Week III

Overview

- Flow Network;
- Cut;
- Maximum Flow;
- Minimal Cut.

Definitions

- A **flow network** is a **directed** graph where each edge has a **capacity** and each edge receives a **flow**, where the amount of flow allowed in each edge cannot surpass its capacity;
- A **cut** in graph theory corresponds to a **partition** of the nodes in a graph splitting them into **disjoint subsets**.

Problem (Maximum Flow Problem (MaxFlow))

Given a flow network represent as a digraph $G = (v, E)$ with capacities u and unique source and unique sink s and t respectively, such that $s, t \in V$.
The goal is to find an s-t-flow of ***maximum*** *value.*

Problem (Minimum Cut Problem (MinCut))

Given a flow network represent as a digraph $G = (v, E)$ with capacities u and unique source and unique sink s and t respectively, such that $s, t \in V$.
The goal is to find an s-t-cut of ***minimum capacity****.*

Ford-Fulkerson's

Algorithm 1: FORD-FULKERSON ALGORITHM

Input: digraph $G = (V, E)$, capacities $u\colon E \to \mathbb{Z}_+$, $s, t, \in V$
Output: maximal s-t-flow f

```
set f(e) = 0 for all e ∈ E
while there exists f-augmenting path in G_f do
    choose f-augmenting path P
    set Δ_f(P) = min_{a∈E(P)} u_f(a)
    augment f along P by Δ_f(P)
    update G_f
return f
```

Emonds-Karp's

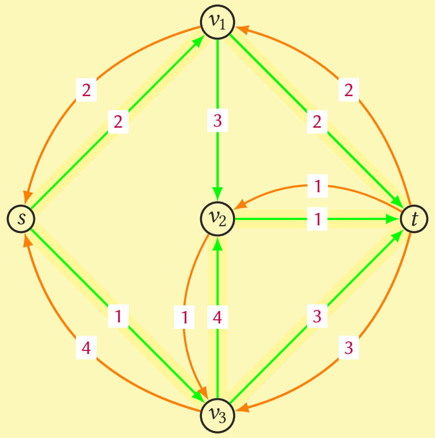
Algorithm 2: EDMONDS-KARP ALGORITHM

Input: digraph $G = (V, E)$, capacities $u\colon E \to \mathbb{R}_+$, $s, t, \in V$
Output: maximal s-t-flow f

```
set f(e) = 0 for all e ∈ E
while there exists f-augmenting path in G_f do
    choose f-augmenting path P with minimal
      number of edges
    set Δ_f(P) = min_{a∈E(P)} u_f(a)
    augment f along P by Δ_f(P)
    update G_f
return f
```

Maximum Flow ILP

$$
\begin{aligned}
\max \quad & \sum_{e \in \delta^+(s)} f_e & & \text{(1a)}\\
\text{s.t.} \quad & \sum_{e \in \delta^-(v)} f_e - \sum_{e \in \delta^+(v)} f_e = 0 & v \in V \setminus \{s, t\} \quad & \text{(1b)}\\
& f_e \le u(e) & e \in E \quad & \text{(1c)}\\
& f_e \ge 0 & e \in E \quad & \text{(1d)}
\end{aligned}
$$



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