Week VI

Overview

- Clique;
- Independent Set;
- Dominant Set;
- Hamiltonian Path;
- How to compare different NP-Problems?

Definitions

- **Clique** is a subset of nodes in a graph where every two nodes is adjacent.
- **Independent Set** is a subset of nodes in a graph where there is no any adjacency between two nodes.
- **Dominant Set** is a subset of nodes where each node is either part of the subset or adjacent to it.
- **Hamiltonian Path** is a path in a graph where every node is visited once.
- **Hamiltonian Cycle** is a cycle in a graph where every node is visited once.

Cliques

$$\text{Maximize} \quad \sum_{v \in V} x_v$$

Subject to:

$$x_u + x_v \leq 1 \qquad \forall (u, v) \in \overline{E}$$
$$x_v \in \{0, 1\} \qquad \forall v \in V$$

Independent Set

$$\text{Maximize} \quad \sum_{v \in V} x_v$$

Subject to:

$$x_u + x_v \leq 1 \qquad \forall (u, v) \in E$$
$$x_v \in \{0, 1\} \qquad \forall v \in V$$

Hamiltonian Cycle

$$\text{Minimize} \quad \sum_{(u,v) \in E} x_{uv}$$

Subject to:

$$\sum_{v \in V} x_{uv} = 2 \qquad \forall u \in V$$
$$\sum_{(u,v): u \in S, v \notin S} x_{uv} \geq 2 \qquad \forall S \subset V, S \neq 0$$
$$x_{uv} \in \{0, 1\} \qquad \forall v \in V$$

Overview

- Require **at least one extra constraint** for subtour elimination;
  either DFJ or MTZ (as examples);
- $^*b_u \in \{-1, 0, 1\}$;
  for all nodes $v \in V$ depending if they are a **source**, a **sink** or an **intermediary nodes**.

Dominant Set

$$\text{Minimize} \quad \sum_{v \in V} x_v$$

Subject to:

$$x_v + \sum_{(u,v) \in E} x_u \geq 1 \qquad \forall v \in V$$
$$x_v \in \{0, 1\} \qquad \forall v \in V$$

Hamiltonian Path

$$\text{Minimize} \quad \sum_{(u,v) \in E} x_{uv}$$

Subject to:

$$\sum_{v \in V} x_{uv} = 2 \qquad \forall u \in V$$
$$\sum_{(uv) \in \delta^+(u)} x_{uv} - \sum_{(vu) \in \delta^-(u)} x_{uv} = b_u \qquad \forall u \in V$$
$$\sum_{(u,v) \in \delta^+(u)} x_{uv} \leq 1 \qquad \forall u \in V$$
$$x_{uv} \in \{0, 1\} \qquad \forall v \in V$$

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