Overview

- Admin Topics;
- Graph Theory;
- Graph Representation;
- Depth-First Search (DFS).

Graph Theory

- directed vs undirected graphs;
- walk, path and edge progression;
- reachability.

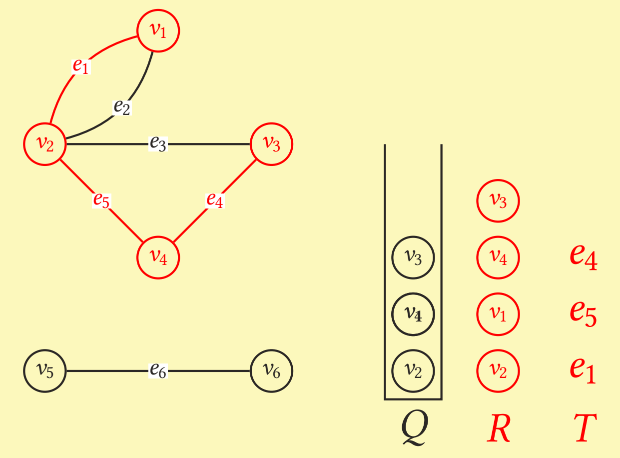
DFS

Algorithm 1: DEPTH FIRST SEARCH (DFS)

Input: undirected graph G, vertex $s \in V(G)$

Output: tree $(R, T) \subseteq G$, R reachable from s

1 set $R := \{s\}$, $Q := \{s\}$ and $T = \emptyset$;
2 **if** $Q = \emptyset$ **then return** R, T;
3 **else** $v :=$ last vertex added to Q;
4 choose $w \in V(G) \setminus R$ with $\{v, w\} \in E(G)$;
5 **if** *there is no such* w **then**
6 set $Q := Q \setminus \{v\}$ and **go to** 2
7 set $R := R \cup \{w\}$, $Q := Q \cup \{w\}$, $T := T \cup \{\{v, w\}\}$, **go to** 2;



Graph Representation

incidence matrix	adjacency matrix	adjacency list
$A \in \{0,1\}^{\lvert V\rvert \times \lvert E\rvert}$,	$A \in \mathbb{Z}^{\lvert V\rvert \times \lvert V\rvert}$,	$L = [\ell(v) \colon v \in V]$,
$a_{v,e} = \begin{cases} 1, & \text{if } v \in e \\ 0, & \text{if } v \notin e \end{cases}$	$a_{v,w} = \lvert\{e = \{v, w\} \in E\}\rvert$	$\ell(v) = [e \colon e = \{u, v\} \in E]$
$\begin{pmatrix} 1&1&0&0&0&0 \\ 1&1&1&0&1&0 \\ 0&0&1&1&0&0 \\ 0&0&0&1&1&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \end{pmatrix}$	$\begin{pmatrix} 0&2&0&0&0&0 \\ 2&0&1&1&0&0 \\ 0&1&0&1&0&0 \\ 0&1&1&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \end{pmatrix}$	$\ell(v_1) = [e_1, e_2]$ $\ell(v_2) = [e_1, e_2, e_3, e_5]$ $\ell(v_3) = [e_3, e_4]$ $\ell(v_4) = [e_4, e_5]$ $\ell(v_5) = [e_6]$ $\ell(v_6) = [e_6]$
$O(\lvert V\rvert \lvert E\rvert)$	$O(\lvert V\rvert^2)$	$O(\lvert E\rvert \log \lvert V\rvert)$

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Combinatorial Optimization

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