Homework -exercises 8.-9.2.2024

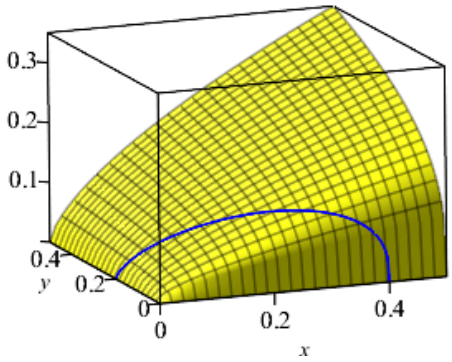
Round 5

To get points from these exercises do them at home before the second exercise session of the week and at the beginning of the class mark them on the list.

1. Evaluate the maximum value of the Cobb-Douglas production function

$$f(x, y) = x^{\alpha} y^{\beta}$$

under the conditions $ax + by = 1$ and $x, y > 0$ using the Lagrange multiplier method. Here a, b, α, β are positive constants.
Some simplified background(cf. Wikipedia): The Cobb-Douglass production function is used in economics to tell how production is tied to $x =$ workload and $y =$ material cost. The constraint arises naturally when a, b are price coefficients.
Hint: Although the problem can be reduced to a single-variable case, use Lagrangemultipliern method, which is a better alternative for more complex models. In the figure, the graph of the function f is shown in yellow and in blue is the curve corresponding the constraint.
(Answer will look like: $x = \alpha/(a(\alpha + \beta))$, $y = \dots$)



2. Calculate the following integrals without using symbolic calculators.

a) $\displaystyle\int_0^{\sqrt{\pi}} \int_{-x}^{x} \cos(x^2) dy\, dx$

b) $\displaystyle\int_1^2 \int_0^{1/y} e^{xy} dx\, dy$

c) $\displaystyle\int_0^c \int_0^b \int_0^a (xy^2 + z^3)\, dx\, dy\, dz$

3. Find the maximum and minimum values of the function $f(x, y) = x^2 - 2x + y^2$ in a triangle

$$D = \{(x, y) \in \mathbf{R}^2 \mid x \geq 0,\ |y| \leq 3 - x\}.$$