Homework -exercises 1.-2.2.2024

Round 4

To get points from these exercises do them at home before the second exercise session of the week and at the beginning of the class mark them on the list.

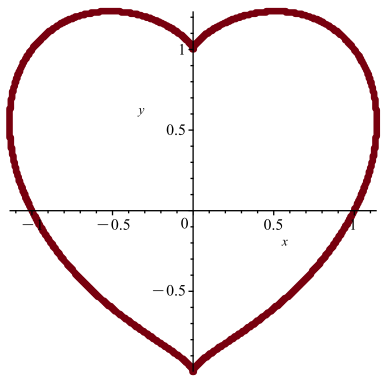
1. (a) Prove that the point $(1,1)$ is located on the heart curve $(x^2+y^2-1)^3 - x^2y^3 = 0$ (picture below) and find the equation of the tangent line at this point.

   Hint: The heart curve can be thought of as the level curve of the function $g(x,y) = (x^2+y^2-1)^3 - x^2y^3$. Thus the gradient of the function gives a normal $\mathbf{N}$ for the curve. The tangent to the curve is perpendicular to the normal, i.e. if the point (x,y) is at the tangent, then the vector $(x-1, y-1)$ is perpendicular to the normal.

   (b) Prove that the points $(\pm 1, 0)$ and $(0, \pm 1)$ are located on the heart curve. And that the gradient of the function $g(x,y) = (x^2+y^2-1)^3 - x^2y^3$ is zero vector on those points.

   With Maple:

```
> with(plots):
> implicitplot((x^2+y^2-1)^3-x^2*y^3=0,x=-1.5..1.5,y=-1..1.5,
scaling=constrained, numpoints=30000,coloring=["Red","White"],
thickness=5)
```



2. Prove that the function $f(x,y) = e^x \sin y$ has no extreme values.

3. Find the local maxima of the function:

$$f(x,y) = \frac{-6x}{2+x^2+y^2}.$$

   Hint: Use Hessian matrix to find out which critical point is maximum.