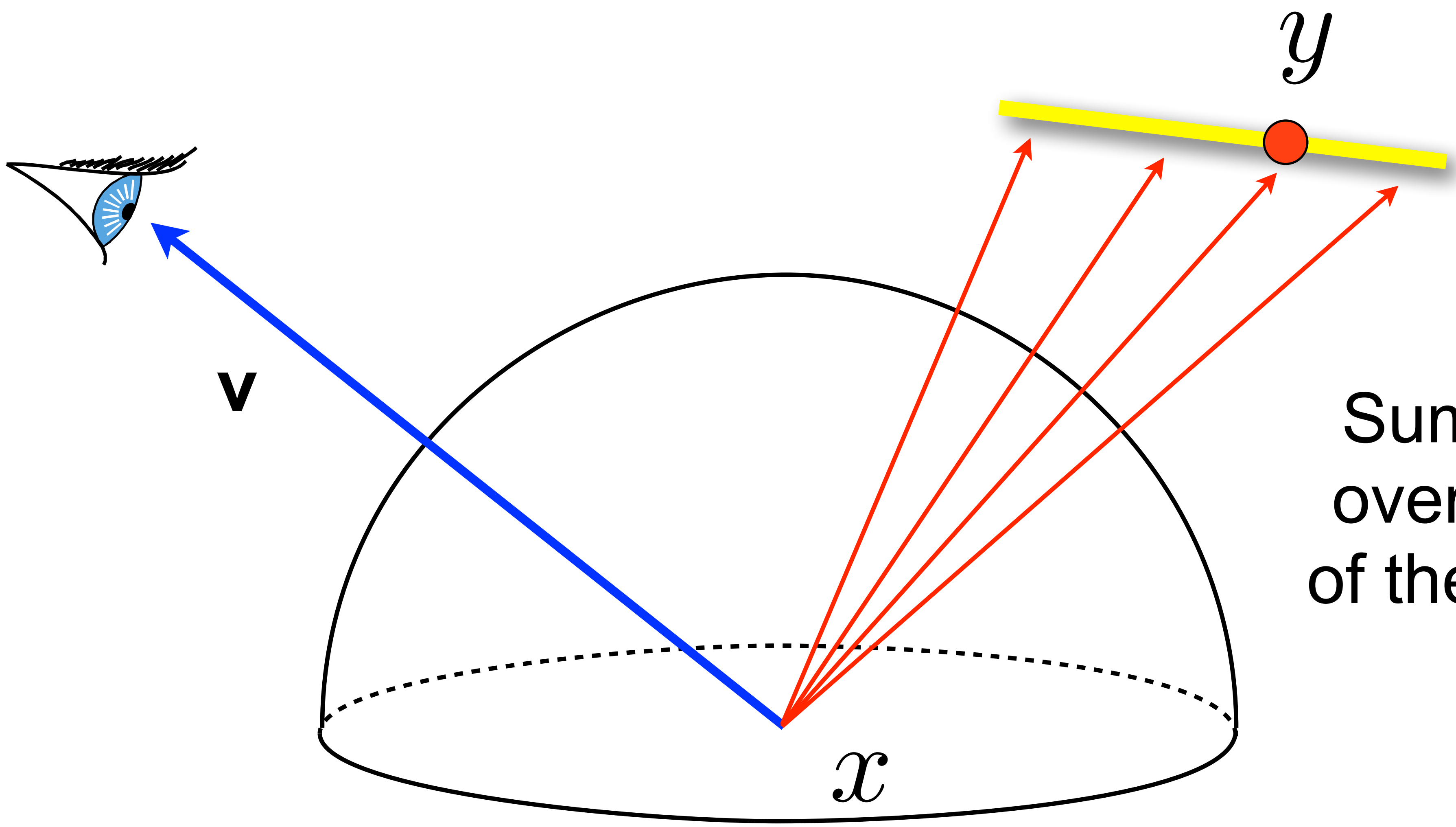
$$L_{\mathrm{out}}(x) = \frac{\rho(x)}{\pi} \int_{\Omega} L_{\mathrm{in}}(x, \omega)\ \cos\theta \, \mathrm{d}\omega$$

incident light
from direction $\omega$




**Only few incident directions contribute!**

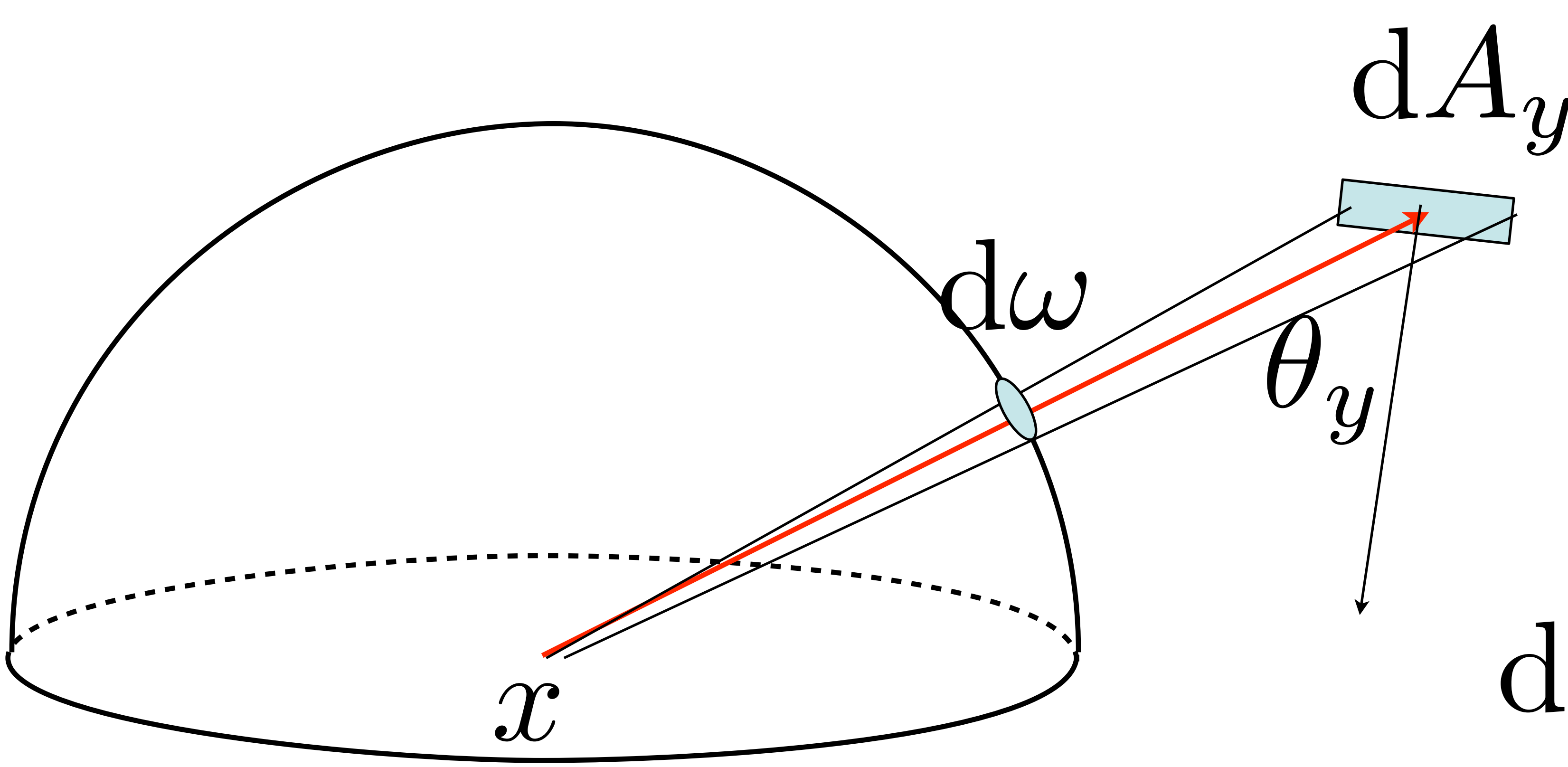
# Fortunately, We Know What To Do!




Sum (integrate) over the surface of the light source instead

# Looks Hairy, But Isn't

- We started today by looking at the solid angle, and how it relates to infinitesimal surface patches
- This really is just a change of integration variables
  - With proper normalization factors (you know this from math), integral over surface <=> integral over solid angle



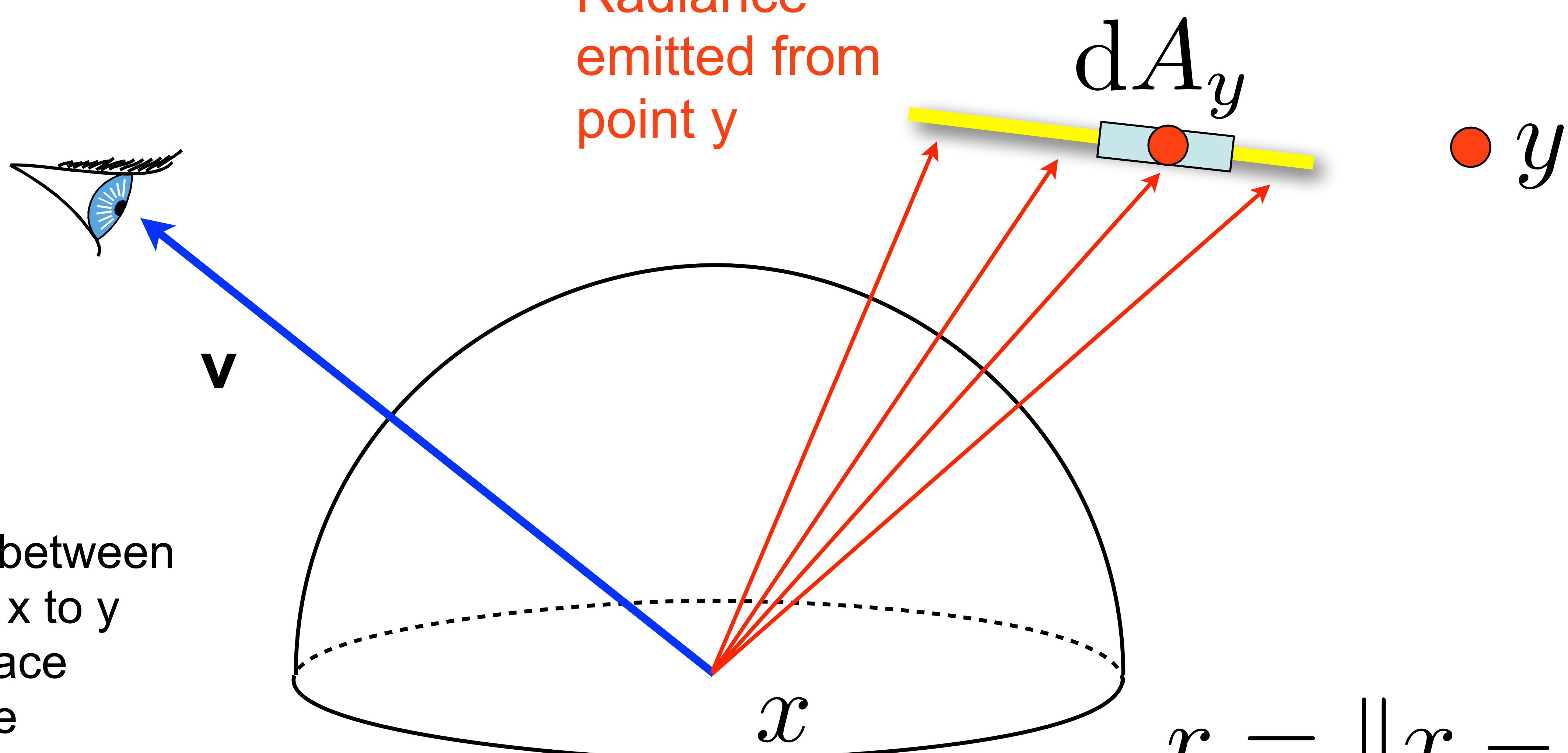

$$\mathrm{d}\omega = \frac{\cos\theta_y}{r^2}\,\mathrm{d}A_y$$

# Change variables and integrate over light

Area <=> solid angle conversion

$$L_{\mathrm{out}} = \frac{\rho(x)}{\pi} \int_{\mathrm{light}} E(y)\, \frac{\cos\theta_y}{r^2}\, \cos\theta \,\mathrm{d}A_y$$

Radiance emitted from point y

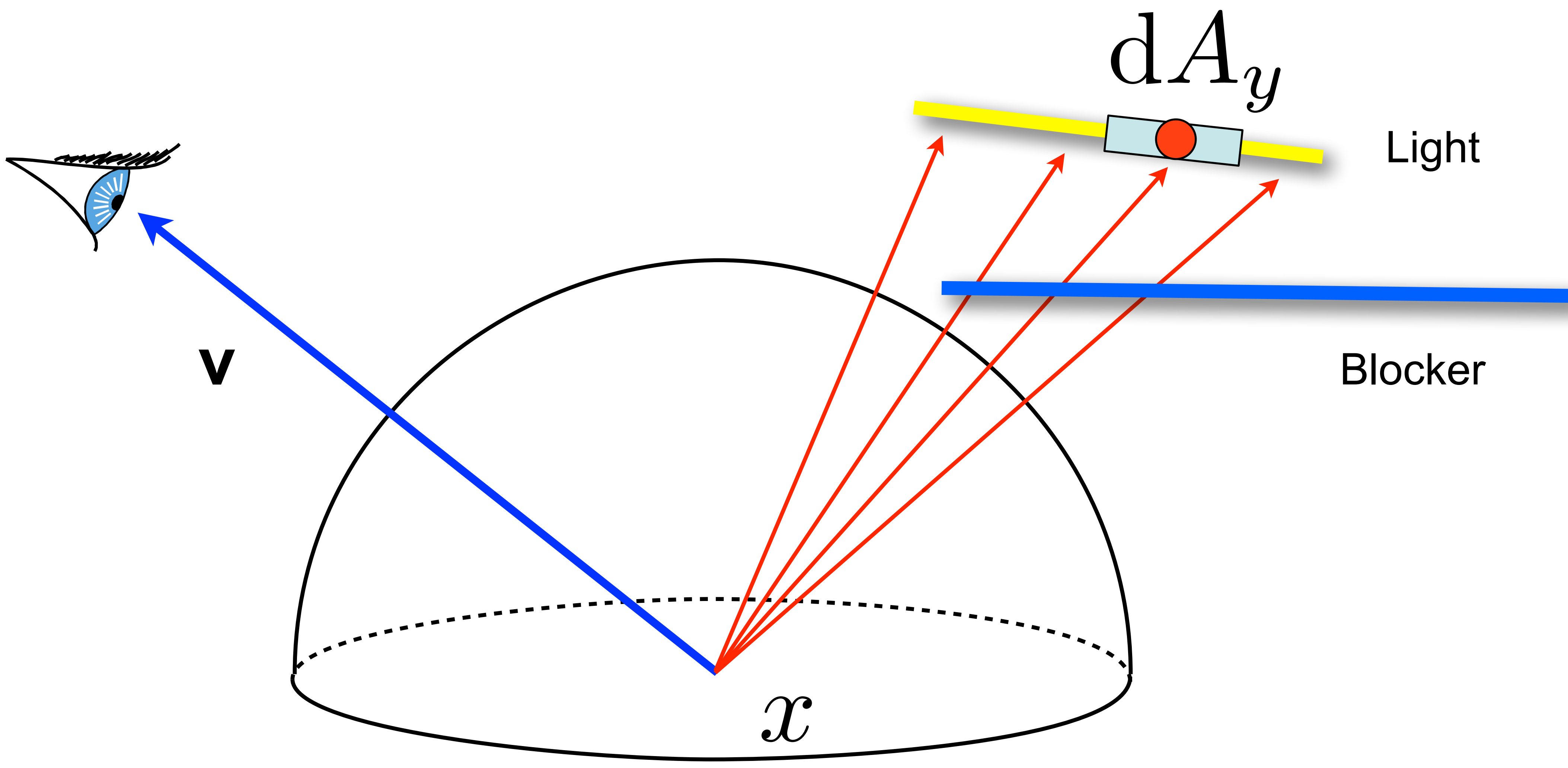



$\theta_y$

is the angle between the ray from x to y and the surface normal of the differential surface patch dA_y.

$$r = \|x - y\|$$

# Still Not Quite There Yet



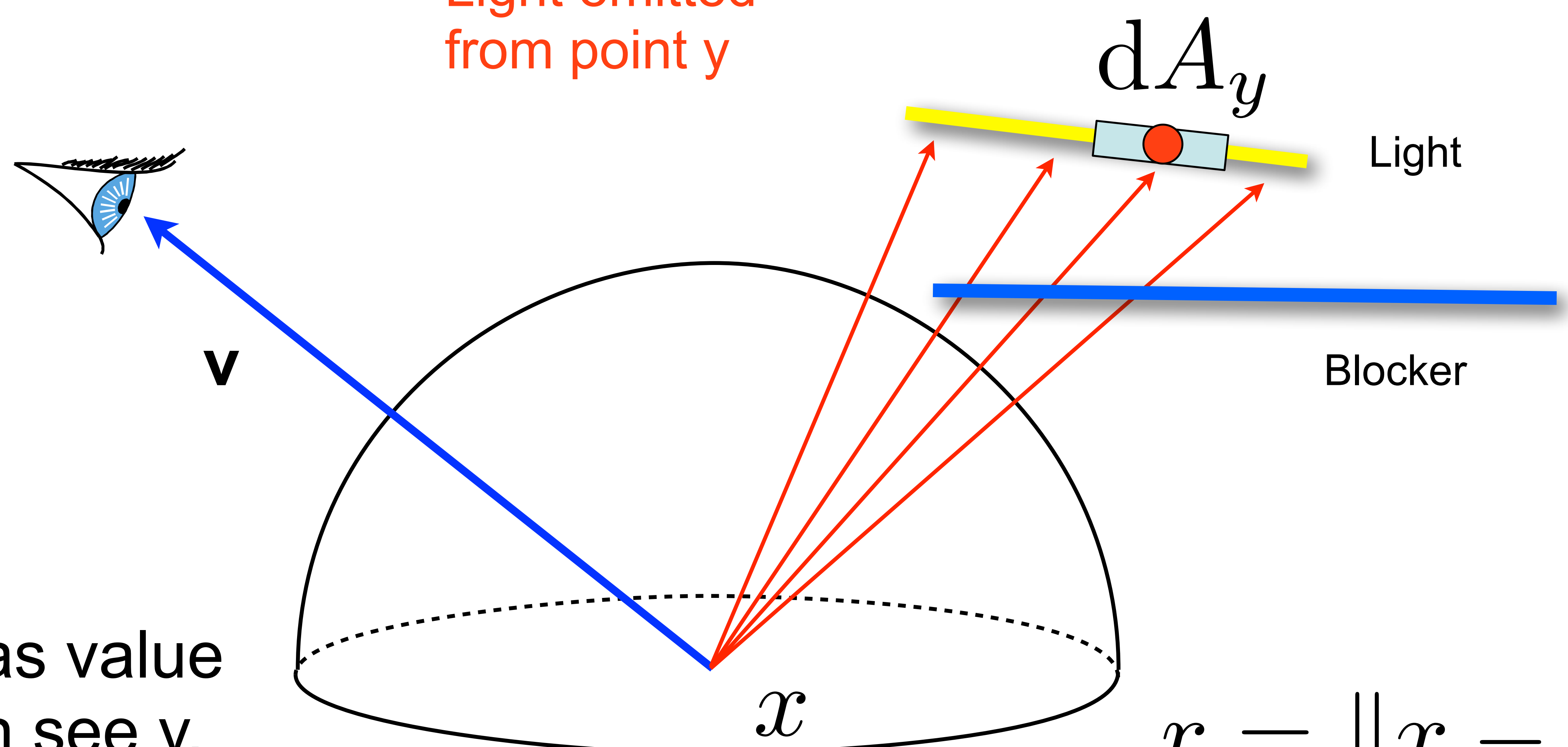
# Visibility Causes Soft Shadows

Area <=> solid angle conversion

Visibility function

$$L_{\text{out}} = \frac{\rho(x)}{\pi} \int_{\text{light}} E(y)\, V(x,y)\, \frac{\cos\theta_y}{r^2}\, \cos\theta\, \mathrm{d}A_y$$

Light emitted from point y




V(x,y) has value
1 if x can see y,
0 if not

$$r = \|x - y\|$$

# Questions?

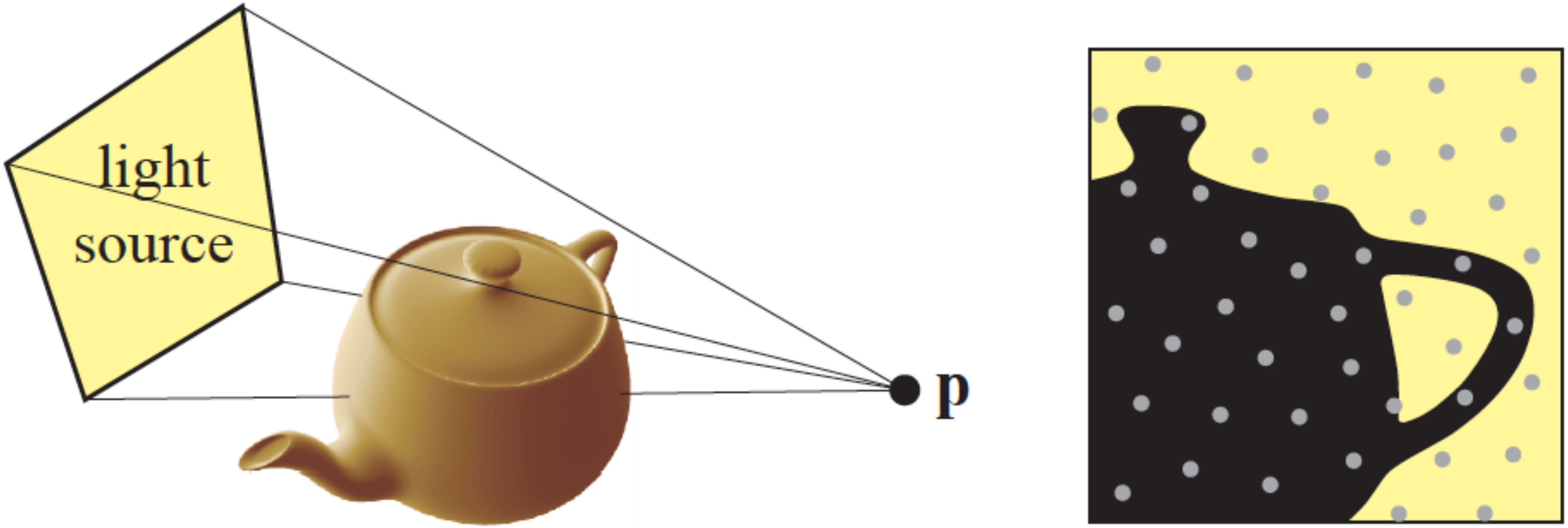
Laine et al., cover of SIGGRAPH 2005 proceedings

# Algorithm for Diffuse Soft Shadows

$$L_{\text{out}} = \frac{\rho(x)}{\pi} \int_{\text{light}} E(y)\, V(x,y)\, \frac{\cos\theta_y}{r^2}\, \cos\theta \,\mathrm{d}A_y$$

```
for each visible point x
 Generate N random points y_i on light source, store
 probabilities p_i as well (uniform: p_i == 1/A)
 est = 0
 for each y_i, i=1,...,N
  Cast shadow ray to evaluate V(x,y_i)
  if visible
   est = est + E(y_i)cos(theta_yi)cos(theta)/r^2/p_i
  endif
 endfor
 L_out(x) = 1/N * est * rho(x)/pi
endfor
```

# Intuitive Picture




(a) (b)

Laine et al. SIGGRAPH 2005

# I've Skipped Ahead of Myself

- Note the use of random numbers
  - We are performing Monte Carlo integration
  - We'll come to that

- **BUT:** Why not write an area light renderer as extra credit for your first programming assignment?
  - After writing code to place the light where you want, you can pretty much translate the pseudocode into actual C++

- Also, note that we haven't talked about non-diffuse surfaces or indirect illumination, yet.

# That's It for Today

- Next week: reflectance equation, rendering equation

- Useful reading
  - Pat Hanrahan's slides on radiometry
    - More detail than what we've covered today, **highly recommended**
  - Monte Carlo integration
  - Phil Dutré's Global Illumination Compendium
    - A handy collection of most math that relates to GI
  - Dutré, Bala, Bekaert: Advanced Global Illumination
  - Cohen, Wallace: Radiosity and Realistic Image Synthesis
  - Pharr, Humphreys, Jakob: Physically Based Rendering, 3rd ed.