

**In class -exercises 5.-7.2.2024**

## Round 5

The In class -exercises are to be done in the exercise session and the assistant will give advice on how to do them if necessary. The correct solutions to the problems will be discussed together. To obtain points for these exercises, you only need to be present.

1. Evaluate the lengths of the semi-axes of the ellipse  $g(x, y) = 3x^2 + 2xy + 3y^2 - 16 = 0$  by finding the extreme values for the square of the distance  $f(x, y) = x^2 + y^2$  using the method of Lagrange multipliers.

2. Evaluate the double integrals

$$\int_{-1}^1 \int_0^2 (2x + 2y) dx dy, \quad \text{ja} \quad \int_0^1 \int_0^{\ln 2} xe^{xy} dx dy.$$

**Hint:** Sometimes changing the order of the integration helps.

3. Evaluate the integral

$$\iiint_R (x^2 + y^2 + z^2) dV$$

over the cube  $R = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq x, y, z \leq 1\}$ . (Hint: Idea here is same than for the double integrals.)

4. Using Newton's method, find the solution with 3-decimal-place accuracy for the following system of equations

$$\begin{cases} x^2 - xy + 2y^2 = 10 \\ xy - 1 = 0 \end{cases}$$

How many solutions does the system of equations have?