

Homework -exercises 15.-16.2.2024

Round 6

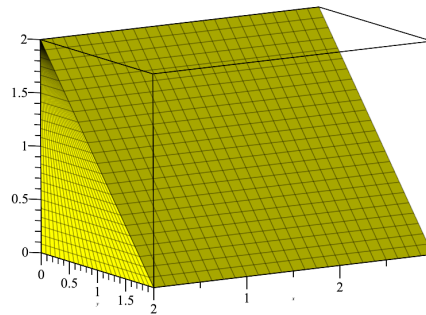
To get points from these exercises do them at home before the second exercise session of the week and at the beginning of the class mark them on the list.

1. The body K (picture below) is given by

$$K = \{(x, y, z) \in \mathbf{R}^3 \mid 0 \leq x \leq 3, 0 \leq y \leq 2, 0 \leq z \leq 2 - y\}.$$

- (a) What is the volume V of the body?
 (b) The temperature in the body is given by function $T = 10 + 30z$. Calculate the average temperature of the body:

$$\bar{T} = \frac{1}{V} \iiint_K T \, dV.$$



2. a) Evaluate the integral

$$\int_0^1 \int_0^{3-3x} \int_{x^2+3y^2}^{8-x^2-y^2} 1 \, dz \, dy \, dx.$$

What is the interpretation of this integral?

- b) Evaluate the integral

$$\int_0^1 \int_0^1 \int_{x^2}^1 12xz e^{zy^2} \, dy \, dx \, dz.$$

Hint: Draw a picture of the object over which the integration takes place and think of other ways to set the integration limits.

3. Evaluate

$$\iint_D \frac{\arctan(y/x)}{x^2 + y^2} \, dA,$$

where $D = \{(x, y) \in \mathbf{R}^2 : 1 < x^2 + y^2 < 4 \text{ and } x > 0, y > 0\}$.