# Week I

### **Overview**

- ► Admin Topics;
- ► Graph Theory;
- ► Graph Representation;
- ▶ Depth-First Search (DFS).

## **Graph Theory**

- directed vs undirected graphs;
- walk, path and edge progression;
- reachability.

#### **DFS**

Algorithm 1: DEPTH FIRST SEARCH (DFS)

**Input:** undirected graph G, vertex  $s \in V(G)$ 

**Output:** tree  $(R, T) \subseteq G$ , R reachable from s

1 set  $R := \{s\}, Q := \{s\} \text{ and } T = \emptyset;$ 

2 if  $Q = \emptyset$  then return R, T;

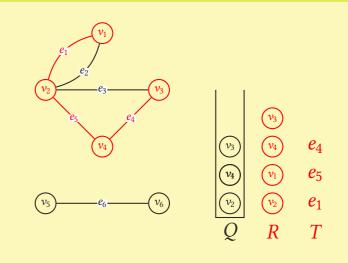
3 **else** v :=last vertex added to Q;

4 choose  $w \in V(G) \setminus R$  with  $\{v, w\} \in E(G)$ ;

5 if there is no such w then

set  $Q := Q \setminus \{v\}$  and **go to** 2

7 set  $R := R \cup \{w\}, Q := Q \cup \{w\},$  $T := T \cup \{\{v, w\}\},$ **go to** 2;



## **Graph Representation**

$A \in \{0,1\}^{ V  \times  E }, \qquad A \in \mathbb{Z}^{ V  \times  V }, \qquad L = [\ell(v) : v \in V],$ $a_{v,e} = \begin{cases} 1, & \text{if } v \in e \\ 0, & \text{if } v \notin e \end{cases}  a_{v,w} =  \{e = \{v, w\} \in E\}   \ell(v) = [e : e = \{u, v\} \in E] \end{cases}$ $\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}  \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}  \begin{cases} \ell(v_1) = [e_1, e_2] \\ \ell(v_2) = [e_1, e_2, e_3, e_5] \\ \ell(v_3) = [e_3, e_4] \\ \ell(v_4) = [e_4, e_5] \\ \ell(v_5) = [e_6] \\ \ell(v_6) = [e_6] \end{cases}$ $O( V  E )  O( V ^2)  O( E \log V )$	incidence matrix	adjacency matrix	adjacency list
$ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} \ell(v_1) = [e_1, e_2] \\ \ell(v_2) = [e_1, e_2, e_3, e_5] \\ \ell(v_3) = [e_3, e_4] \\ \ell(v_4) = [e_4, e_5] \\ \ell(v_5) = [e_6] \\ \ell(v_6) = [e_6] \end{pmatrix} $	$A \in \{0,1\}^{ V  \times  E },$	$A \in \mathbb{Z}^{ V  \times  V },$	$L = [\ell(v) \colon v \in V],$
$ \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} \ell(v_2) = [e_1, e_2, e_3, e_5] \\ \ell(v_3) = [e_3, e_4] \\ \ell(v_4) = [e_4, e_5] \\ \ell(v_5) = [e_6] \\ \ell(v_6) = [e_6] $	$a_{v,e} = \begin{cases} 1, & \text{if } v \in e \\ 0, & \text{if } v \notin e \end{cases}$	$a_{v,w} =  \{e = \{v, w\} \in E\} $	$\ell(v) = [e \colon e = \{u, v\} \in E]$
$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} \ell(v_3) = [e_3, e_4] \\ \ell(v_4) = [e_4, e_5] \\ \ell(v_5) = [e_6] \\ \ell(v_6) = [e_6] \end{bmatrix}$	1	(0 2 0 0 0 0)	$\ell(v_1) = [e_1, e_2]$
$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{cases} \ell(v_4) = [e_4, e_5] \\ \ell(v_5) = [e_6] \\ \ell(v_6) = [e_6] \end{cases}$	1 1 1 0 1 0	2 0 1 1 0 0	$\ell(v_2) = [e_1, e_2, e_3, e_5]$
$ \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}                $	0 0 1 1 0 0	0 1 0 1 0 0	$\ell(v_3) = [e_3, e_4]$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0 0 0 1 1 0	0 1 1 0 0 0	$\ell(v_4) = [e_4, e_5]$
	0 0 0 0 0 1	0 0 0 0 0 1	$\ell(v_5) = [e_6]$
$O( V  E )$ $O( V ^2)$ $O( E \log V )$	$(0 \ 0 \ 0 \ 0 \ 0 \ 1)$	$(0 \ 0 \ 0 \ 0 \ 1 \ 0)$	$\ell(v_6) = [e_6]$
- (1 · 11 · 1) - (1 · 1 · 1 · 1 · 1 · 1 · 1 · 1 · 1 · 1	O( V  E )	$O( V ^2)$	$O( E \log V )$