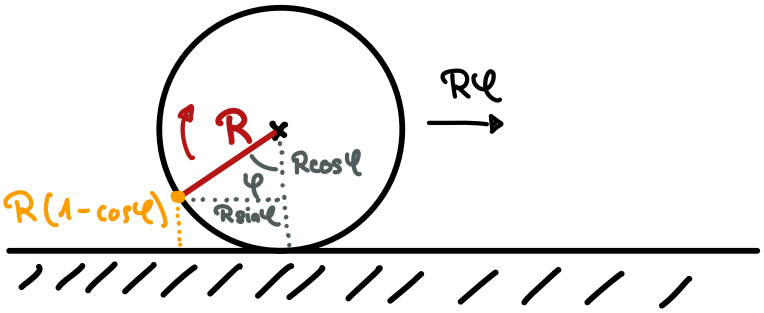
K1



Kiven käyrä parametrisoidussa muodossa:

$$\begin{cases} x(\varphi) = R(1-\cos\varphi) \\ y(\varphi) = R(\varphi - \sin\varphi) \end{cases} \implies \begin{cases} \dot{x}(\varphi) = R\sin\varphi \\ \dot{y}(\varphi) = R(1-\cos\varphi) \end{cases}$$

Käyrän pituus

$$\ell = \int_0^{2\pi} \sqrt{\dot{x}^2 + \dot{y}^2}\, d\varphi = \int_0^{2\pi} R\sqrt{1 - 2\cos\varphi + \cos^2(\varphi) + \sin^2(\varphi)}\, d\varphi$$

$$= R\int_0^{2\pi} \sqrt{2 - 2\cos\varphi}\, d\varphi = 2R\int_0^{2\pi} \sin\frac{\varphi}{2}\, d\varphi = -4R\cos\frac{\varphi}{2}\Big|_0^{2\pi}$$

$$= 8R$$

joten

$$\frac{\ell}{2\pi R} = \frac{8R}{2\pi R} = \underline{\underline{\frac{4}{\pi}}}$$

K2 a) $\underline{r}(t) = at^2\,\underline{i} + bt\,\underline{j} + c\ln(t)\,\underline{k} \implies \dot{\underline{r}}(t) = 2at\,\underline{i} + b\,\underline{j} + \frac{c}{t}\,\underline{k}$

Käyrän pituus, kun $t \in [1, T]$

$$\ell = \int_1^T \|\dot{\underline{r}}(t)\|\,dt = \int_1^T \sqrt{(2at)^2 + b^2 + \left(\frac{c}{t}\right)^2}\,dt$$

ja kun $b^2 = 4ac$

$$\ell = \int_1^T \sqrt{4a^2t^2 + 4ac + \frac{c^2}{t^2}}\,dt = \int_1^T \sqrt{\left(2at + \frac{c}{t}\right)^2}\,dt$$

$$= \int_1^T \left(2at + \frac{c}{t}\right)dt = \Big[at^2 + c\ln(t)\Big]_1^T = \underline{\underline{a(T^2 - 1) + c\ln(T)}}$$

b) Käyrä on heliksin muotoinen.

$$\begin{cases} \dot{x}(t) = a(\cos^2 t - \sin^2 t) \\ \dot{y}(t) = 2a\cos t \sin t \\ \dot{z}(t) = b \end{cases}$$

Sen pituus, kun $t \in [0, T]$:

$$\ell = \int_0^T \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\,dt$$

$$= \int_0^T \sqrt{a^2(\cos^4 t - 2\cos^2 t \sin^2 t + \sin^4 t) + 4a^2\cos^2 t \sin^2 t + b^2}\,dt$$

$$= \int_0^T \sqrt{a^2(\cos^2 t + \sin^2 t)^2 + b^2}\,dt = \int_0^T \sqrt{a^2 + b^2}\,dt = \underline{\underline{T\sqrt{a^2 + b^2}}}$$