Recommend exercises 15.-16.2.2024

These exercises are recommend but not submitted for grading. The model solutions will be published in MyCourses.

1. The integral
$$\int_0^{\pi/2} \int_y^{\pi/2} \frac{\sin x}{x}\, dx\, dy$$
cannot be calculated in this order. Sketch the set over which the integral is calculated, change the order of integration and evaluate the integral.
Note: The function $\frac{\sin x}{x}$ can be extended continuously to the point $x = 0$ as 1, so this is not an improper integral.

2. Consider the half of the ring area
$$D = \{(x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 4, y \geq 0\}.$$
Is the center of mass $(\overline{x}, \overline{y})$ of set D located inside or outside the set assuming that the mass is uniformly distributed?
Hint: Integration in polar coordinates.

3. Consider homogeneous half ball $W = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq R^2, z \geq 0\}$. The volume of this half ball is $V = \iiint_W dV = \frac{2\pi}{3} R^3$. Because of symmetry the center of mass for this object lies on z-axis. The z-coordinate of the center of mass is obtained by calculating the integral $\bar{z} = \frac{1}{V} \iiint_W z\, dv$.
   (a) Evaluate the integral using cylinder coordinates.
   (b) Evaluate the integral using spherical coordinates.