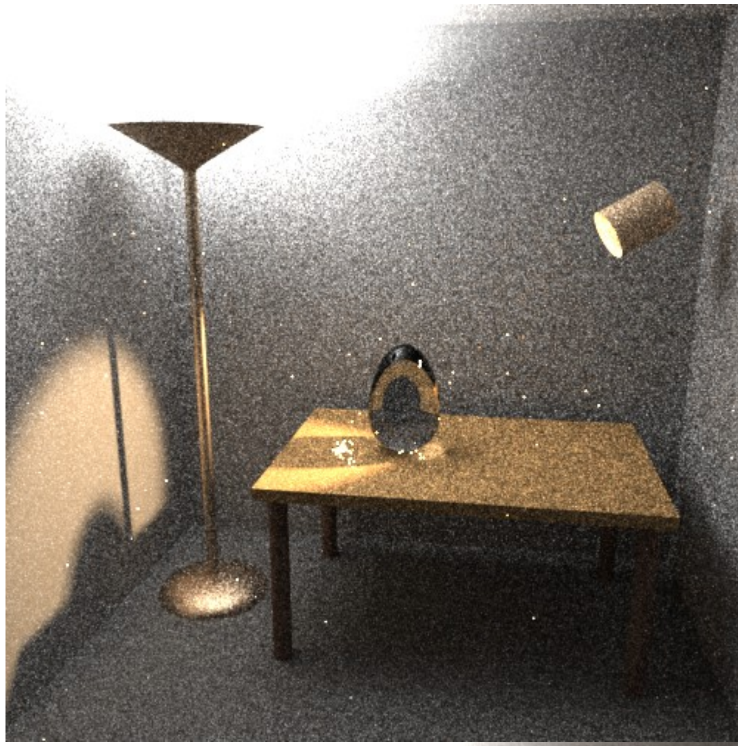
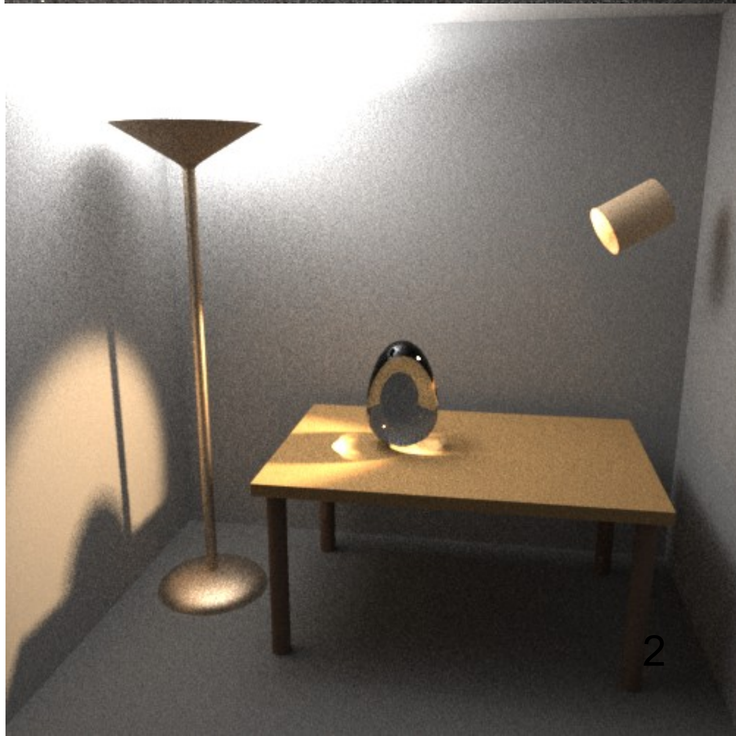
Surface Reflectance, BRDFs

Aalto CS-E5520 Spring 2024 Jaakko Lehtinen
with some slides from Frédo Durand of M.I.T.

Today

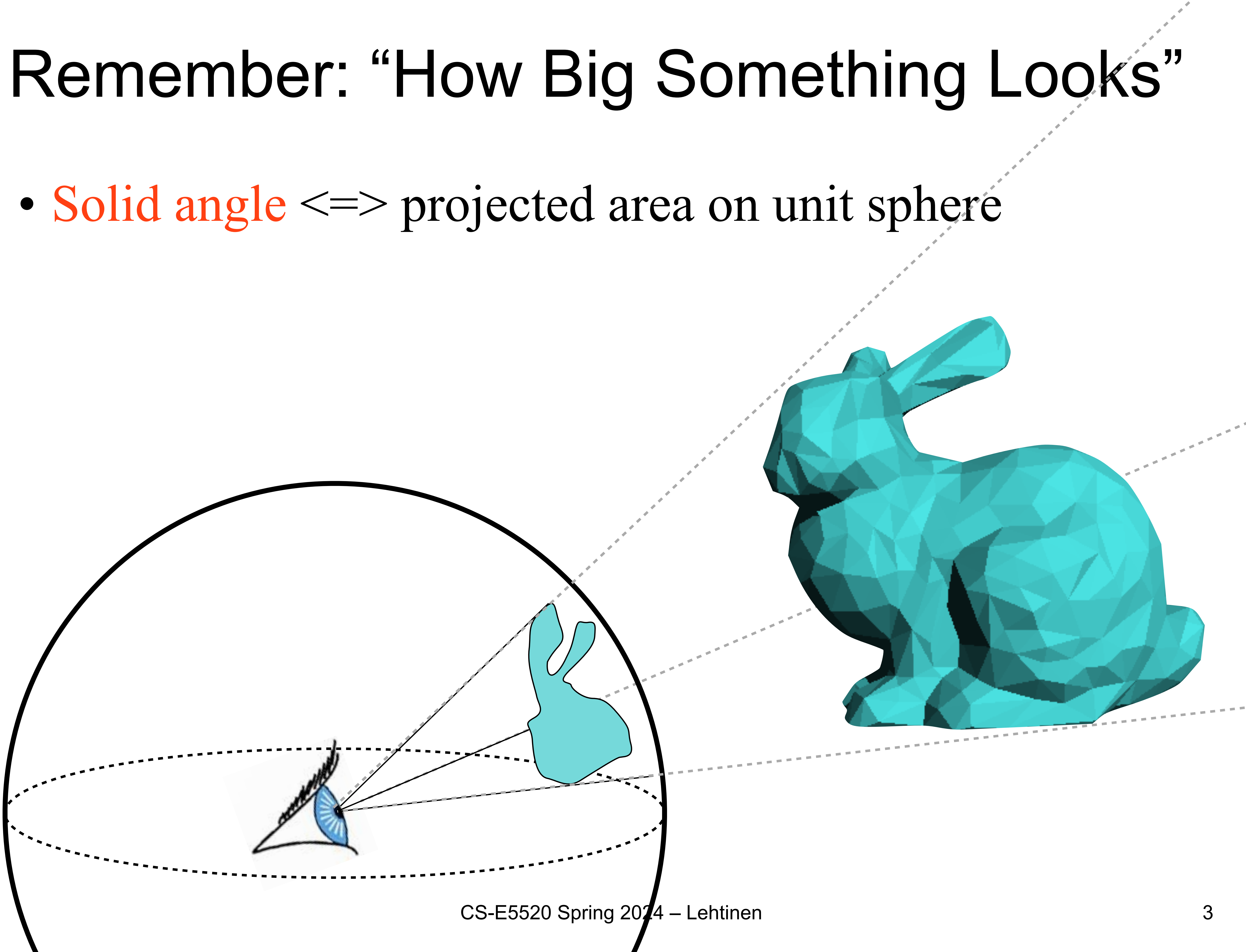
- Reflectance Equation
  - Recap of the BRDF
  - plus details





Remember: “How Big Something Looks”

- Solid angle <=> projected area on unit sphere



Recap: Radiance

- **Sensors are sensitive to radiance**
  - It’s what you assign to pixels
  - The fundamental quantity in image synthesis

- “Intensity does not attenuate with distance”
  <=> **radiance stays constant along straight lines****

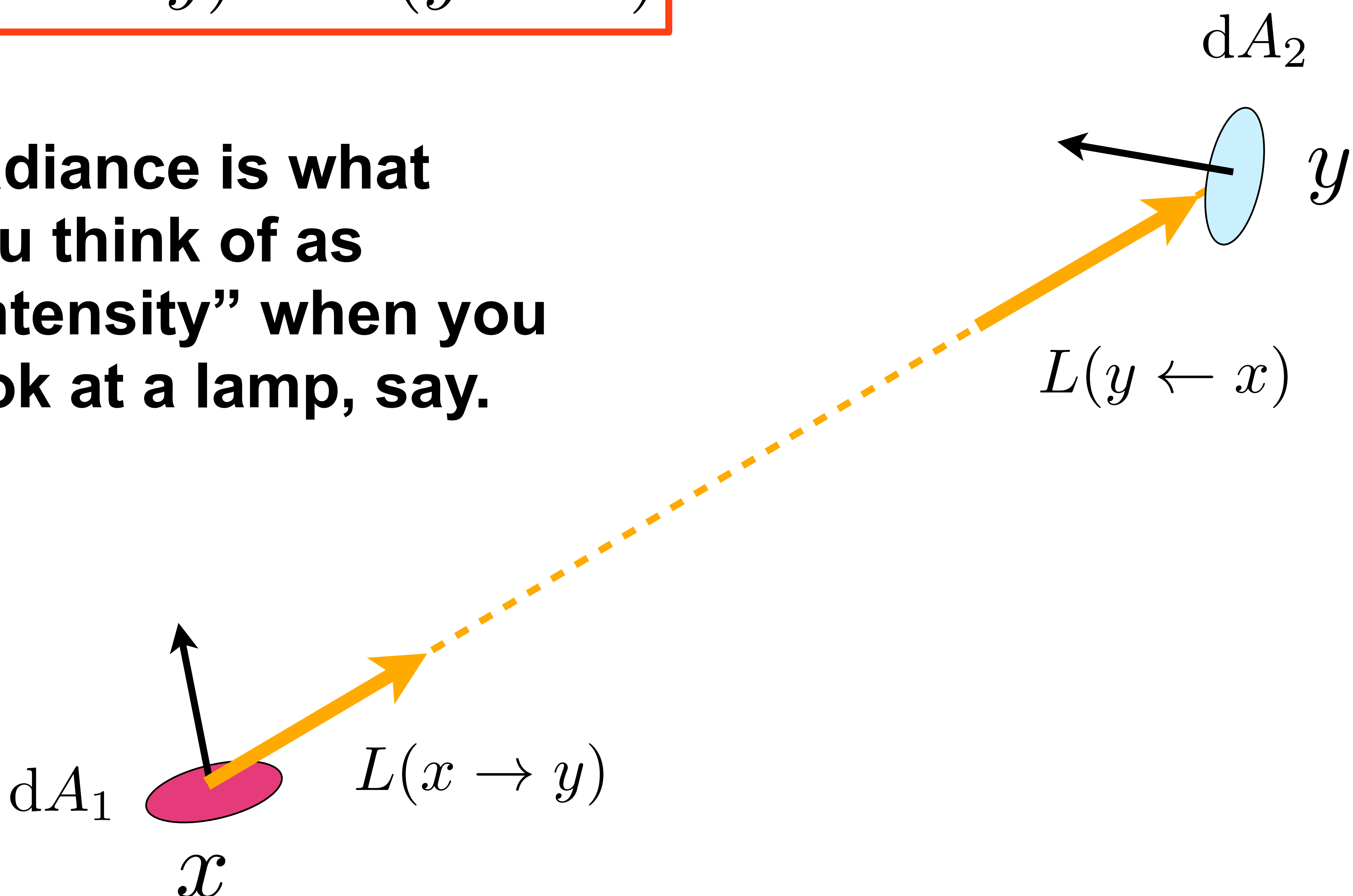
- **All relevant quantities (irradiance, etc.) can be derived from radiance**

**unless the medium is participating, e.g., smoke, fog

Constancy Along Straight Lines

$$L(x \to y) = L(y \leftarrow x)$$

Radiance is what you think of as "intensity" when you look at a lamp, say.



Recap: Radiance

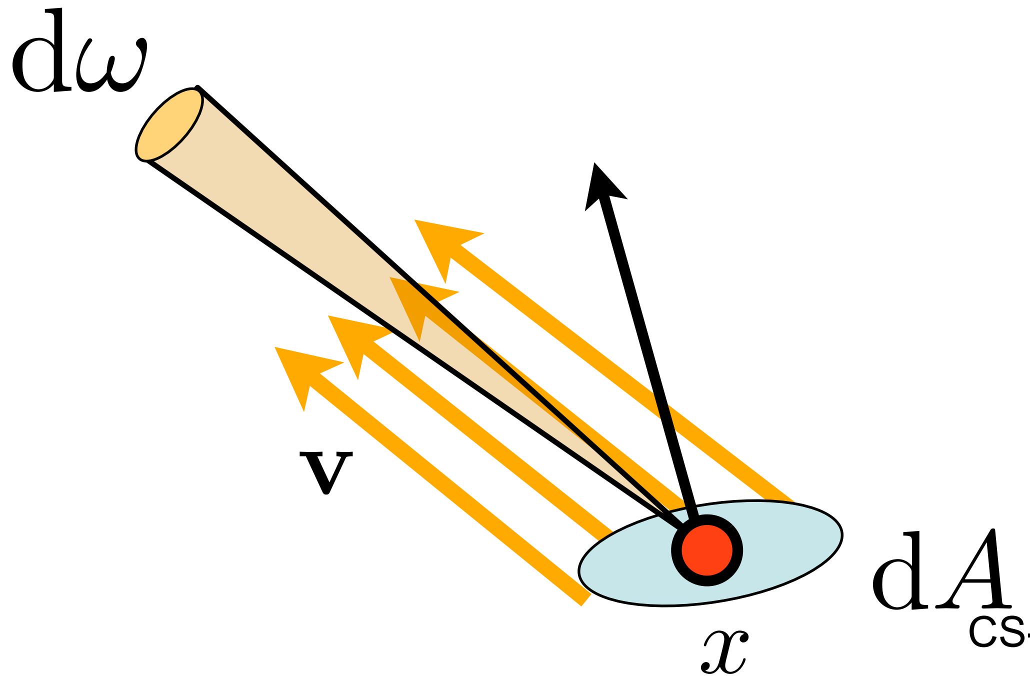
- Radiance L = **flux per unit projected area per unit solid angle**

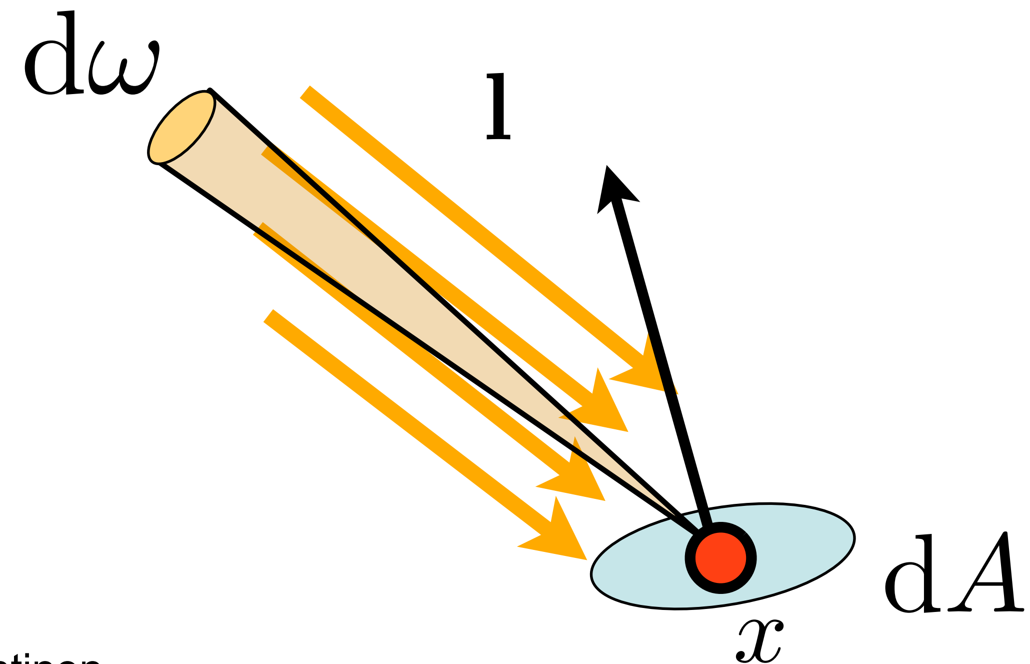
$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$

$$[L] = \left[\frac{W}{m^2\ sr}\right]$$

Recap: Radiance Notation

- $L(x \rightarrow \mathbf{v})$ denotes radiance leaving dA located at point x towards direction $\mathbf{v}$
  - Alternative notation: $L_{\mathrm{out}}(x, \mathbf{v})$
- $L(x \leftarrow \mathbf{l})$ denotes radiance impinging on dA located at point x from direction $\mathbf{l}$
  - Alternative notation: $L_{\mathrm{in}}(x, \mathbf{l})$





Recap: Irradiance

- Integrate incident radiance times cosine over the hemisphere Ω

$$E = \int_{\Omega} L(\omega)\ \cos\theta\ \mathrm{d}\omega$$

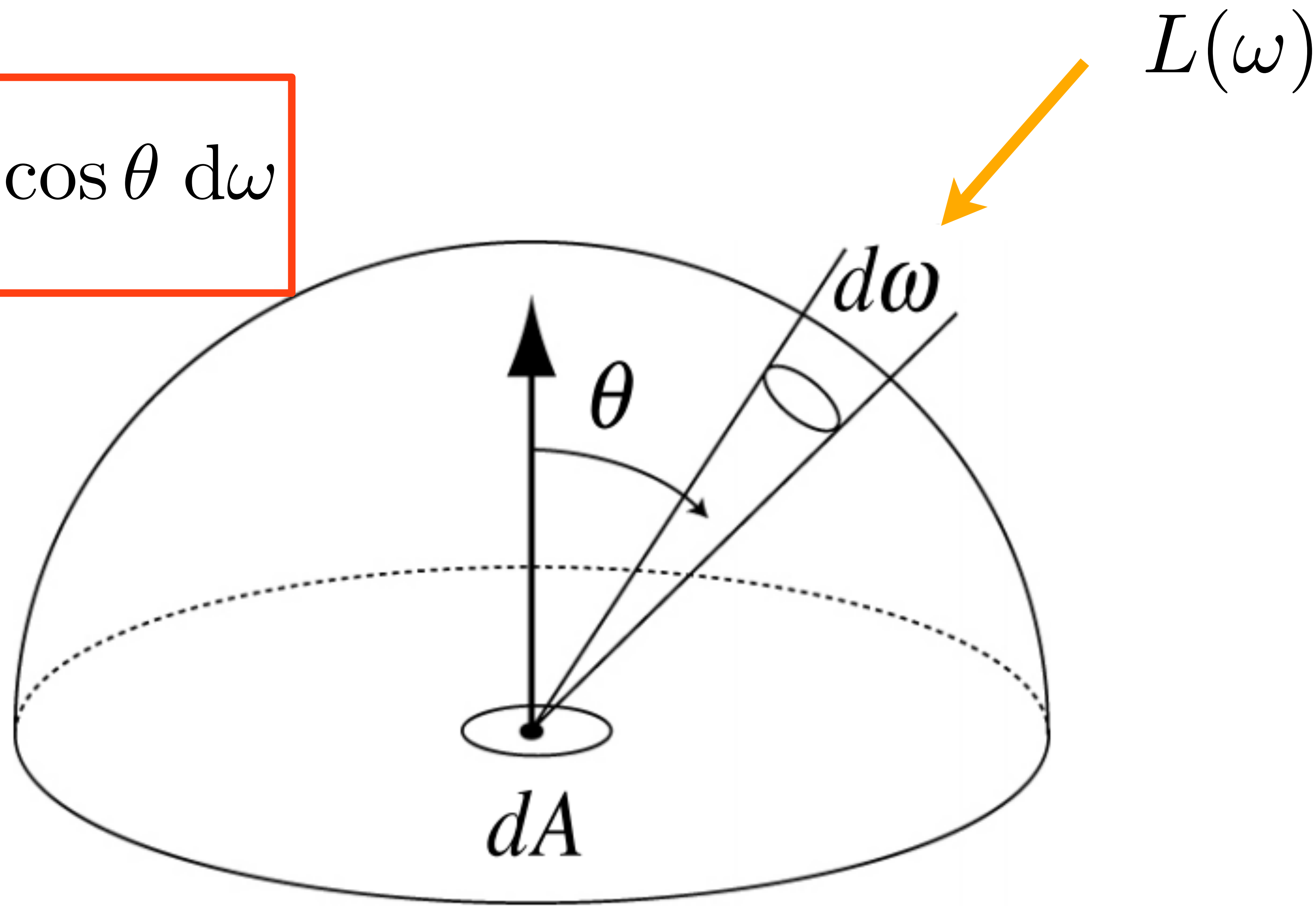
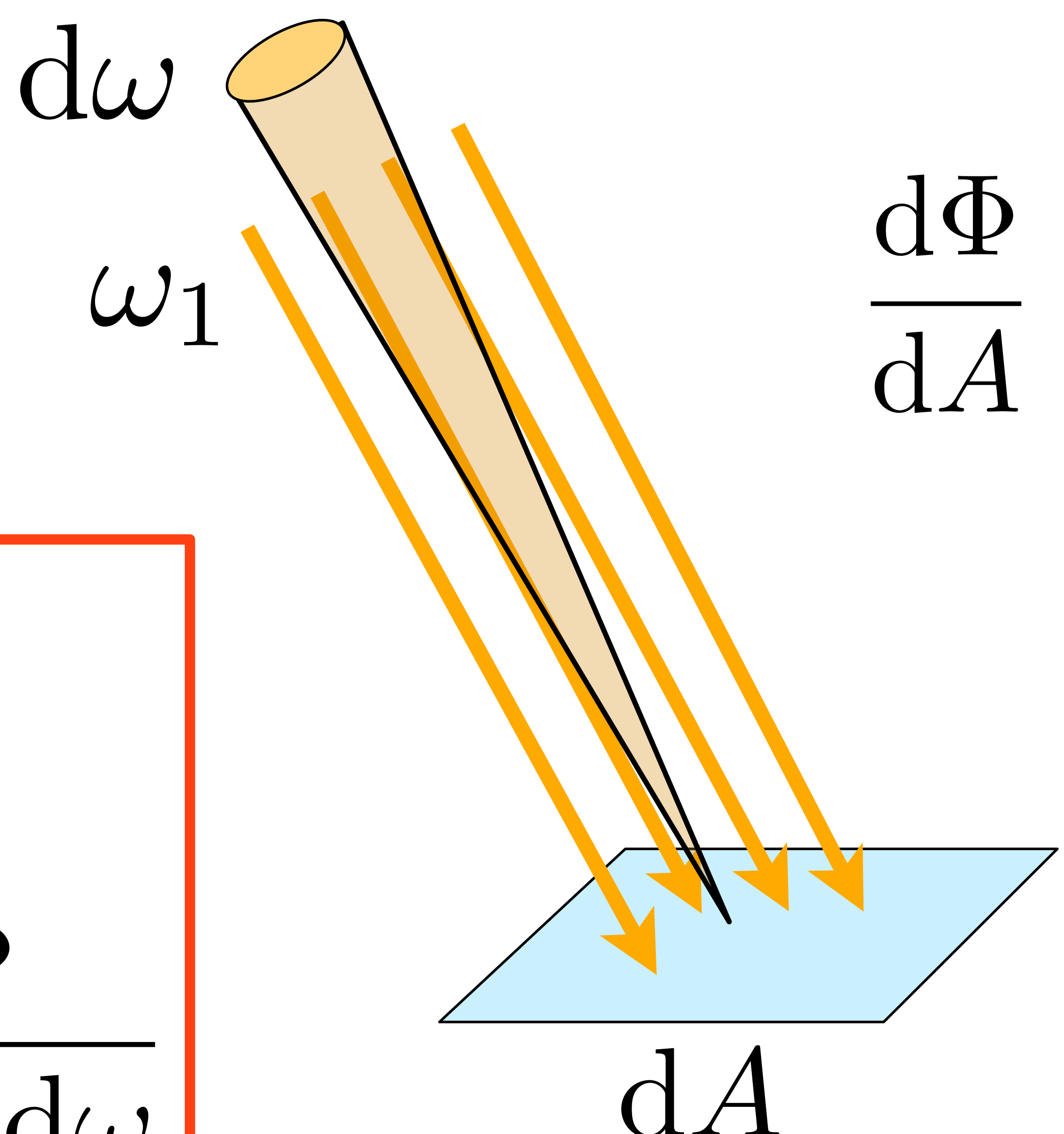


Figure adapted from Pat Hanrahan

Recap: Differential Irradiance

- To measure irradiance, add up the radiance from all the differential beams from all directions

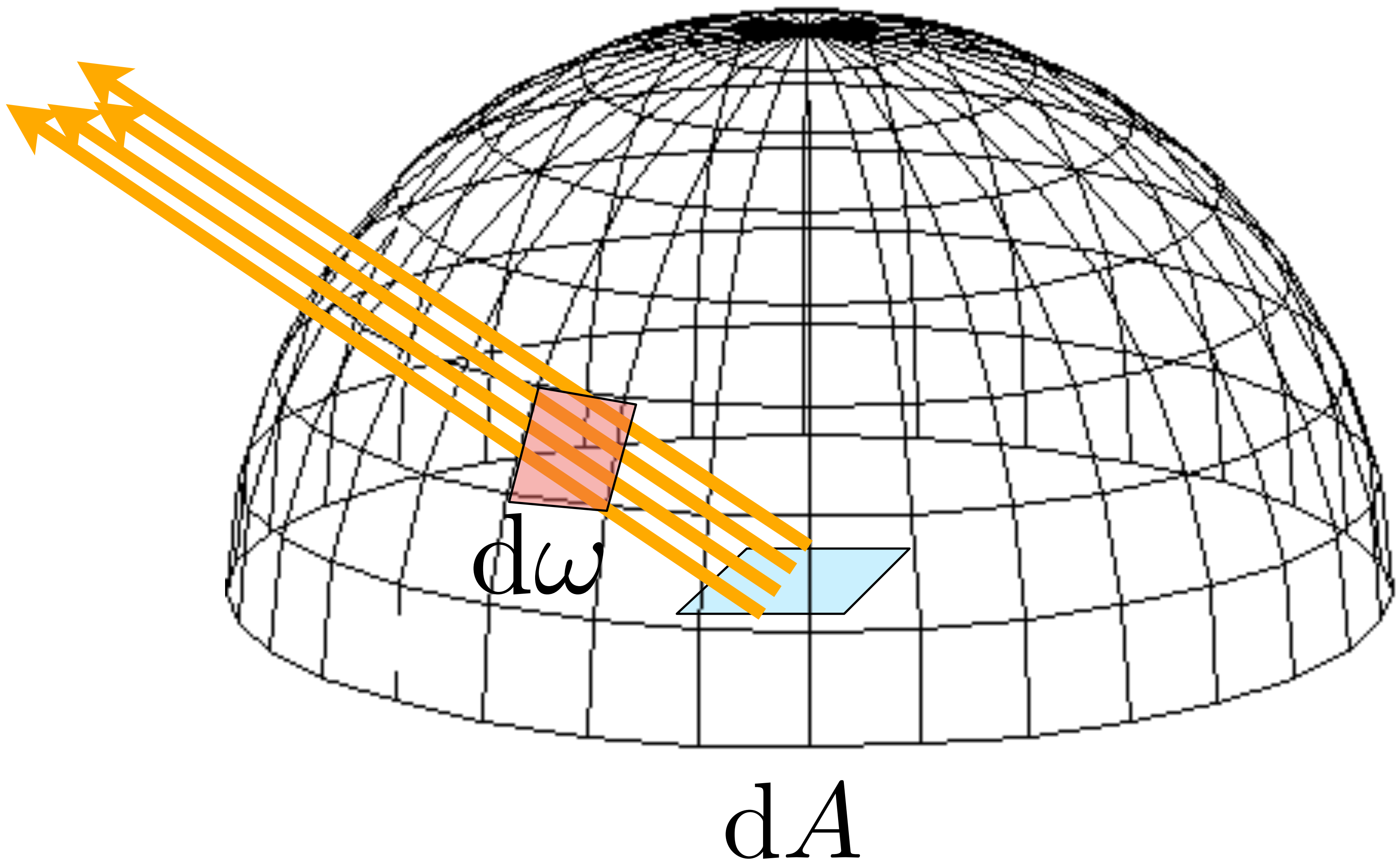


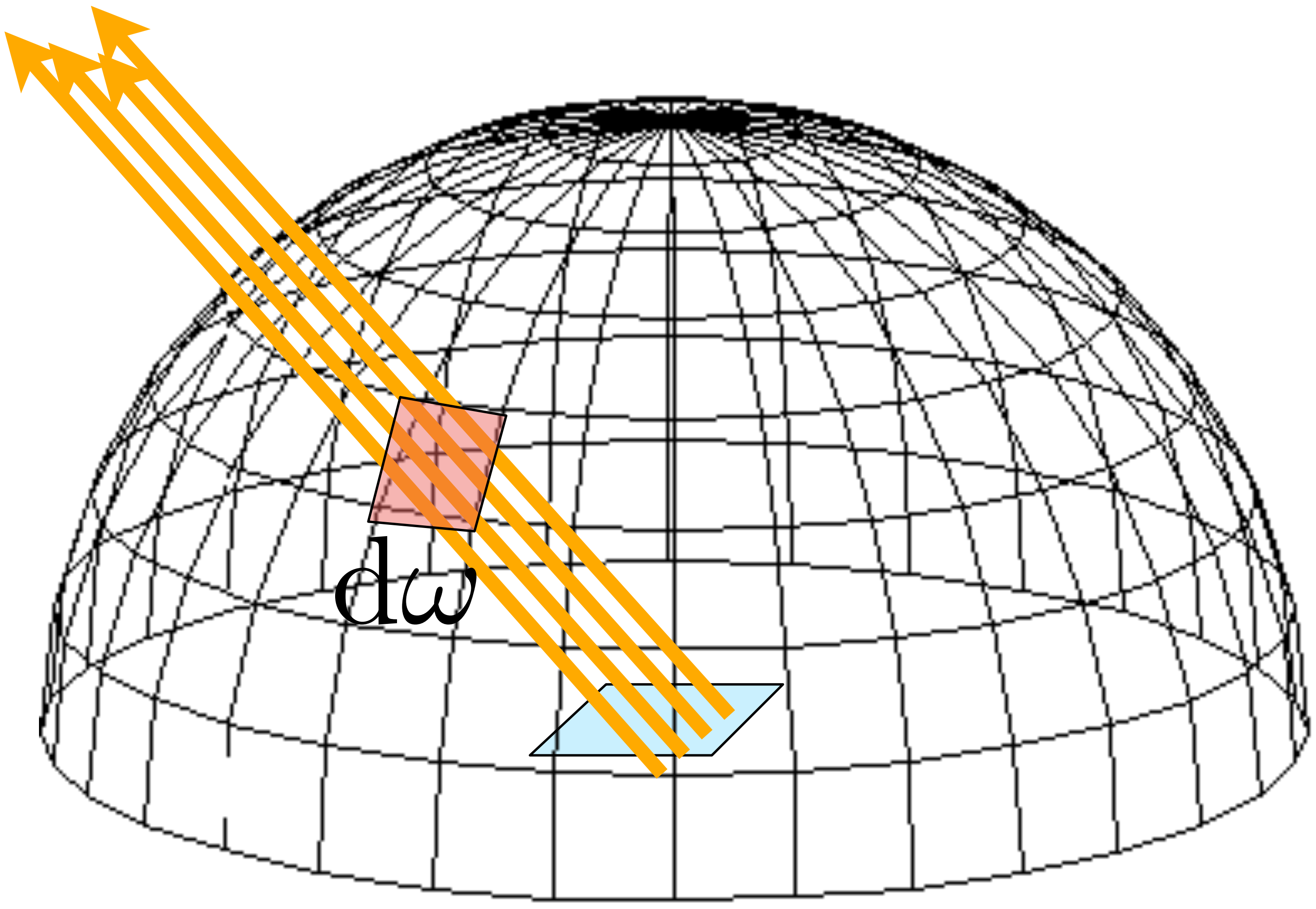
$$\frac{\mathrm{d}\Phi}{\mathrm{d}A} = \underbrace{L(\omega_1)\cos\theta}\,\mathrm{d}\omega$$

Differential irradiance

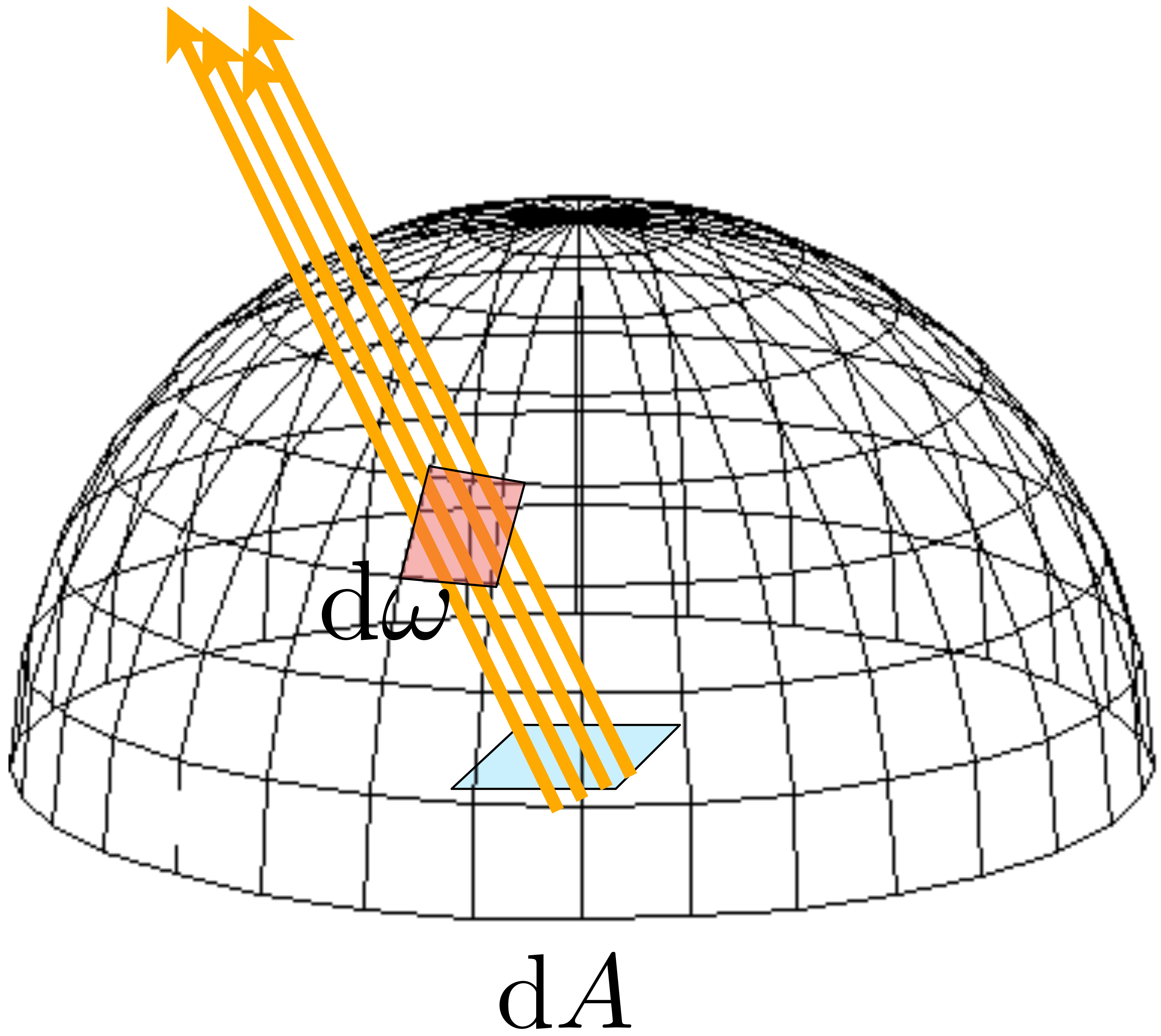
$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A}$$

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\,\mathrm{d}\omega}$$


dω
dA


$d\omega$
dA

- ...



$\mathrm{d}\omega$
$\mathrm{d}A$

Recap: Irradiance to Radiosity

- The reflectivity of a diffuse surface is determined by its *albedo* $\rho \in [0, 1)$
  - This is the “diffuse color k_d” from your ray tracer in 4310
- The flux emitted by a diffuse surface per unit area is called *radiosity* B
  - Same units as irradiance, [B] = [W/m^2]
  - Hence

$$B = \frac{\rho E}{\pi}$$

Recap: Lambertian Soft Shadows

differential solid angle

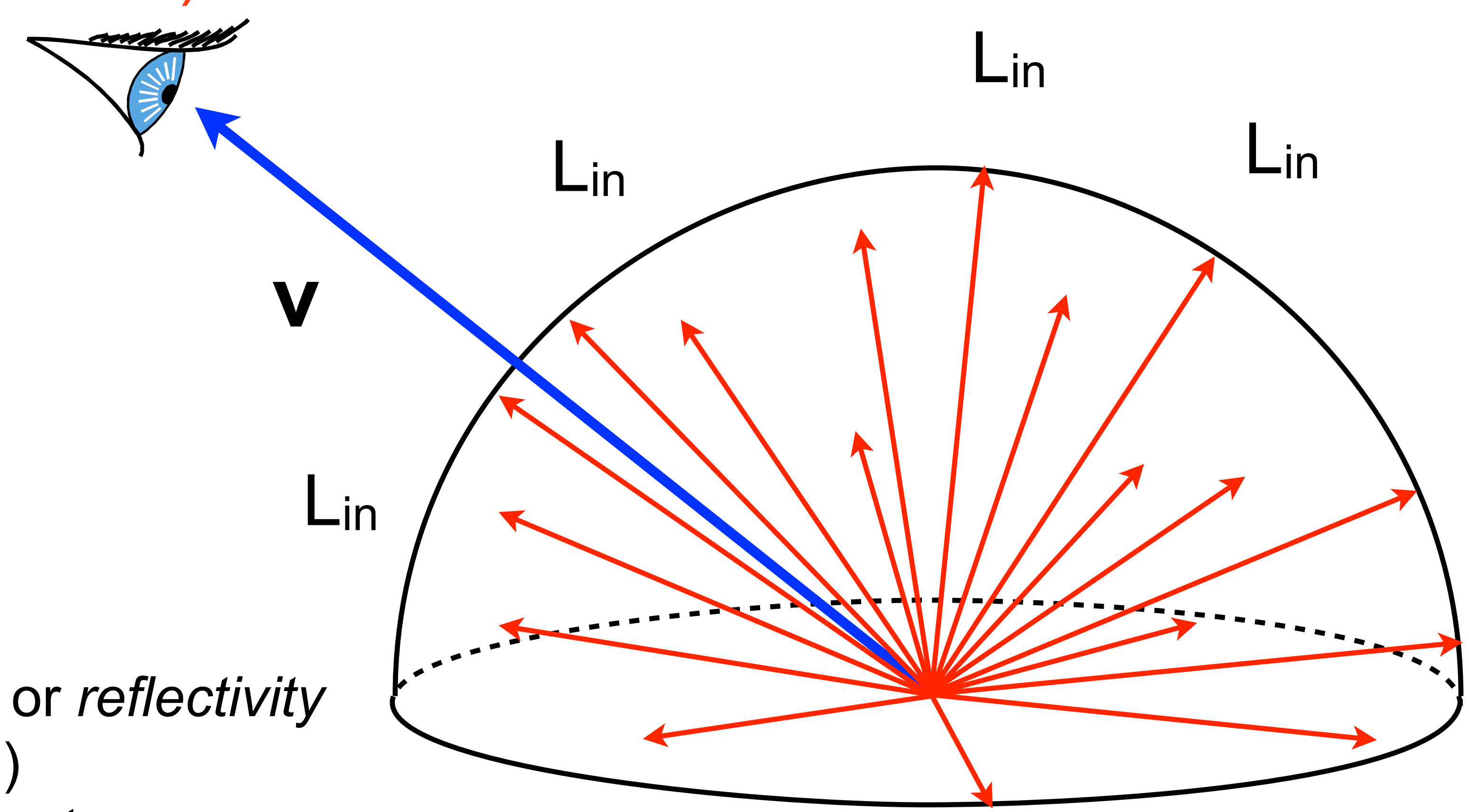
$$L_{\text{out}}(x) = \frac{\rho(x)}{\pi} \int_{\Omega} L_{\text{in}}(x, \omega) \cos\theta \, \mathrm{d}\omega$$

outgoing light (diffuse => independent of direction v)

albedo/pi

incident radiance

cosine term



Sum (integrate) over every direction on the hemisphere, modulate incident illumination by cosine, albedo/pi

$\rho(x)$

is the albedo or *reflectivity* (between 0,1) of the surface at x

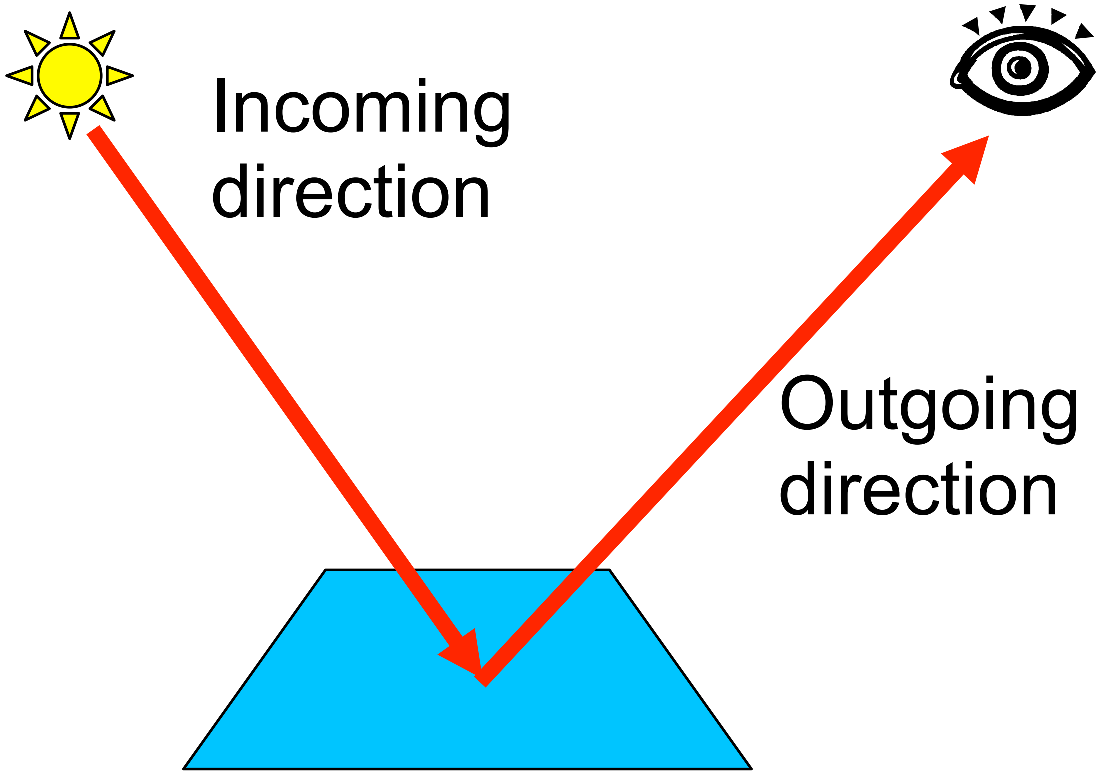
Last Time: Diffuse Reflectance Only

None of these surfaces are diffuse!

Luxology user dreamweaver

Quantifying Reflection – BRDF

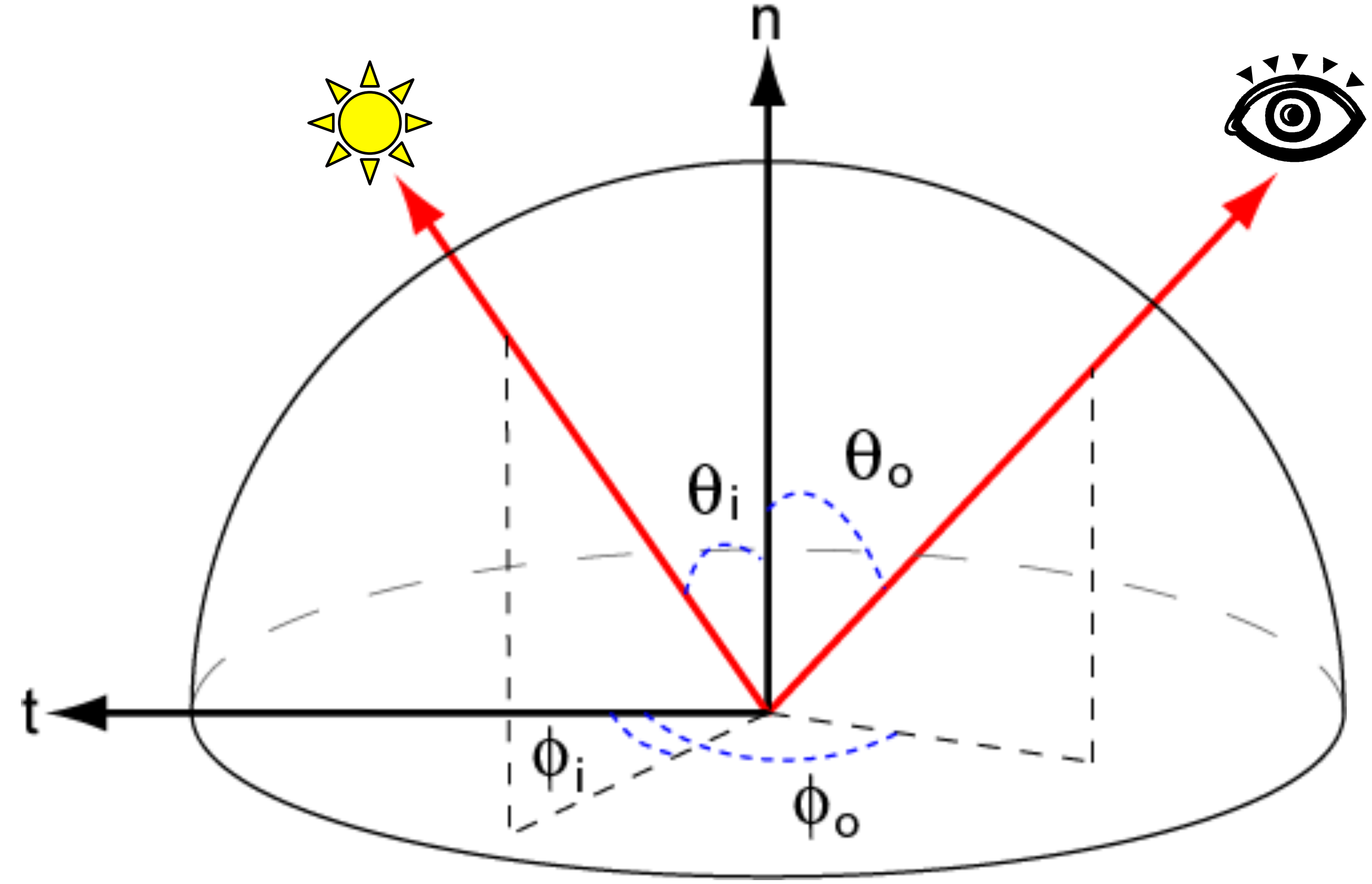
- Bidirectional Reflectance Distribution Function
- "Ratio of light coming from one direction that gets reflected in another direction"
  - Pure reflection, assumes no light scatters into the material



- Focuses on angular aspects, not spatial variation of the material
- **How many dimensions?**

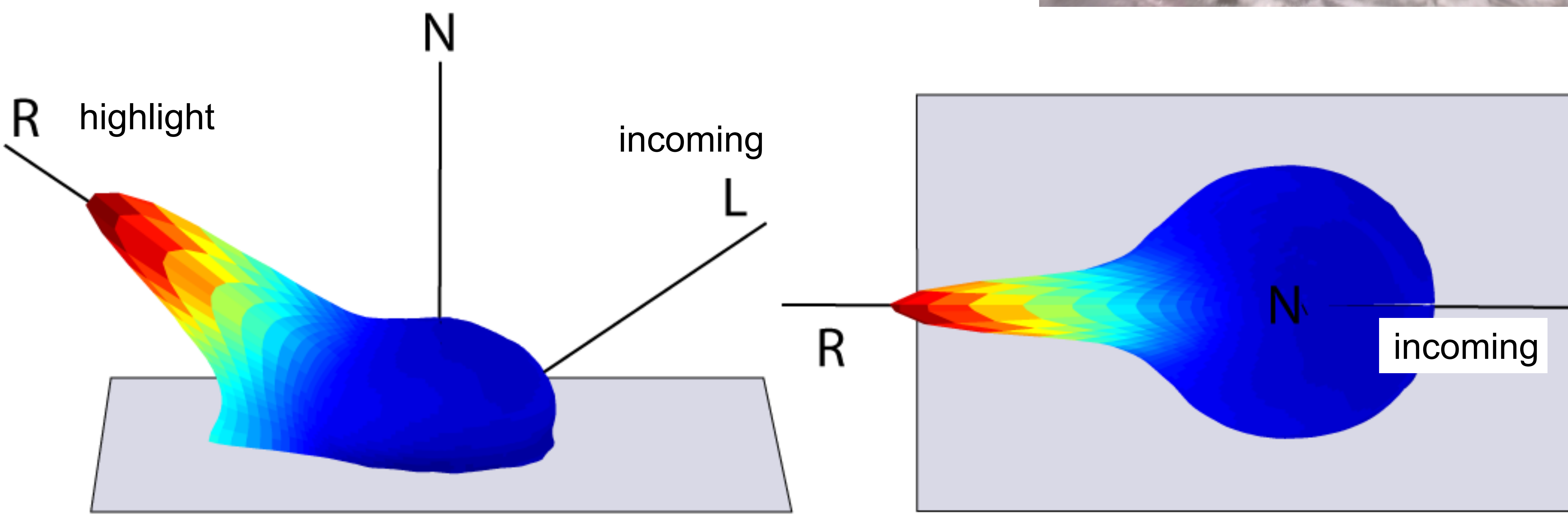
BRDF f_r

- Bidirectional Reflectance Distribution Function
  - 4D: 2 angles for each direction
  - BRDF = $f_r(\theta_i, \phi_i; \theta_o, \phi_o)$
  - Or just two unit vectors: BRDF = $f_r(\mathbf{l}, \mathbf{v})$
    - **l** = light direction
    - **v** = view direction



2D Slice at Constant Incidence

- For a fixed incoming direction **l**, view dependence is a 2D spherical function
  - Here a moderate glossy component towards mirror direction R



Example: Plot of “PVC” BRDF at 55° incidence

BRDF f_r

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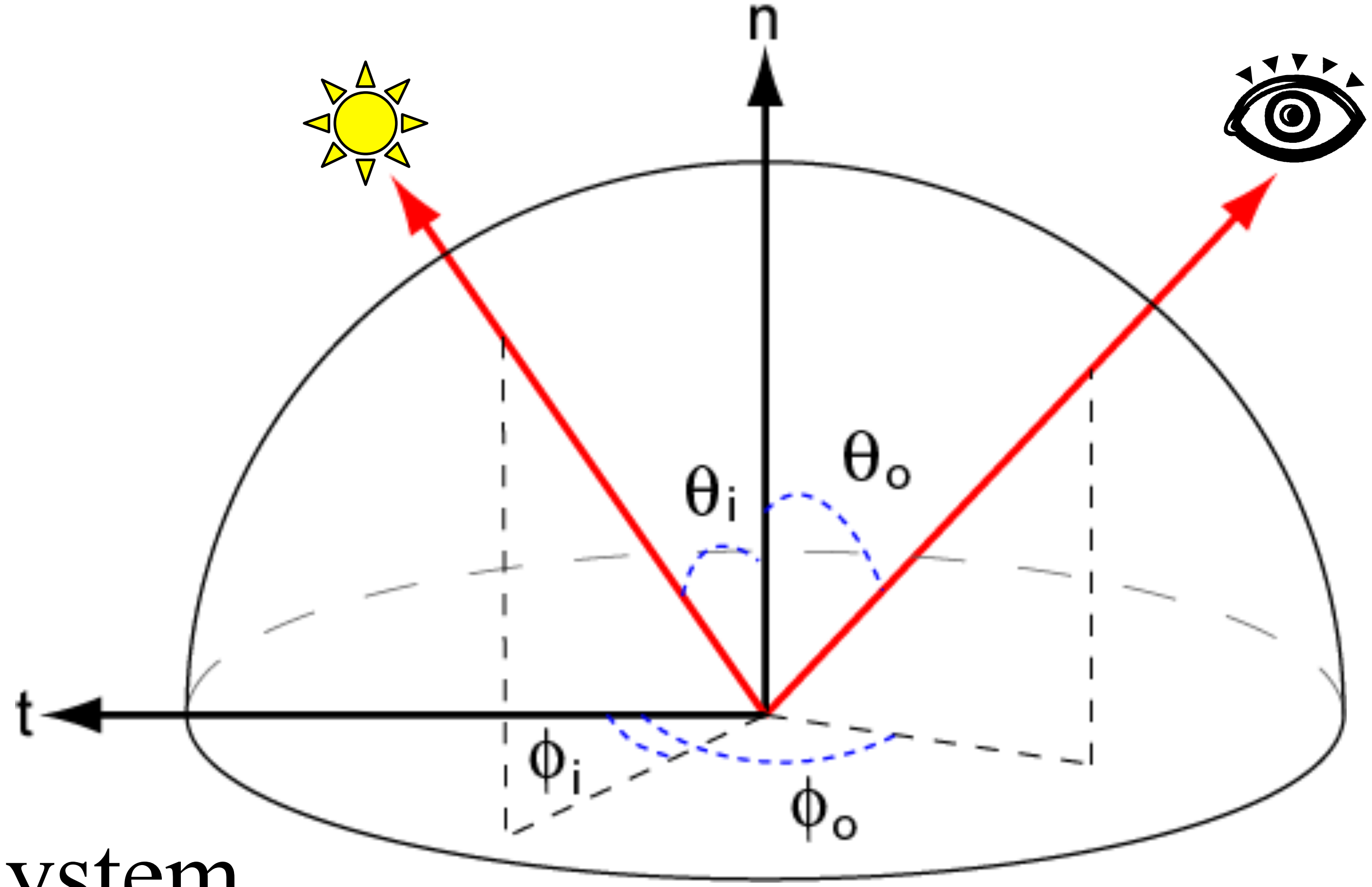
  - Or just two unit vectors: BRDF = $f_r(\mathbf{l}, \mathbf{v})$
    - **l** = light direction
    - **v** = view direction
  - The BRDF is aligned with the surface; the vectors **l** and **v** must be in a local coordinate system

Mirror BRDF: Infinitely thin and tall spike ("Dirac delta") in mirror direction



BRDF Definition, For Real This Time

- Relates **incident differential irradiance** from every direction **to outgoing radiance**

$$\text{BRDF}(\mathbf{l}, \mathbf{v}) = \frac{\text{radiance to direction } \mathbf{v}}{\text{differential irradiance from direction } \mathbf{l}}$$

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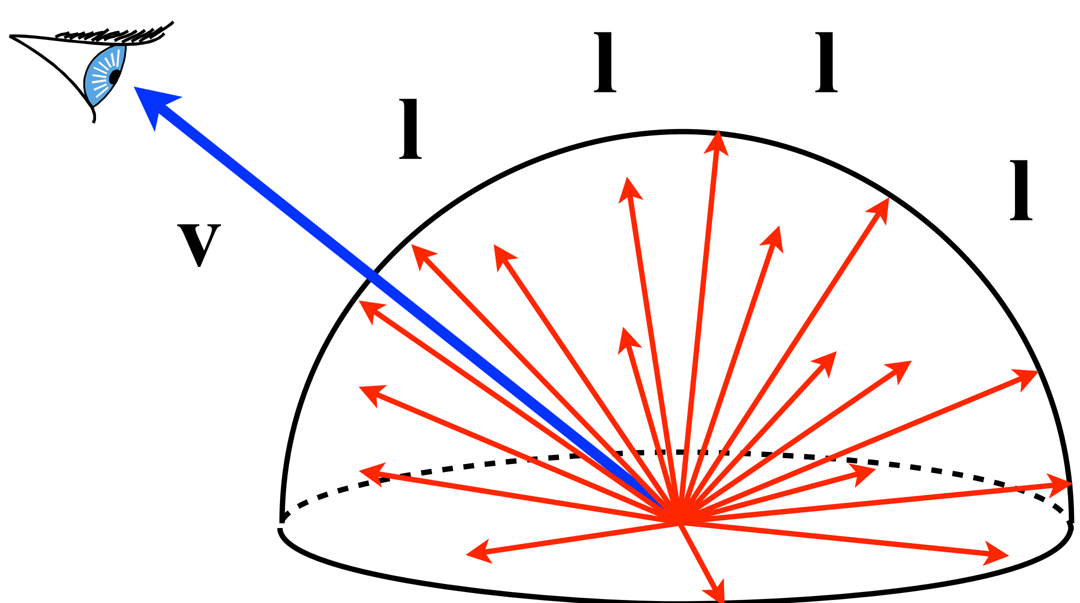
How are we going to use this in order to compute reflected radiance that accounts for light coming in from every direction?

Reflectance Equation

$$L(x \to \mathbf{v}) = \int_\Omega f_r(x, \mathbf{l} \to \mathbf{v}) \, L(x \leftarrow \mathbf{l}) \, \cos\theta \, \mathrm{d}\mathbf{l}$$

- $L(x \to \mathbf{v})$: **outgoing radiance**
- $\int_\Omega$: **integral over hemisphere**
- $f_r(x, \mathbf{l} \to \mathbf{v})$: **BRDF**
- $L(x \leftarrow \mathbf{l})$: **incoming radiance**
- $\cos\theta$: **cosine of incident angle**
- $L(x \leftarrow \mathbf{l}) \cos\theta$: **incident differential irradiance**



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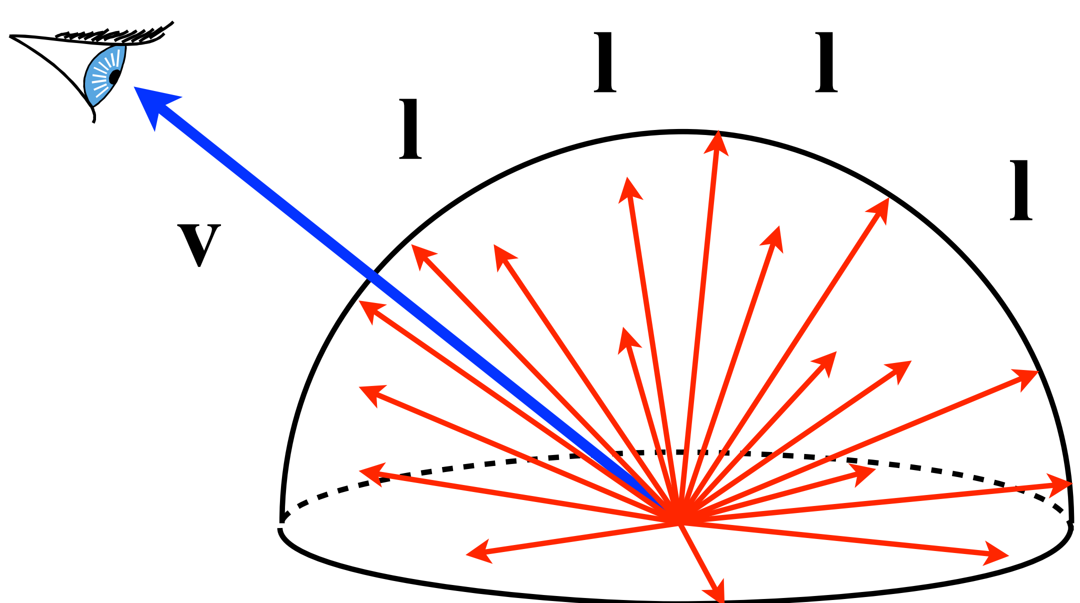

$$\text{BRDF}(\mathbf{l}, \mathbf{v}) = \frac{\text{radiance to direction } \mathbf{v}}{\text{differential irradiance from direction } \mathbf{l}}$$

Compare to Diffuse Case

$$L(x \to \mathbf{v}) =$$

$$\int_\Omega f_r(x, \mathbf{l} \to \mathbf{v})\, L(x \leftarrow \mathbf{l}) \cos\theta \, \mathrm{d}\mathbf{l}$$

incident differential irradiance

BRDF

$$L_{\text{out}}(x) = \frac{\rho(x)}{\pi} \int_\Omega L_{\text{in}}(x, \omega) \cos\theta \, \mathrm{d}\omega$$

Diffuse BRDF

$$L_{\text{out}}(x) = \frac{\rho(x)}{\pi} \int_{\Omega} L_{\text{in}}(x, \omega) \, \cos\theta \, \mathrm{d}\omega$$

- Diffuse reflectance independent of outgoing angle
- Hence, the diffuse BRDF is

$$f_r(x) = \frac{\rho}{\pi}$$

  - (ρ is the albedo, remember)

- Note: no cosine, it's included in the reflectance eq.!

BRDF Properties

- Reciprocity: $f_r(\mathbf{l} \rightarrow \mathbf{v}) = f_r(\mathbf{v} \rightarrow \mathbf{l})$

- Energy conservation: $\int f_r(\mathbf{l} \rightarrow \mathbf{v}) \cos\theta_v \, \mathrm{d}\mathbf{v} \leq 1$

  – **Intuitive**: the BRDF tells you how a single beam of incident illumination from direction **l** is spread into all reflected directions **v**; you can't have more energy coming out than going in.

  – **Note: This *does not* imply** $f_r(\mathbf{l} \rightarrow \mathbf{v}) \leq 1$ **!!**

  – It's an "unnormalised density"

BRDF Properties

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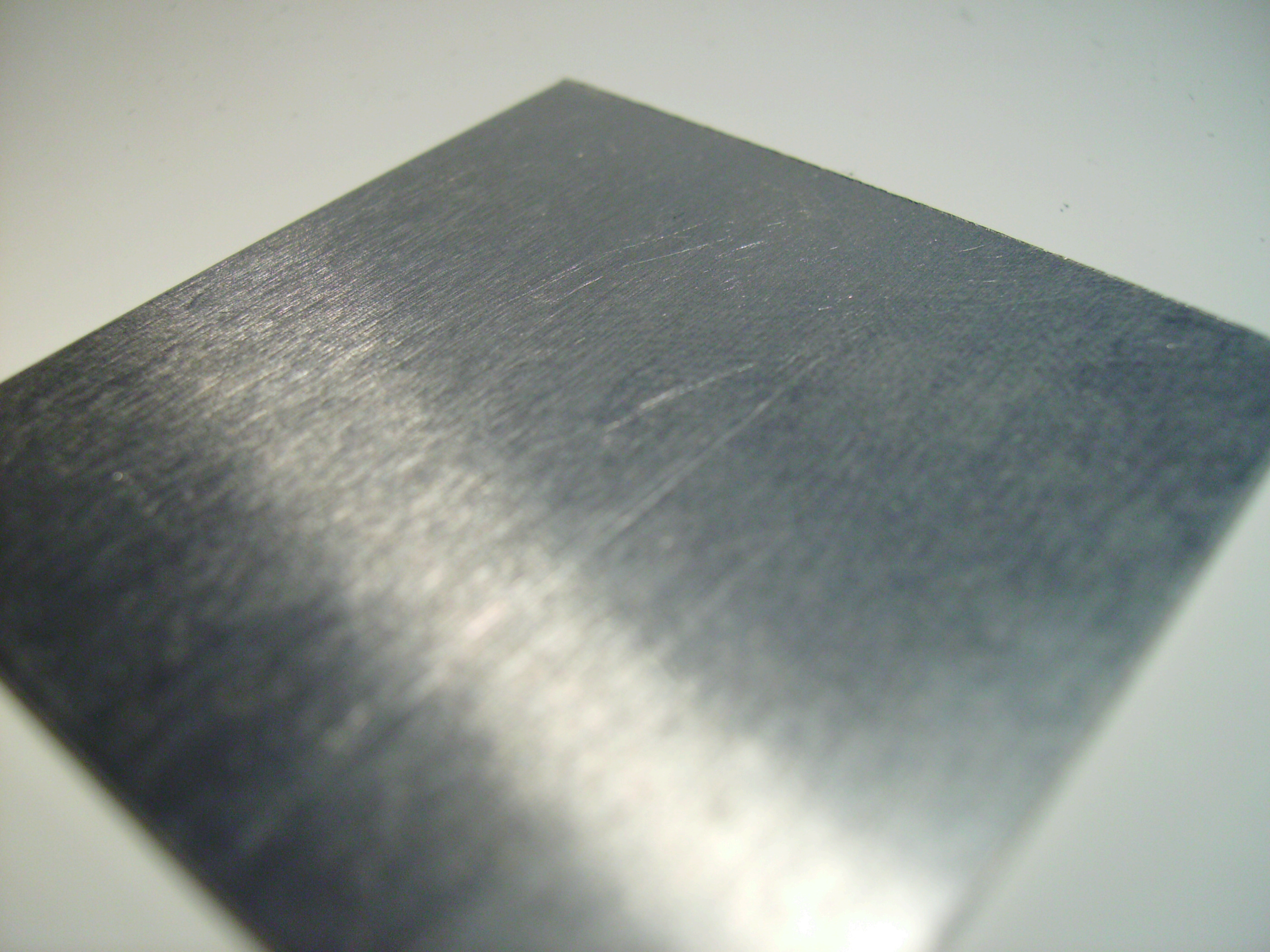
- Energy conservation: $\int f_r(\mathbf{l} \to \mathbf{v}) \cos\theta_v \, \mathrm{d}\mathbf{v} \leq 1$
  - **Intuitive**: the BRDF tells you how a single beam of incident illumination from direction **l** is spread into all reflected directions **v**; you can't have more energy coming out than going in.
  - But also, due to reciprocity, the same must hold if you swap the incident and outgoing directions.
- Non-negativity: $f_r(\mathbf{l} \to \mathbf{v}) \geq 0$

Isotropic vs. Anisotropic

- When keeping **l** and **v** fixed, if rotation of surface around the normal doesn't change the reflection, the material is called *isotropic*
- Surfaces with strongly oriented microgeometry elements are *anisotropic*
- Examples:
  - brushed metals,
  - hair, fur, cloth, velvet

Westin et.al 92



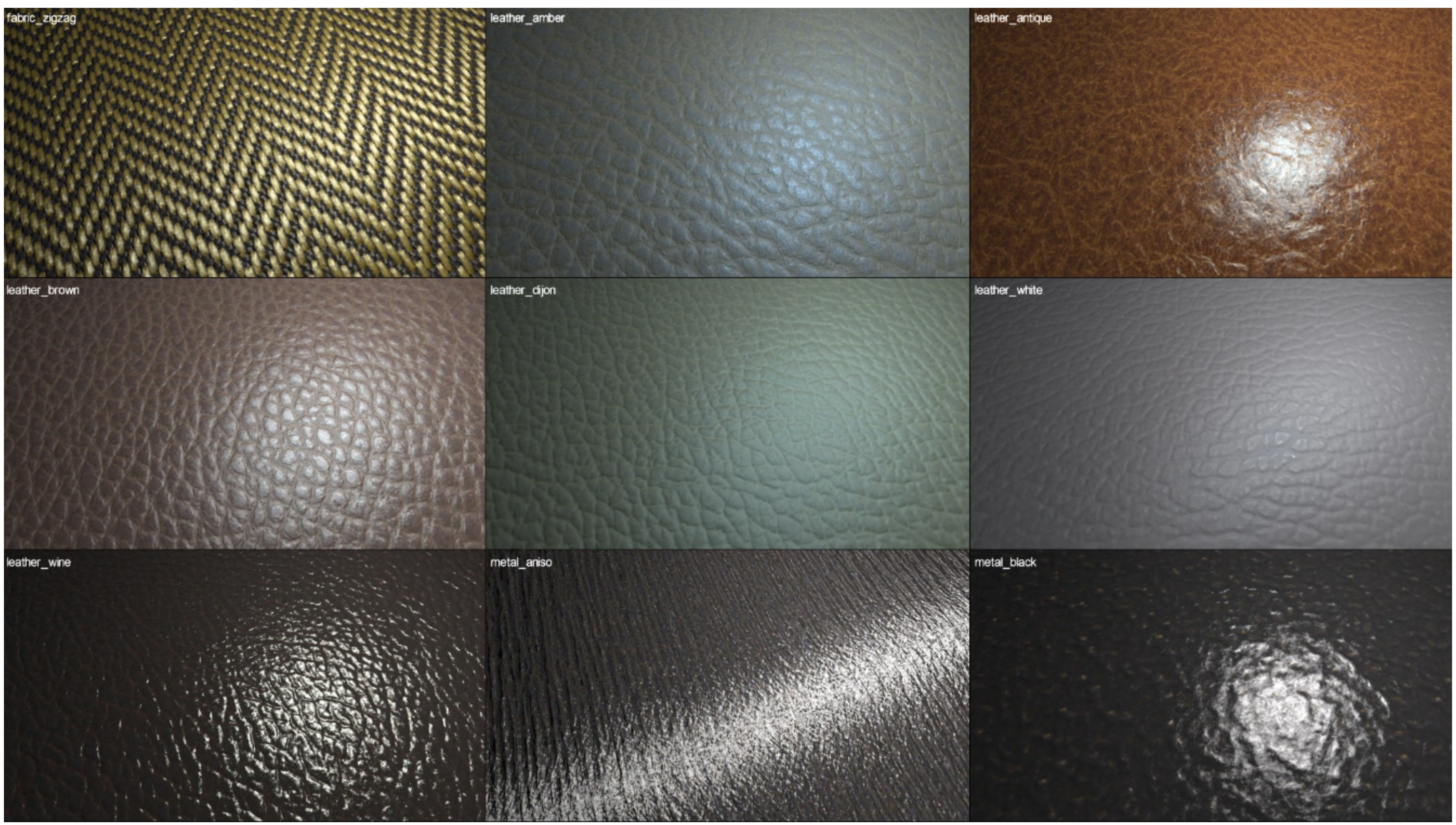
Hmmh

- The BRDF is a 4D function for a single surface point
- When you make it vary over surfaces, you add two more dimensions
  - The Spatially Varying BRDF (SVBRDF) is 6D!

Spatially Varying Reflectance

- Very, very, VERY important for realistic surface appearance
- VIDEO

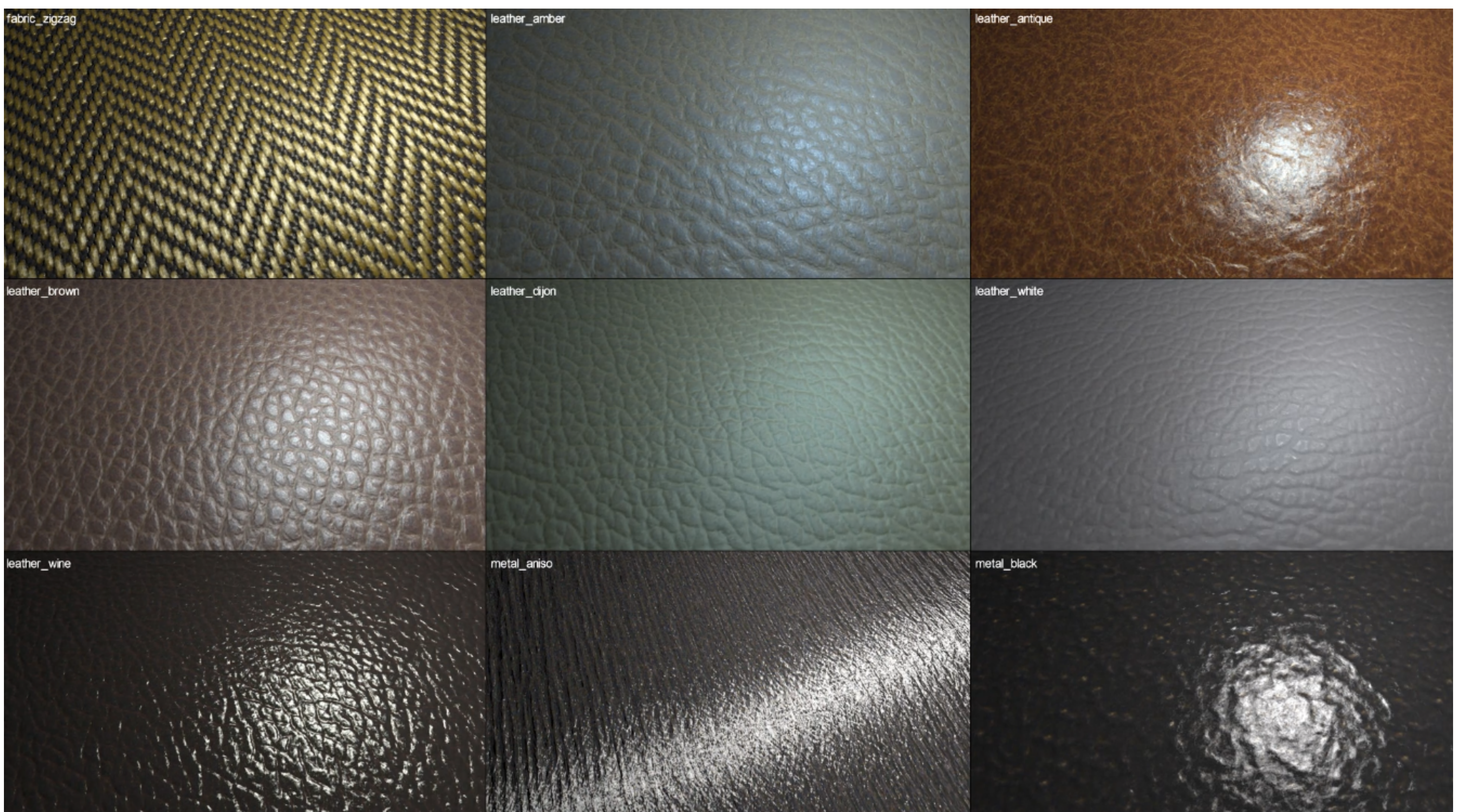
Aittala, Weyrich, Lehtinen 2015



Spatially Varying Reflectance

- You can find these SVBRDF material models online and use them in your assignments!

Aittala, Weyrich, Lehtinen 2015



Parametric BRDF Models

- BRDFs can be measured from real data
  - But storage and computation using arbitrary 4D or 6D functions is unwieldy, must do something smarter

Parametric BRDF Models

- BRDFs can be measured from real data
  - But storage and computation using arbitrary 4D or 6D functions is unwieldy, must do something smarter
- **Solution: parametric models**
  - What this means: use a small set of (hopefully intuitive) parameters that determine reflectance at each point
- We've seen one model already: diffuse reflectance determined by one parameter, the albedo
  - Well, 3 actually (RGB)

Parametric BRDF Models

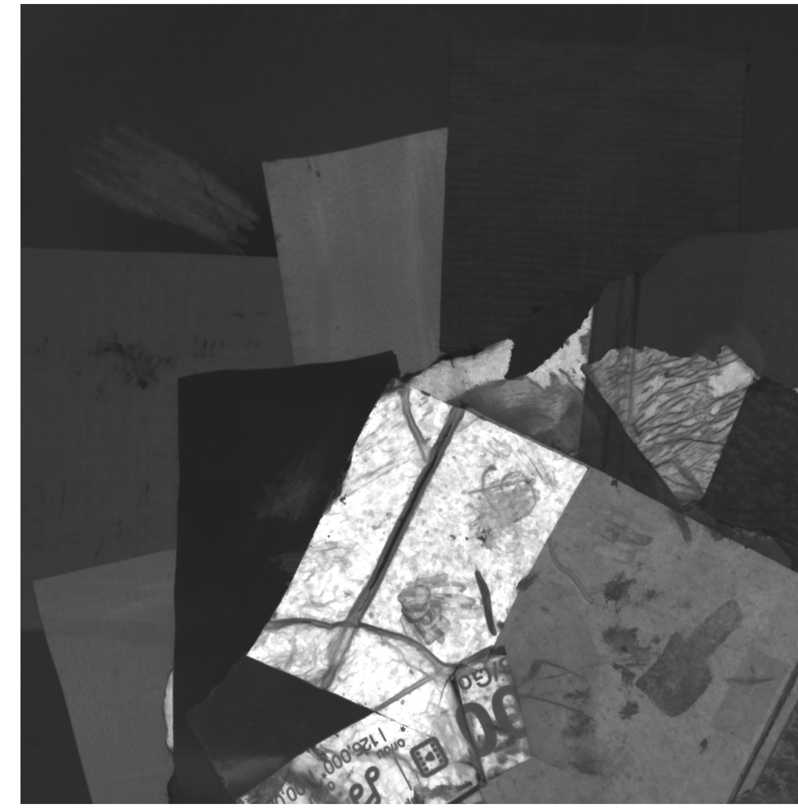
- Parametric BRDF models represent the relationship between incident and outgoing light by some mathematical formula with tunable parameters
  - The appearance can then be tuned by setting parameters
    - “Color”, “Shininess”, “anisotropy”, etc.
  - Many ways of coming up with these
  - Can models with measured data (examples later)

- Popular models: Diffuse, Blinn-Phong, Cook-Torrance, Lafortune, Ward, Oren-Nayar, etc.

Parametric SVBRDF Example

Diffuse albedo (color)

Specular albedo (color)



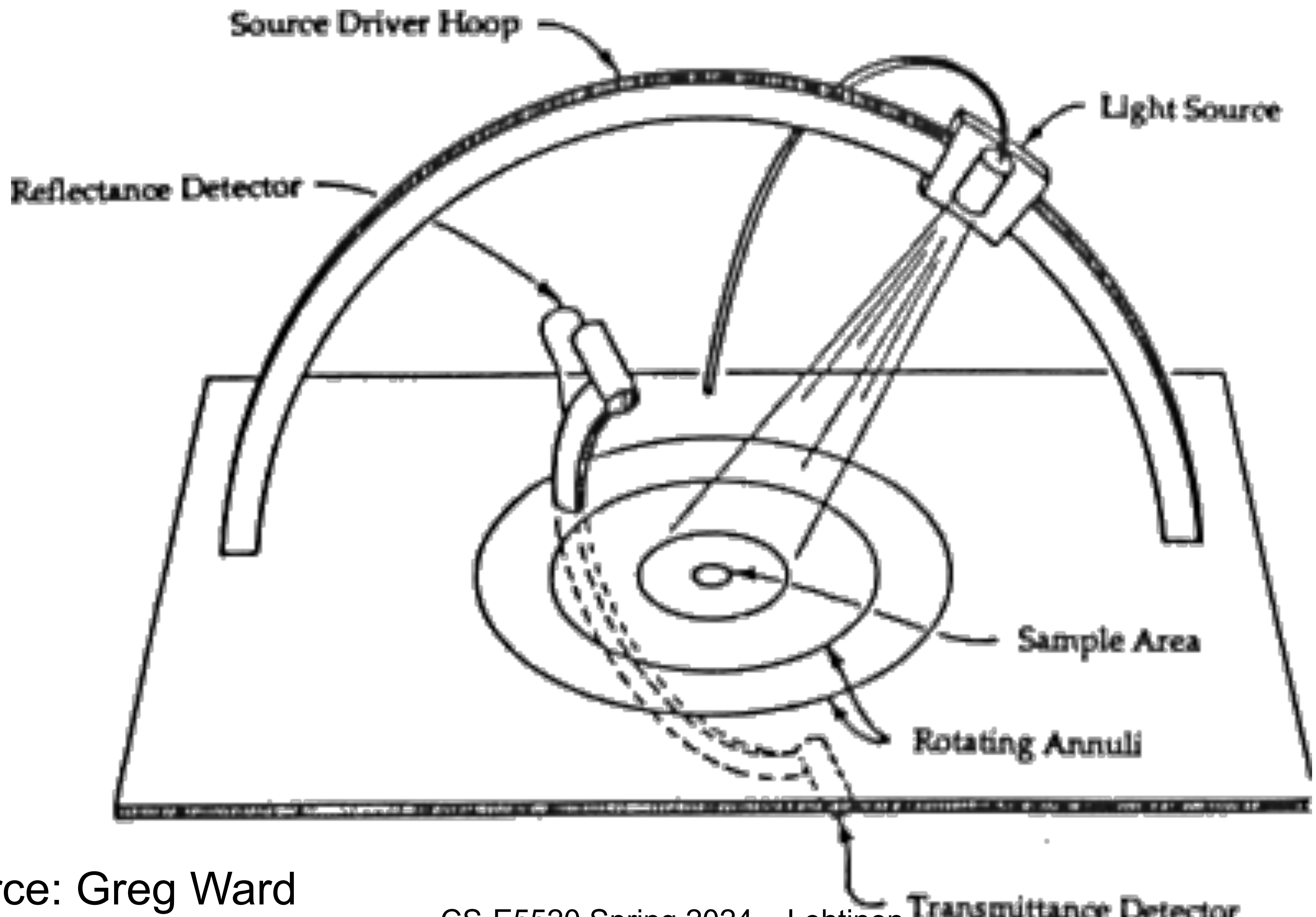
Glossiness

These are just parameters to a Fresnel-modulated Blinn-Phong model!

Surface normal

How do we obtain BRDFs?

- One possibility: Gonioreflectometer
  - 4 degrees of freedom



Source: Greg Ward

How do we obtain BRDFs?

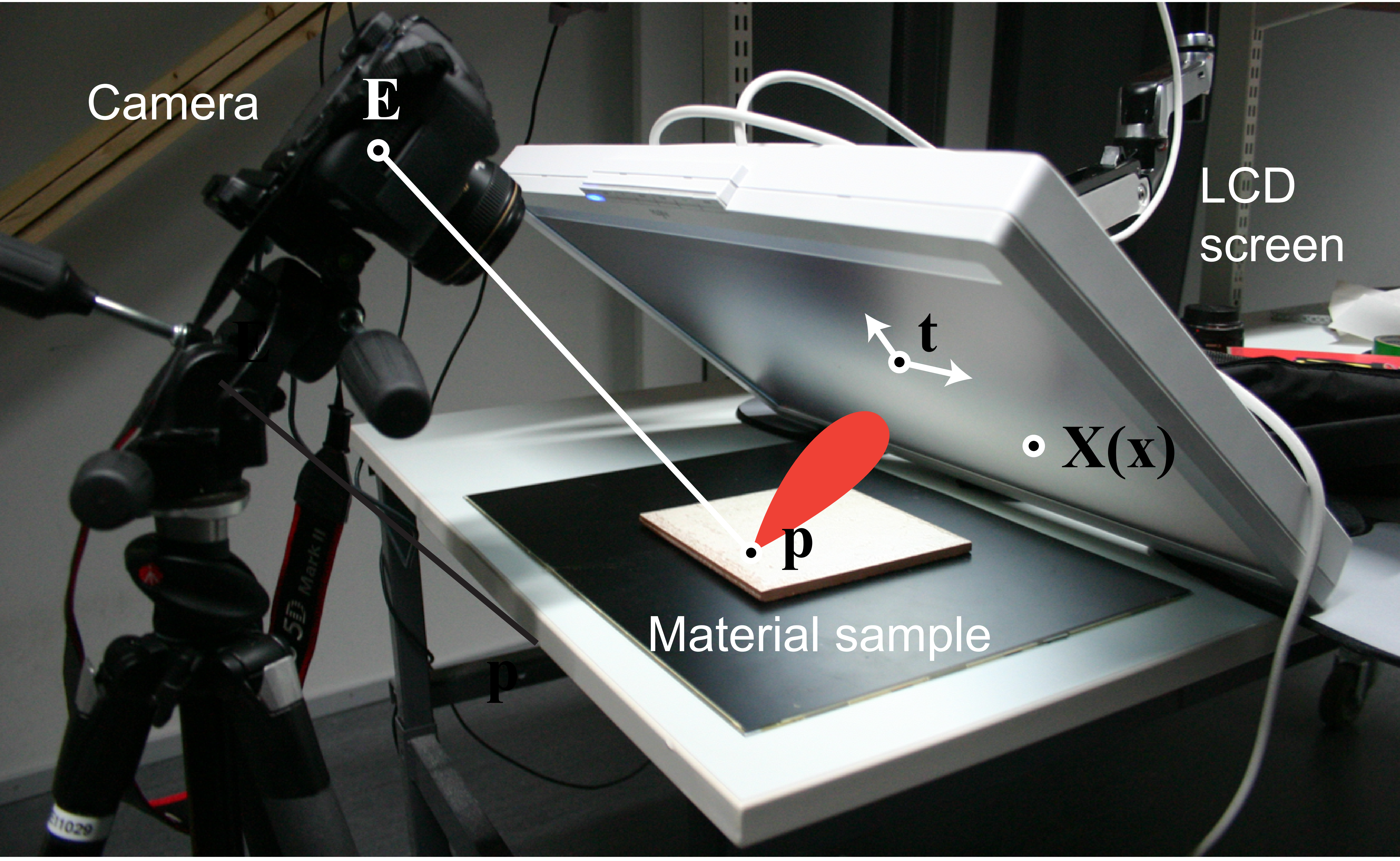
Matusik et al.

Image-Based Acquisition

- See W. Matusik et al. for how
  - A Data-Driven Reflectance Model, SIGGRAPH 2003
  - The data is available from MERL

We’ve Pushed State of The Art

Aittala, Weyrich, Lehtinen, *Practical SVBRDF Capture in the Frequency Domain*, SIGGRAPH 2013



Even less effort…

with some restrictions on what materials can be captured

- SIGGRAPH 2015, http://tinyurl.com/TwoShotSVBRDF

Two-Shot SVBRDF Capture for Stationary Materials

Miika Aittala
Aalto University

Tim Weyrich
University College London

Jaakko Lehtinen
Aalto University, NVIDIA

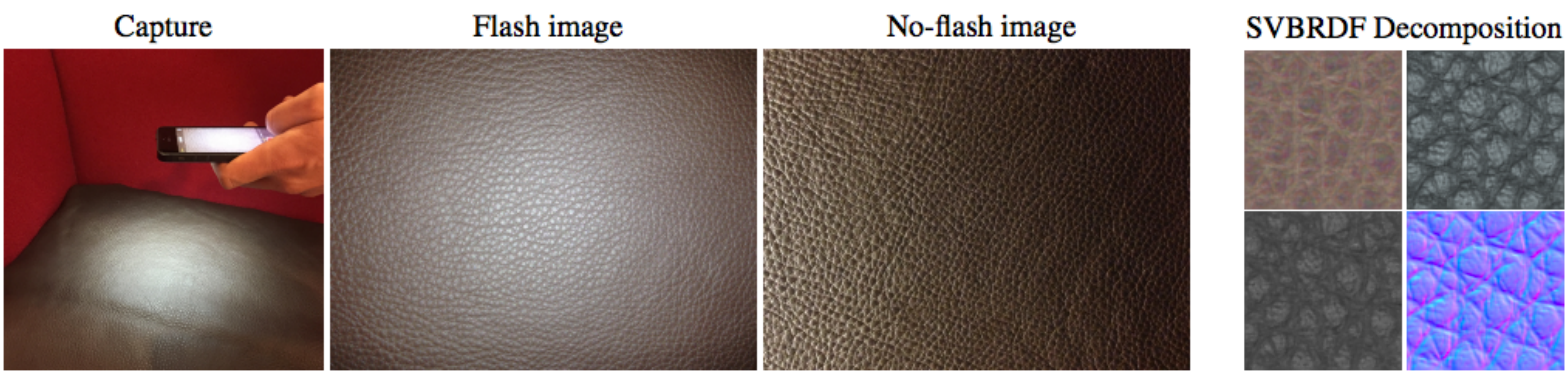
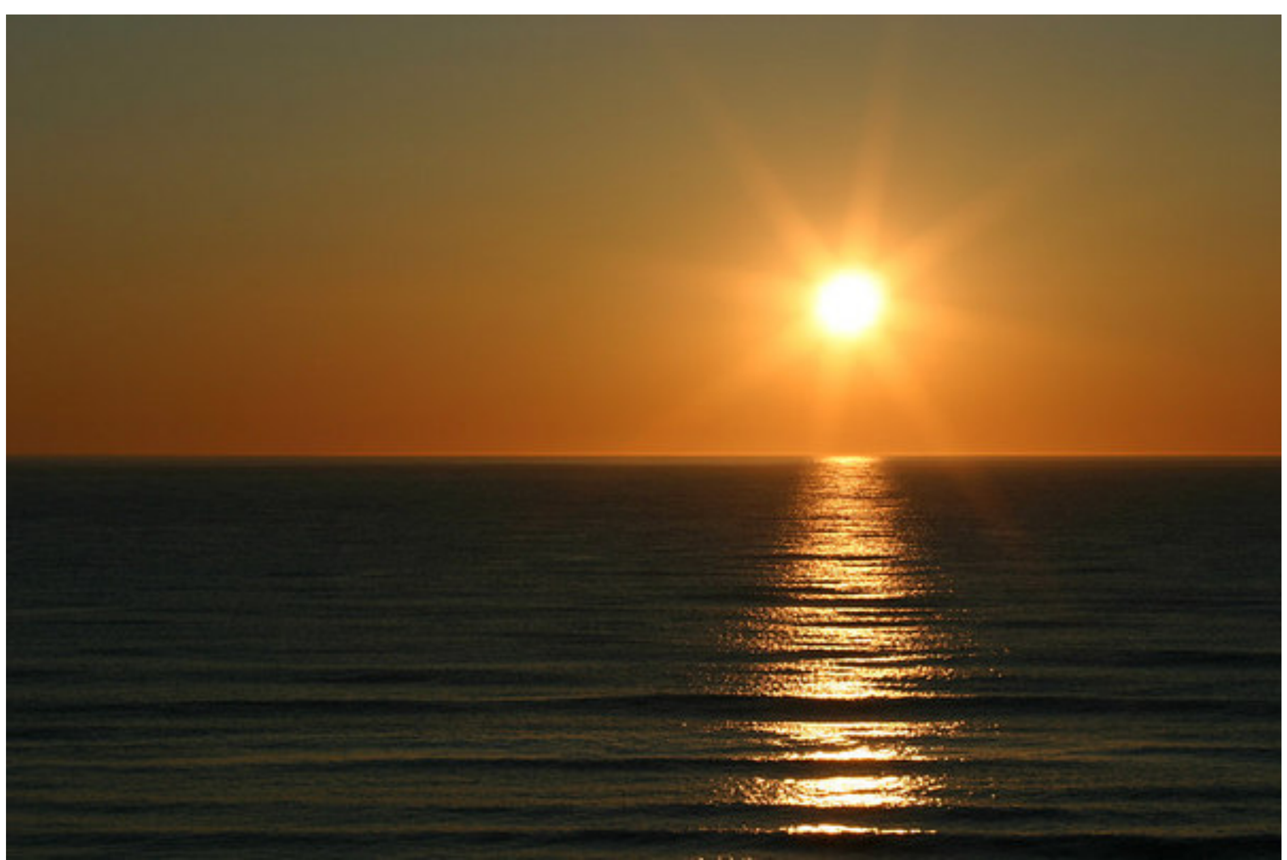


Figure 1: *Given an flash-no-flash image pair of a "textured" material sample, our system produces a set of spatially varying BRDF parameters (an SVBRDF, right) that can be used for relighting the surface. The capture (left) happens in-situ using a mobile phone.*

Questions?

Microfacet Theory

- Example
  - Think of water surface as lots of tiny mirrors (microfacets)
  - “Bright” pixels are
    - Microfacets aligned with the vector between sun and eye
    - But not the ones in shadow
    - And not the ones that are occluded

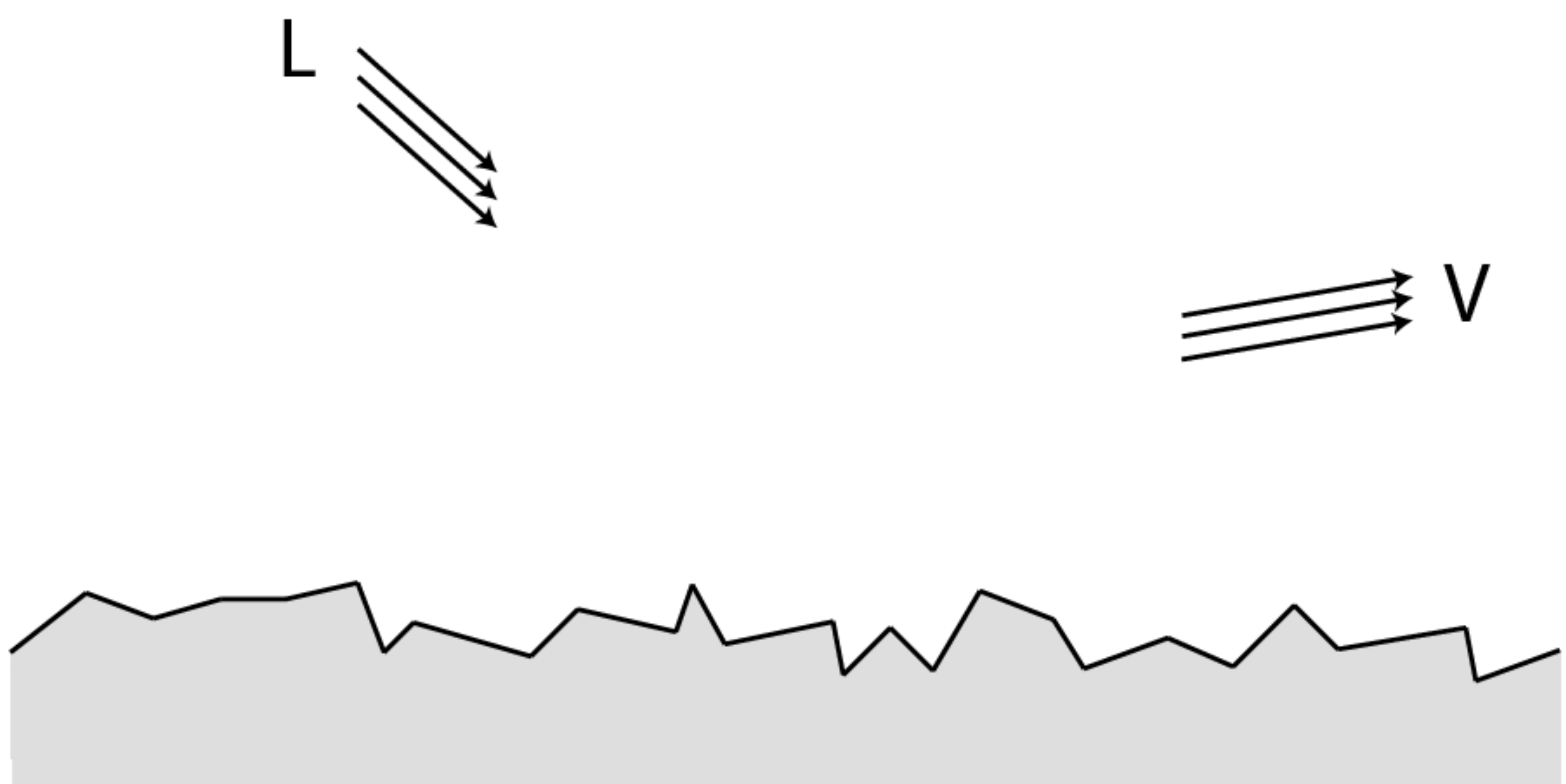


Microfacet Theory

- Model surface by tiny mirrors
  [Torrance & Sparrow 1967]

Microfacet Theory

- Value of BRDF at (L,V) is a product of
  - number of mirrors oriented halfway between L and V



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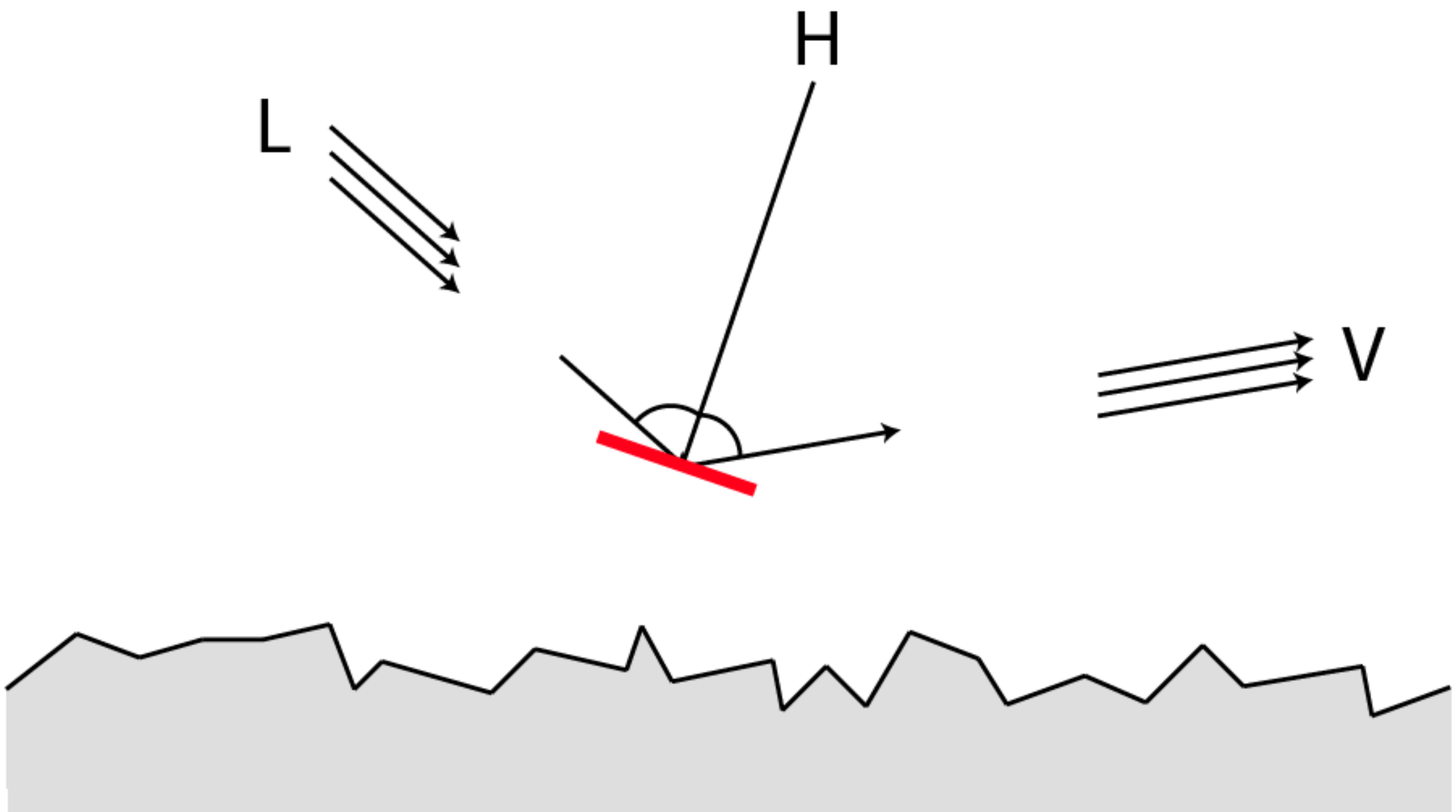

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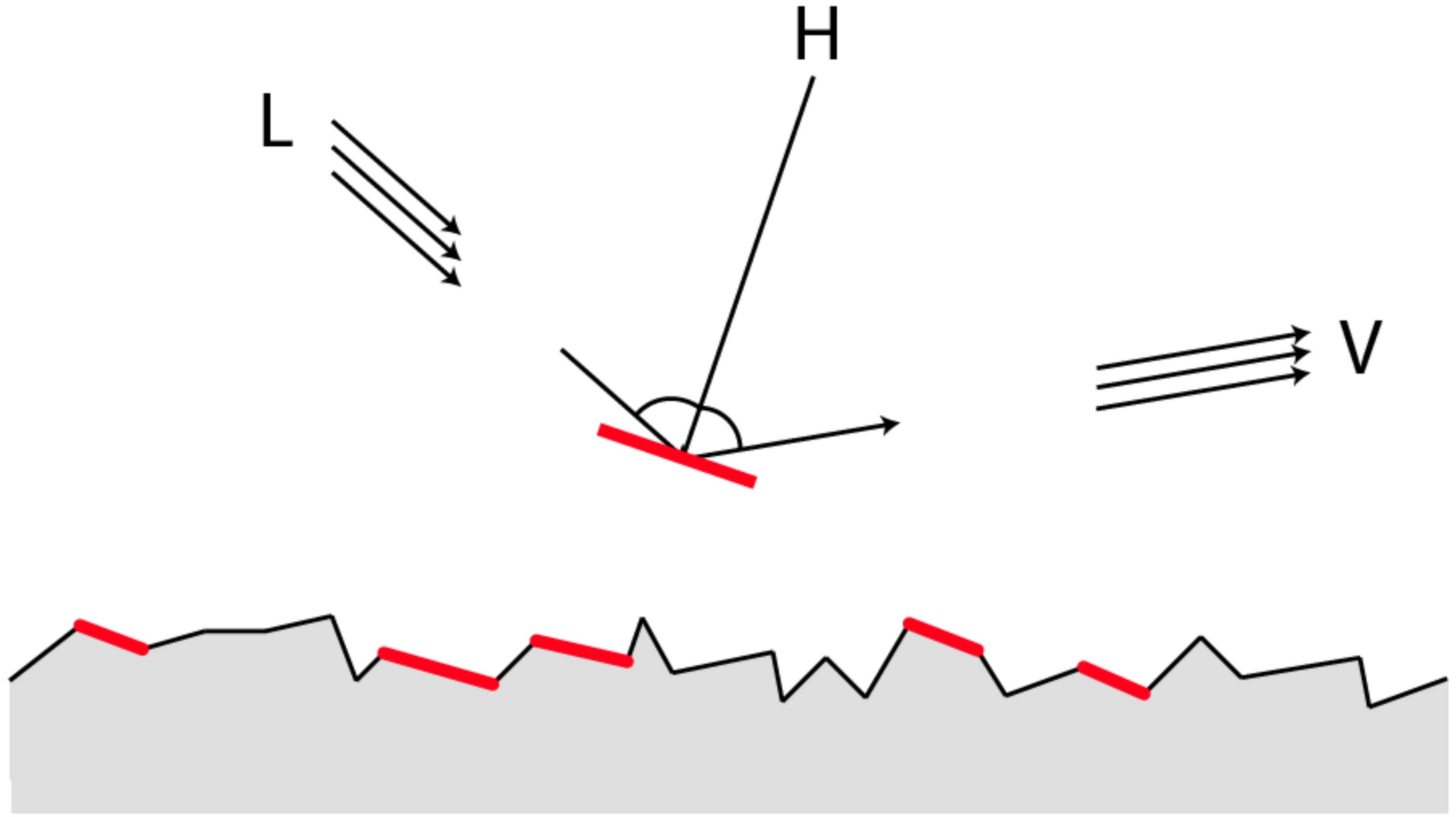
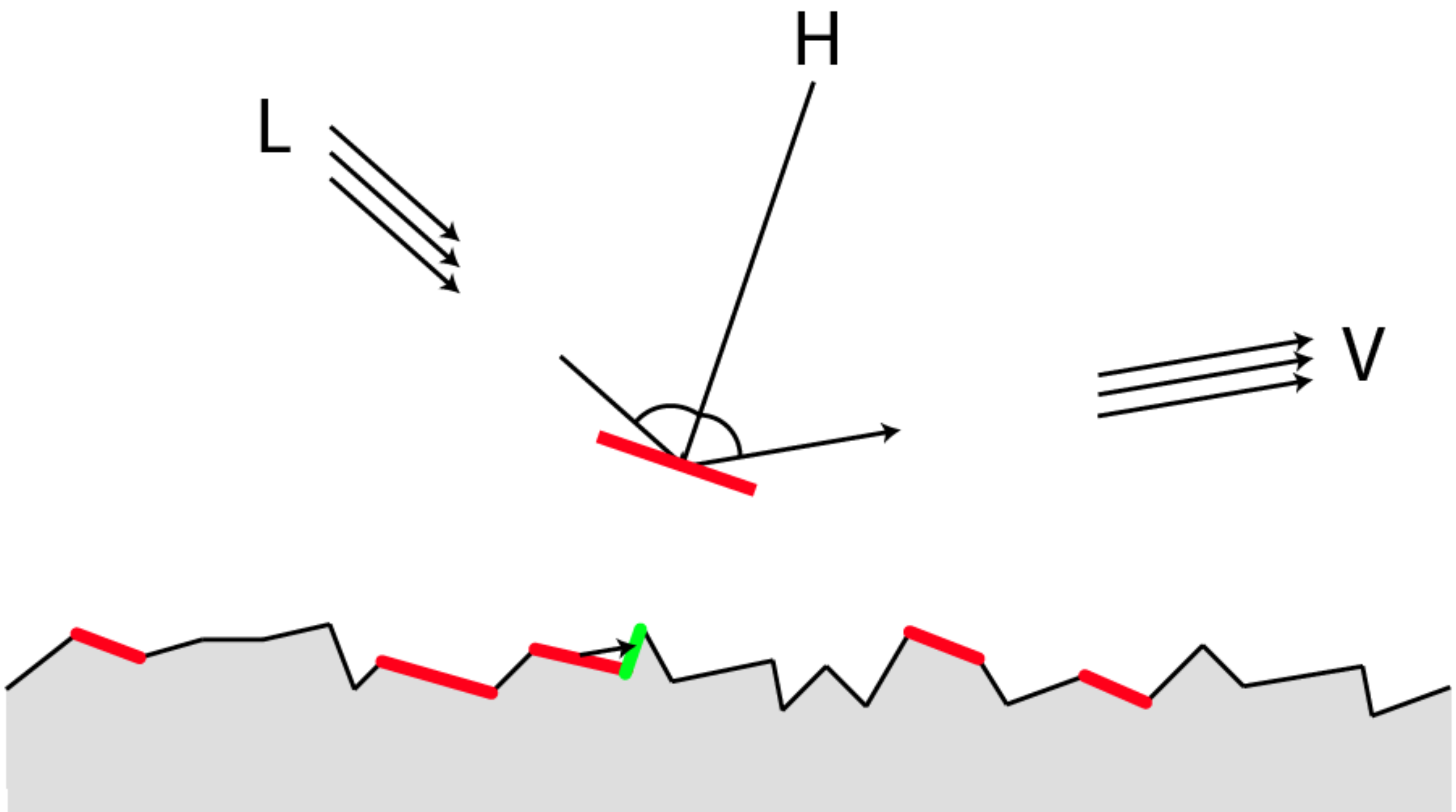


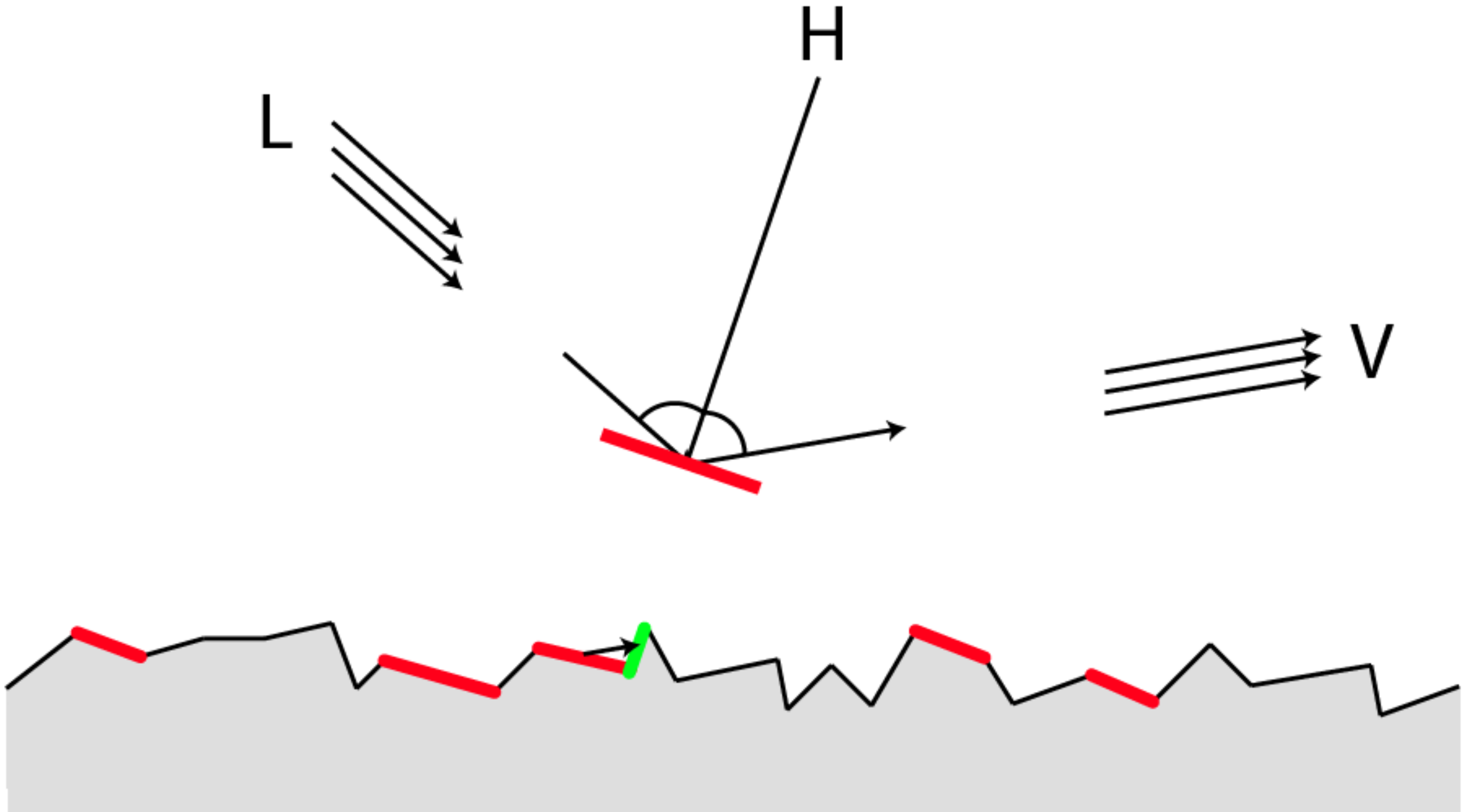
Microfacet Theory

- Value of BRDF at (L,V) is a product of
  - number of mirrors oriented halfway between L and V
  - ratio of the un(shadowed/masked) mirrors



Microfacet Theory

- Value of BRDF at (L,V) is a product of
  - number of mirrors oriented halfway between L and V
  - ratio of the un(shadowed/masked) mirrors
  - Fresnel coefficient

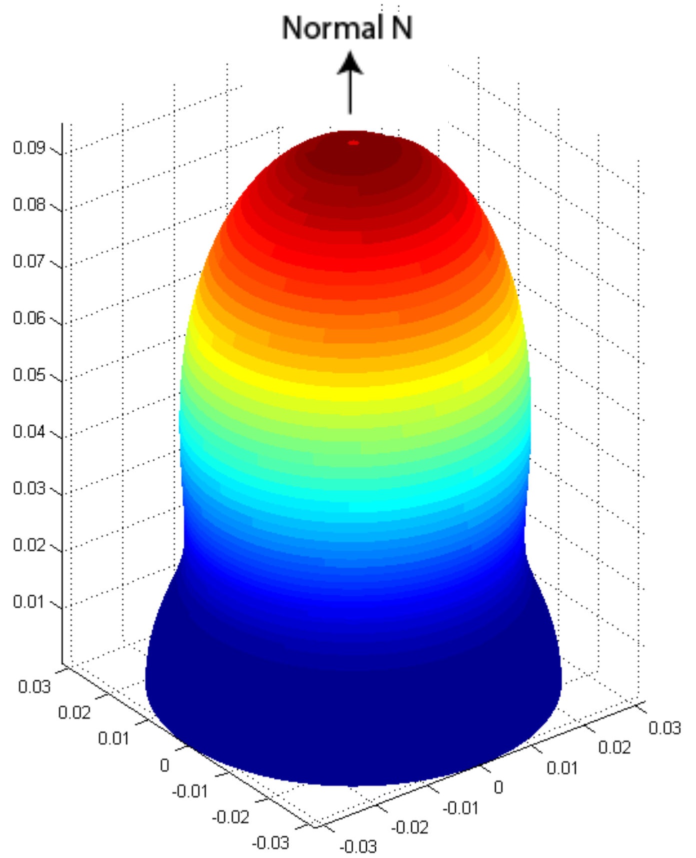


Microfacet Theory-based Models

- Develop BRDF models by imposing simplifications [Torrance-Sparrow 67], [Blinn 77], [Cook-Torrance 81], [Ashikhmin et al. 2000]

- Model the distribution D(**h**) of microfacet normals
  - Also, statistical models for shadows and masking

- As always, $\mathbf{h} = \dfrac{\mathbf{l} + \mathbf{v}}{\|\mathbf{l} + \mathbf{v}\|}$



spherical plot of a Gaussian-like p(H)

General Microfacet BRDF (Cook-Torrance)

- Sum of Diffuse and Specular terms:

$$f_r = \frac{\rho_d}{\pi} + \frac{\rho_s}{\pi} \frac{F(\mathbf{l} \cdot \mathbf{h})\, D(\mathbf{h})\, G(\mathbf{l}, \mathbf{v})}{(\mathbf{n} \cdot \mathbf{l})(\mathbf{n} \cdot \mathbf{v})}$$

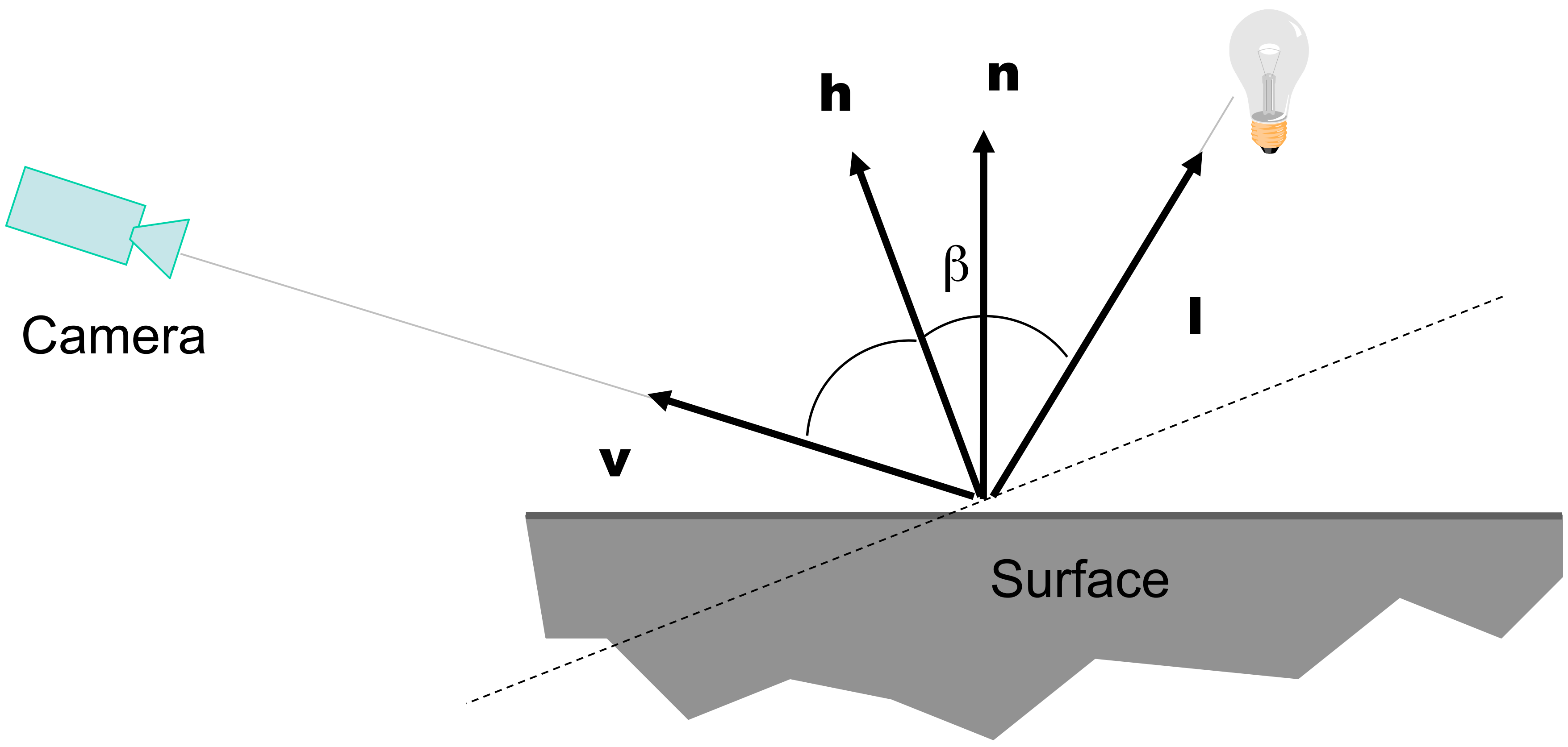
- F is the Fresnel term that accounts for increasing reflection towards grazing angle
- D is the microfacet distribution (common models include Gaussian, Blinn-Phong, Beckmann
  - Shifted Gamma is the new king of the hill
- G is the geometric (shadowing, masking) term
- See linked papers for details

Blinn-Torrance Variation of Phong

- Uses the "halfway vector" **h** between **l** and **v**.

$$D(\mathbf{h}) = N_q \left(\mathbf{n} \cdot \mathbf{h}\right)^q$$

$$\boldsymbol{h} = \frac{\boldsymbol{l} + \boldsymbol{v}}{\|\boldsymbol{l} + \boldsymbol{v}\|}$$



$$N_q = \frac{n+1}{2\pi}$$

is a normalization factor

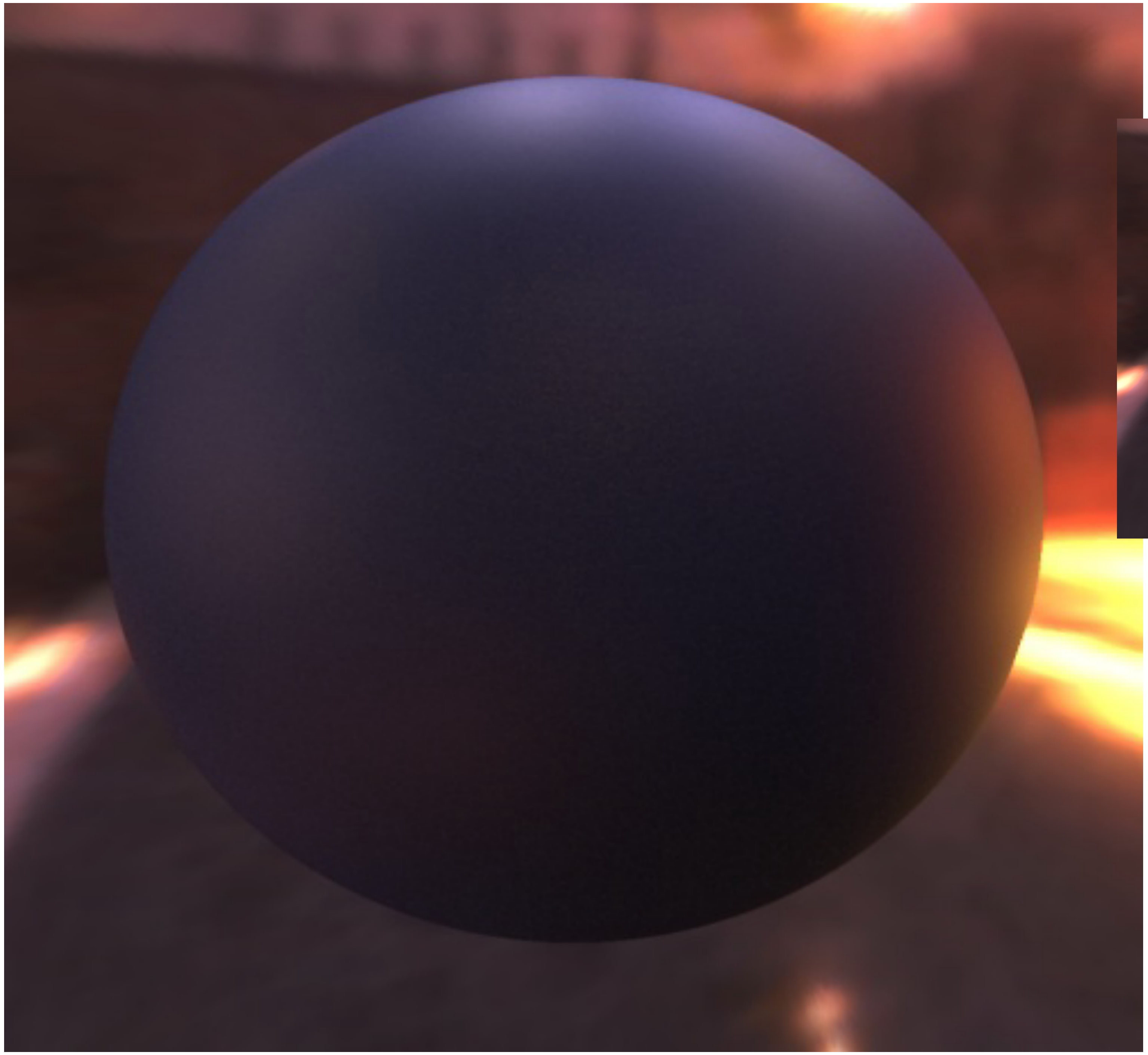
Geometric (Shadowing, Masking) Term

- Can be computed from microfacet distribution by integration
- Cook and Torrance used a heuristic formula

$$G = \min\left\{1, \frac{2(\mathbf{N}\cdot\mathbf{H})(\mathbf{N}\cdot\mathbf{V})}{(\mathbf{V}\cdot\mathbf{H})}, \frac{2(\mathbf{N}\cdot\mathbf{H})(\mathbf{N}\cdot\mathbf{L})}{(\mathbf{V}\cdot\mathbf{H})}\right\}$$

- Current models are more well-founded than this, see e.g. this paper

BRDF Examples: see Ngan et al.

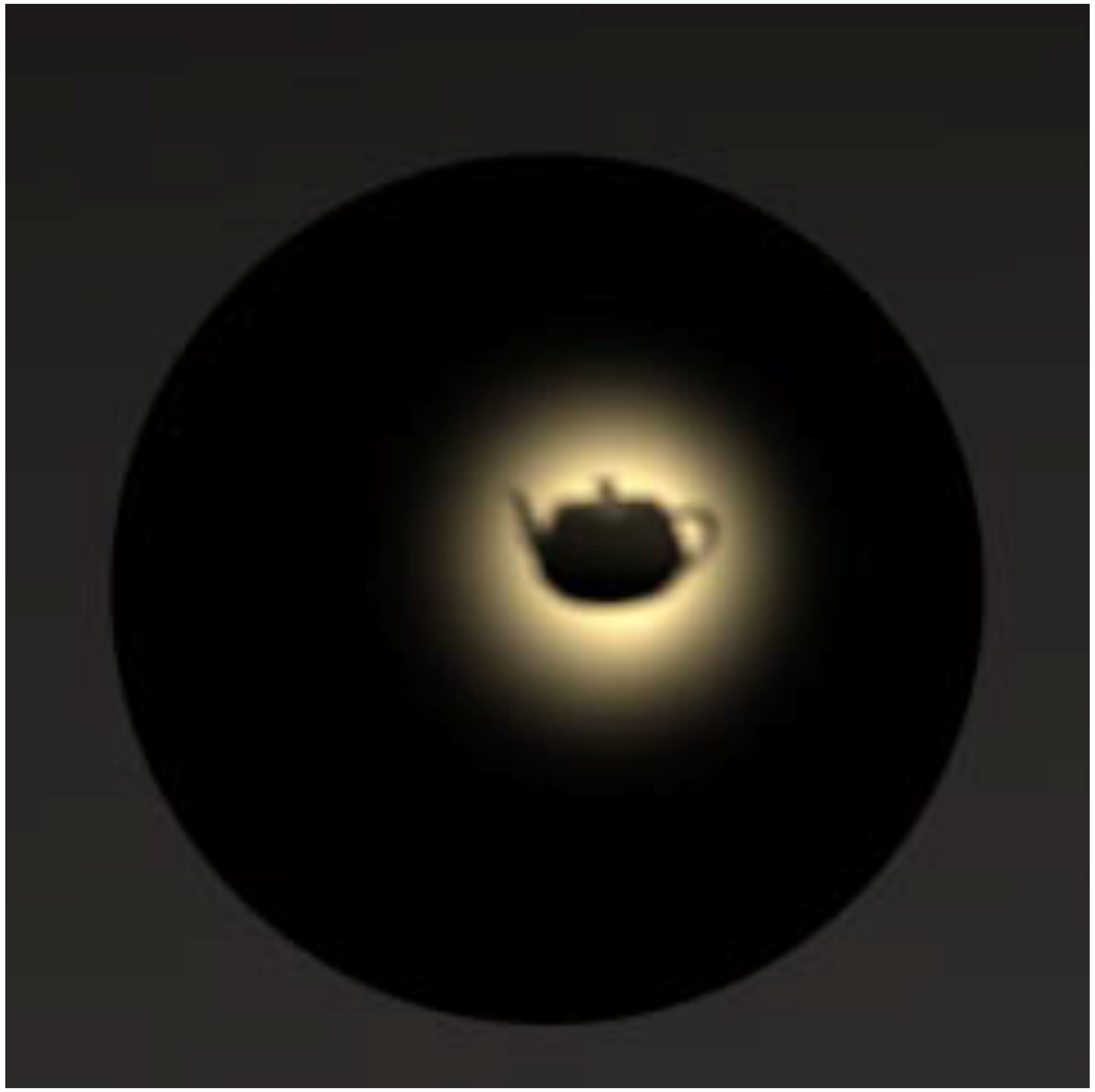


Lighting

Material – Dark blue paint

Questions?

- “Designer BRDFs” by Ashikhmin et al.



Reflectance

- Careful optimization + milling allows one to create a surface that reflects light in such funky ways
- Weyrich, Peers, Matusik, Rusinkiewicz SIGGRAPH 2009, Fabricating Microgeometry for Custom Surface Reflectance

Fabricating Microgeometry for Custom Surface Reflectance

Tim Weyrich
University College London

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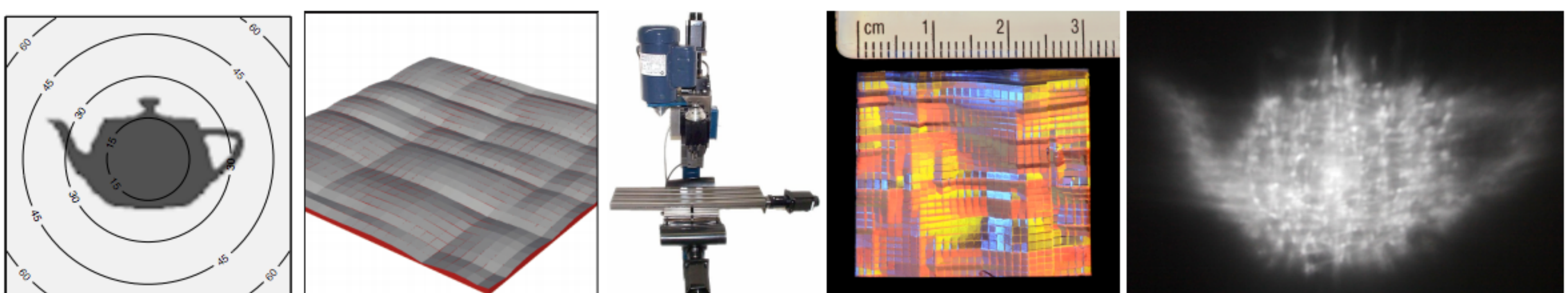


***Figure 1:** From left: a user-designed highlight is converted to an optimized microfacet height field. A computer-controlled milling machine is used to manufacture the surface (30 × 30 facets, each approximately 1 mm × 1 mm), which exhibits the desired reflectance.*

Pure Reflection (BRDF)

BRDF: Light reflects off exactly the same point

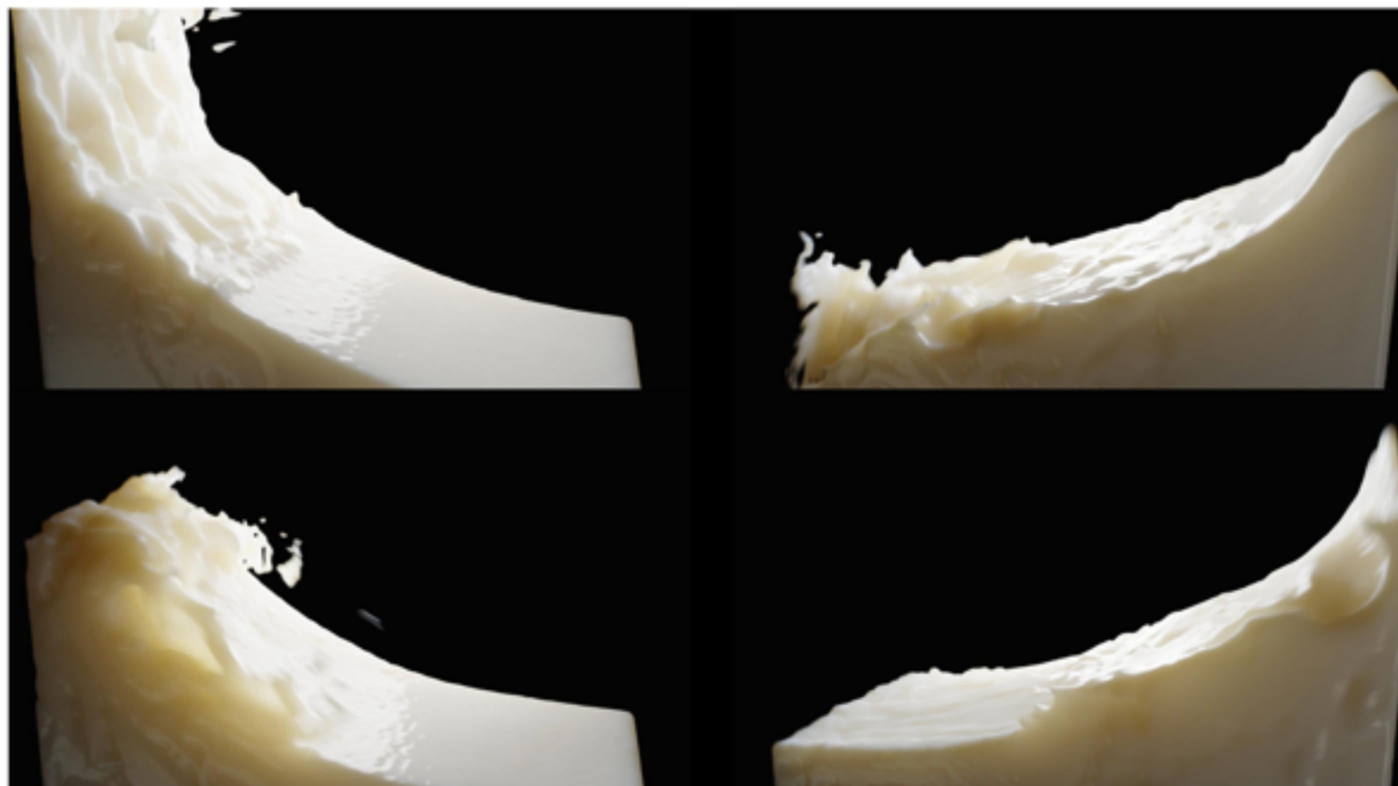
Subsurface Scattering (BSSRDF)

Some light enters material, exits at another point
BSSRDF = Bidirectional Surface Scattering Distribution Function

(See Henrik's paper linked to the title)

Subsurface State of the Art: Weta Digital

See Eugene's paper



BRDF vs. BSSRDF

Jensen et al. SIGGRAPH 2001

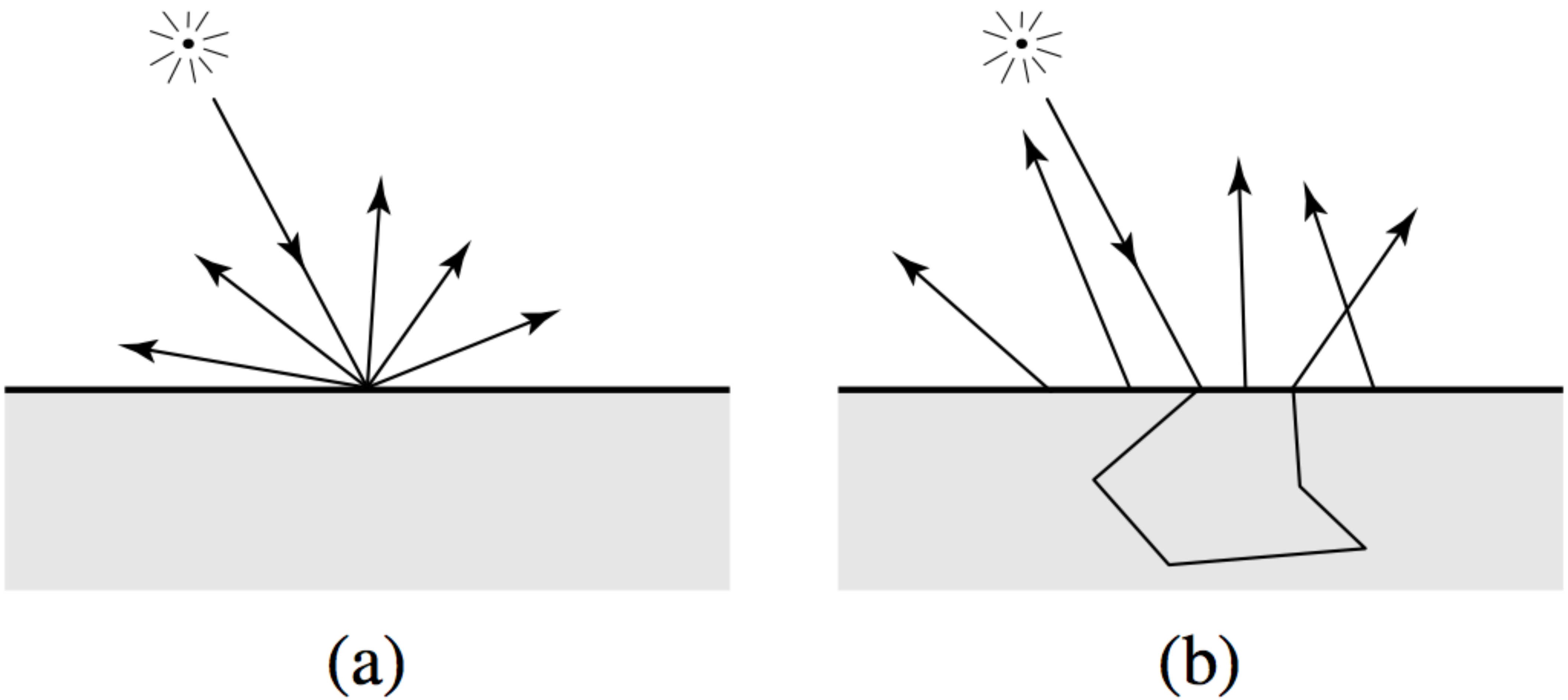


Figure 1: Scattering of light in (a) a BRDF, and (b) a BSSRDF.

BSSRDF Definition

- Relates differential irradiance *at all points* and all directions to outgoing radiance *at every other point* and all outgoing directions
  - 8D! Ouch!

$$L(x \to \mathbf{v}) = \int_A \int_\Omega L(y \leftarrow \mathbf{l}) f_r(x, y, \mathbf{l}, \mathbf{v}) \cos\theta \, \mathrm{d}\mathbf{l} \, \mathrm{d}A_y$$

- To get outgoing light at point x, integrate over all other points y and all incident directions at those points
  - Crazy complicated! Must do something smarter, i.e., cache incident illumination, assume diffuse scattering, etc. (See Henrik)

Questions?

Markus Otto/Winzenrender, Rendered using Maxwell

The Way To Global Illumination

$$L(x \rightarrow \mathbf{v}) = \int_{\Omega} L(x \leftarrow \mathbf{l})\, f_r(x, \mathbf{l} \rightarrow \mathbf{v})\, \cos\theta \, \mathrm{d}\mathbf{l}$$

reflectance equation

- Where does incident L come from?

- Next lecture…