Combinatorial Optimization

NP Problems II

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§ Week VI §

Problem 1: TSP alternative

For the following graph, propose an algorithm to find a solution for TSP.

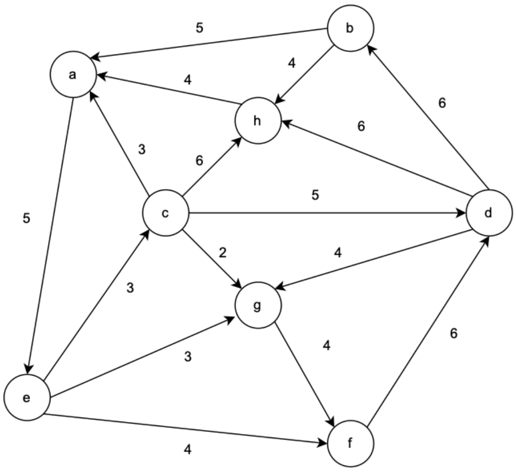


Figure 1: Direct, weighted graph.

Give examples of at least three subtours and their corresponding subtour constraint elimination.

Problem 2: Vertex Cycle Cover

Given a directed graph $G = (V, E)$, a vertex cycle cover is a subset of vertices such that every simple cycle in G passes through at least one of these vertices. For example, the graph shown in Fig 1 has a vertex cycle cover of size 2 (shaded).

Vertex Cycle Cover (VCC): Given a digraph G and an integer k, does G contain a vertex cycle cover of size at most k?

Is this problem NP? Provide a possible algorithm to solve such problem.

Problem 3: Cliques

When finding cliques, it is natural to look for vertices of high degree. Suppose, however, that you want to find cliques consisting of vertices of relatively low degree. We will show that even this problem is NP-complete.

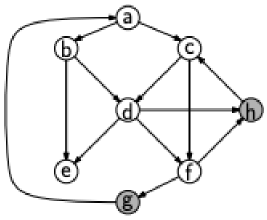


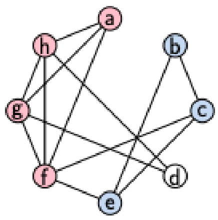
Figure 2: A graph and a vertex cycle cover consisting of $\{g, h\}$.

Low-Degree Clique (LDC): Given a graph $G = (V, E)$ and an integer k, does G have a clique of size at least k consisting entirely of vertices who degree is not greater than the median vertex degree of the entire graph?

By the *median vertex degree*, we mean the median value of the degrees of all n vertices of the graph.

For example, the graph shown in Fig. 2 has median vertex degree of 3. There exists an LDC of size 3 (vertices b, c, e) since all these vertices have degree at most 3. Even though there is a clique of size 4 (vertices a, f, g, h) it is not an LDC since it contains (at least one) vertex of degree higher than 3.

Show that the LDC problem is NP.



vertex:	a	b	c	d	e	f	g	h
degree:	3	2	3	2	3	5	4	4
median degree:	3							

Figure 3: Low-Degree clique

Problem 4: Hamiltonian Path vs TSP

What is the main difference between Hamiltonian Cycle and Travelling Salesman Problem. Can we solve one using the other?

Problem 5: Paths and Cycles

The graph shown below is the Petersen graph. Does it have a Hamilton cycle? Justify your answer. Does it have a Hamilton path? Justify your answer.

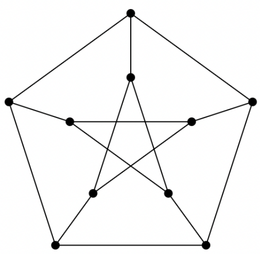


Figure 4: Peterson Graph