

In class -exercises 29.-31.1.2024

Round 4

The In class -exercises are to be done in the exercise session and the assistant will give advice on how to do them if necessary. The correct solutions to the problems will be discussed together. To obtain points for these exercises, you only need to be present.

1. Decide in some "rudimentary" way that the following functions have the smallest value in \mathbf{R}^2 and determine these values:

(a) $f(x, y) = \arctan(1 + 5x^2 + 4y^2)$.

(b) $g(x, y) = x^2 - 2xy + 4y^2$.

(c) $h(x, y) = \ln(6 - 2\sin(xy))$.

Hint: monotonicity of an external function, completing the square, etc.?

2. Let $f(x, y) = \sin(x) + \cos(y)$. Determine the absolute maximum and minimum values of f . At what points do these extreme values occur?
3. The function $f(x, y) = x^2 - 4xy + y^4 + 2x$ has (at least one) critical point $(1, 1)$.
- (a) Check the statement.
- (b) Determine the type of this critical point (local maximum, minimum or saddle point?).

4. Consider the function

$$f(x, y) = \frac{\sqrt{1 - x^2 - y^2}}{x^2}$$

and let D be its domain.

- (a) Find and sketch D .
- (b) Does the function have an absolute minimum on D ? If so, then find it. If not, explain why not.
- (c) Does the function have an absolute maximum on D ? If so, then find it. If not, explain why no.