In class -exercises 12.-14.2.2024

Round 6

The In class -exercises are to be done in the exercise session and the assistant will give advice on how to do them if necessary. The correct solutions to the problems will be discussed together. To obtain points for these exercises, you only need to be present.

1. Calculate the improper integral
$$\iint_D \frac{y}{x^3}\, dA,$$
where $D = \{(x, y) \in \mathbb{R}^2 : x \geq 1, 0 \leq y \leq \sqrt{x}\}$.
Hint: Start by sketching the set D.

2. Find the area of the region in the first quadrant bounded by the curves $xy = 1$, $xy = 4$, $y = x$, and $y = 2x$.

3. Consider the double integral
$$\int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{0} \frac{y}{x^2 + y^2 + 1}\, dxdy.$$
   (a) Sketch the region of integration. What is the name of the shape?
   (b) Do the following change of variables $(r, \theta) \mapsto (r\cos\theta, r\sin\theta)$ i.e. $x = r\cos\theta$ and $y = r\sin\theta$ (so we are changing to polar coordinates), and calculate the integral.
   (c) State one possible explanation for the value of the integral.

4. Let $T = \{(x, y) \in \mathbb{R}^2 \mid x \geq 0,\ y \geq 0,\ 2x^2 + y^2 \leq 9\}$. Evaluate the integral
$$\iint_T \frac{1}{\sqrt{2x^2 + y^2}} dA$$
using "streched polar coordinate"- change of variables:
$$(r, \varphi) \mapsto (\frac{1}{\sqrt{2}} r\cos\varphi, r\sin\varphi).$$