Chapter Seven

INTRODUCTION TO RISK, RETURN, AND THE OPPORTUNITY COST OF CAPITAL

We have managed to go through six chapters without directly addressing the problem of risk, but now the jig is up. We can no longer be satisfied with vague statements like "The opportunity cost of capital depends on the risk of the project." We need to know how risk is defined, what the links are between risk and the opportunity cost of capital, and how the financial manager can cope with risk in practical situations.

In this chapter we concentrate on the first of these issues and leave the other two to Chapters 8 and 9. We start by summarizing more than 70 years of evidence on rates of return in capital markets. Then we take a first look at investment risks and show how they can be reduced by portfolio diversification. We introduce you to beta, the standard risk measure for individual securities.

The themes of this chapter, then, are portfolio risk, security risk, and diversification. For the most part, we take the view of the individual investor. But at the end of the chapter we turn the problem around and ask whether diversification makes sense as a *corporate* objective.

7.1 SEVENTY-TWO YEARS OF CAPITAL MARKET HISTORY IN ONE EASY LESSON

Financial analysts are blessed with an enormous quantity of data on security prices and returns. For example, the University of Chicago's Center for Research in Security Prices (CRSP) has developed a file of prices and dividends for each month since 1926 for every stock that has been listed on the New York Stock Exchange (NYSE). Other files give data for stocks that are traded on the American

Stock Exchange and the over-the-counter market, data for bonds, for options, and so on. But this is supposed to be one easy lesson. We, therefore, concentrate on a study by Ibbotson Associates which measures the historical performance of five portfolios of securities:

1. A portfolio of Treasury bills, i.e., United States government debt securities maturing in less than one year.
2. A portfolio of long-term United States government bonds.
3. A portfolio of long-term corporate bonds.[1]
4. Standard and Poor's Composite Index (S&P 500), which represents a portfolio of common stocks of 500 large firms. (Although only a small proportion of the 7,000 or so publicly traded companies are included in the S&P 500, these companies account for roughly 70 percent of the *value* of stocks traded.)
5. A portfolio of the common stocks of small firms.

These investments offer different degrees of risk. Treasury bills are about as safe an investment as you can make. There is no risk of default, and their short maturity means that the prices of Treasury bills are relatively stable. In fact, an investor who wishes to lend money for, say, three months can achieve a perfectly certain payoff by purchasing a Treasury bill maturing in three months. However, the investor cannot lock in a *real* rate of return: There is still some uncertainty about inflation.

By switching to long-term government bonds, the investor acquires an asset whose price fluctuates as interest rates vary. (Bond prices fall when interest rates rise and rise when interest rates fall.) An investor who shifts from government to corporate bonds accepts an additional *default* risk. An investor who shifts from corporate bonds to common stocks has a direct share in the risks of the enterprise.

Figure 7.1 shows how your money would have grown if you had invested $1 at the start of 1926 and reinvested all dividend or interest income in each of the five portfolios.[2] Figure 7.2 is identical except that it depicts the growth in the *real* value of the portfolio. We will focus here on nominal values.

Portfolio performance coincides with our intuitive risk ranking. A dollar invested in the safest investment, Treasury bills, would have grown to just over $14 by 1997, barely enough to keep up with inflation. An investment in long-term Treasury bonds would have produced $39 and corporate bonds, a pinch more. Common stocks were in a class by themselves. An investor who placed a dollar in the stocks of large U.S. firms would have received $1,828. The jackpot, however, went to investors in stocks of small firms, who walked away with $5,520 for each dollar invested.

Ibbotson Associates also calculated the rate of return from these portfolios for each year from 1926 to 1997. This rate of return reflects both cash receipts—dividends or interest—and the capital gains or losses realized during the year. Averages of the 72 annual rates of return for each portfolio are shown in Table 7.1.

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[1]The two bond portfolios were revised each year in order to maintain a constant maturity.

[2]Portfolio values are plotted on a log scale. If they were not, the ending values for the two common stock portfolios would run off the top of the page.

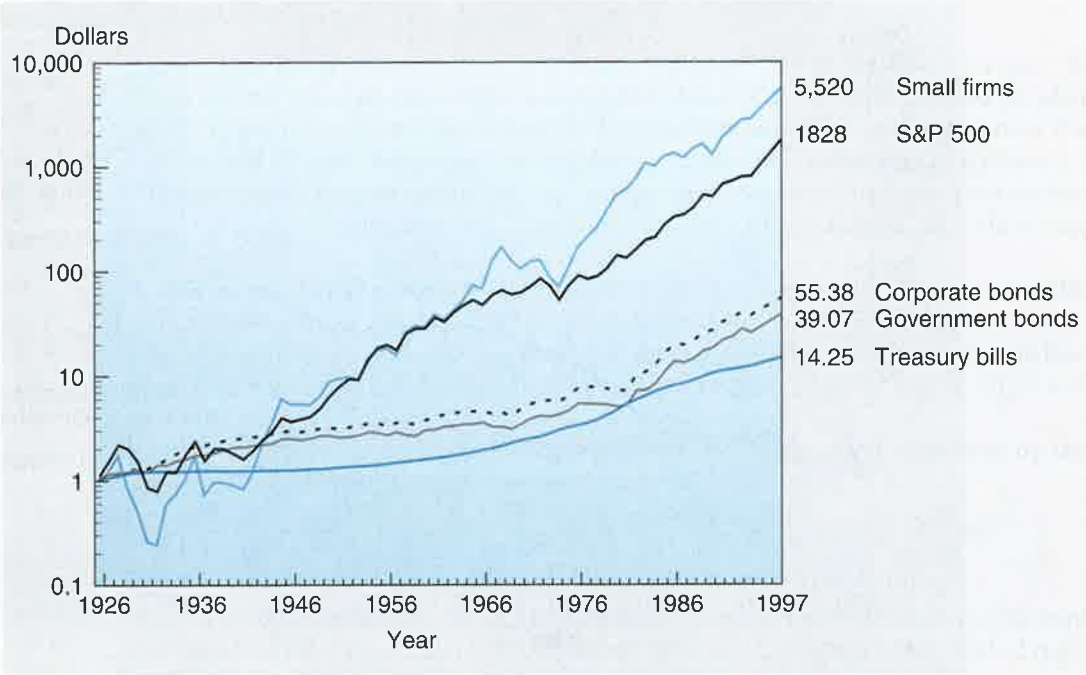

Figure 7.1

How an investment of $1 at the start of 1926 would have grown, assuming reinvestment of all dividend and interest payments.

Source: Ibbotson Associates, Inc., *Stocks, Bonds, Bills, and Inflation, 1998 Yearbook,* Chicago, 1998; cited hereafter in this chapter as the *1998 Yearbook.* © 1998 Ibbotson Associates, Inc.

Since 1926 Treasury bills have provided the lowest average return—3.8 percent per year in *nominal* terms and .7 percent in *real* terms. In other words, the average rate of inflation over this period was just over 3 percent per year. Common stocks were again the winners. Stocks of major corporations provided on average a *risk premium* of 9.2 percent a year over the return on Treasury bills. Stocks of small firms offered an even higher premium.

You may ask why we look back over such a long period to measure average rates of return. The reason is that annual rates of return for common stocks fluctuate so much that averages taken over short periods are meaningless. Our only hope of gaining insights from historical rates of return is to look at a very long period.[3]

[3]We cannot be sure that this period is truly representative and that the average is not distorted by a few unusually high or low returns. The reliability of an estimate of the average is usually measured by its *standard error.* For example, the standard error of our estimate of the average risk premium on common stocks is 2.4 percent. There is a 95 percent chance that the *true* average is within plus or minus 2 standard errors of the 9.2 percent estimate. In other words, if you said that the true average was between 4.4 and 14.0 percent, you would have a 95 percent chance of being right. (*Technical note:* The standard error of the mean is equal to the standard deviation divided by the square root of the number of observations. In our case the standard deviation is 20.3 percent, and therefore the standard error is $20.3/\sqrt{72} = 2.4$.)

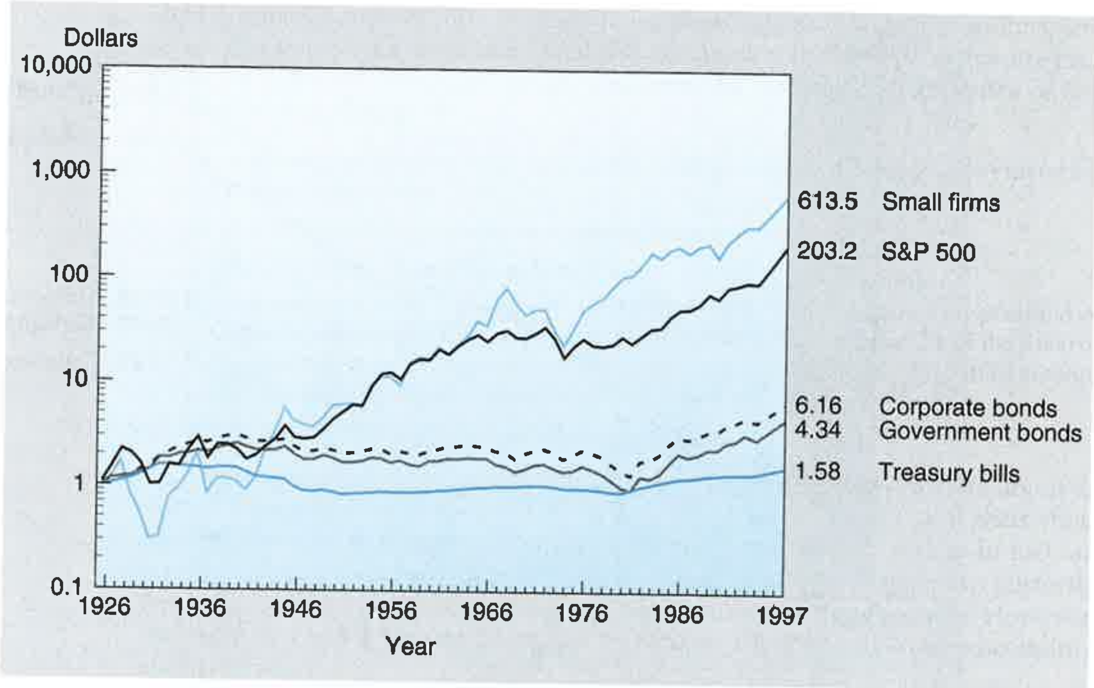

Figure 7.2

How an investment of $1 at the start of 1926 would have grown in real terms, assuming reinvestment of all dividend and interest payments. Compare this plot to Figure 7.1, and note how inflation has eroded the purchasing power of returns to investors.

Source: Ibbotson Associates, Inc., *1998 Yearbook.* © Ibbotson Associates, Inc.

TABLE 7.1

Average rates of return on Treasury bills, government bonds, corporate bonds, and common stocks, 1926–1997 (figures in percent per year)

PORTFOLIO	AVERAGE ANNUAL RATE OF RETURN		AVERAGE RISK PREMIUM (EXTRA RETURN VERSUS TREASURY BILLS)
	NOMINAL	REAL	
Treasury bills	3.8	.7	0
Government bonds	5.6	2.6	1.8
Corporate bonds	6.1	3.0	2.3
Common stocks (S&P 500)	13.0	9.7	9.2
Small-firm common stocks	17.7	14.2	13.9

Source: Ibbotson Associates, Inc., *1998 Yearbook.*

Arithmetic Averages and Compound Annual Returns

Notice that the average returns shown in Table 7.1 are arithmetic averages. In other words, Ibbotson Associates simply added the 72 annual returns and divided by 72. The arithmetic average is higher than the compound annual return over the period. The 72-year compound annual return for the S&P index was 11.0 percent.[4]

The proper uses of arithmetic and compound rates of return from past investments are often misunderstood. Therefore, we call a brief time-out for a clarifying example.

Suppose that the price of Big Oil's common stock is $100. There is an equal chance that at the end of the year the stock will be worth $90, $110, or $130. Therefore, the return could be –10 percent, +10 percent, or +30 percent (we assume that Big Oil does not pay a dividend). The *expected* return is $\frac{1}{3}(-10 + 10 + 30) = +10$ percent.

If we run the process in reverse and discount the expected cash flow by the expected rate of return, we obtain the value of Big Oil's stock:

$$PV = \frac{110}{1.10} = \$100$$

The expected return of 10 percent is therefore the correct rate at which to discount the expected cash flow from Big Oil's stock. It is also the opportunity cost of capital for investments which have the same degree of risk as Big Oil.

Now suppose that we observe the returns on Big Oil stock over a large number of years. If the odds are unchanged, the return will be –10 percent in a third of the years, +10 percent in a further third, and +30 percent in the remaining years. The arithmetic average of these yearly returns is

$$\frac{-10 + 10 + 30}{3} = +10\%$$

Thus the arithmetic average of the returns correctly measures the opportunity cost of capital for investments of similar risk to Big Oil stock.

The average compound annual return on Big Oil stock would be

$$(.9 \times 1.1 \times 1.3)^{1/3} - 1 = .088, \text{ or } 8.8\%,$$

less than the opportunity cost of capital. Investors would not be willing to invest in a project that offered an 8.8 percent expected return if they could get an expected return of 10 percent in the capital markets. The net present value of such a project would be

$$NPV = -100 + \frac{108.8}{1.1} = -1.1$$

Moral: If the cost of capital is estimated from historical returns or risk premiums, use arithmetic averages, not compound annual rates of return.

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[4]This was calculated from $(1 + r)^{72} = 1{,}828$, which implies $r = .11$. *Technical note:* For lognormally distributed returns the annual compound return is equal to the arithmetic average return minus half the variance. For example, the annual standard deviation of returns on the U.S. market was about .20, or 20 percent. Variance was therefore $.20^2$, or .04. The compound annual return is $.04/2 = .02$, or 2 percentage points less than the arithmetic average.

Using Historical Evidence to Evaluate Today's Cost of Capital

Suppose there is an investment project which you *know*—don't ask how—has the same risk as Standard and Poor's Composite Index. We will say that it has the same degree of risk as the *market portfolio,* although this is speaking somewhat loosely, because the index does not include all risky securities. What rate should you use to discount this project's forecasted cash flows?

Clearly you should use the currently expected rate of return on the market portfolio; that is the return investors would forgo by investing in the proposed project. Let us call this market return r_m. One way to estimate r_m is to assume that the future will be like the past and that today's investors expect to receive the same "normal" rates of return revealed by the averages shown in Table 7.1. In this case, you would set r_m at 13 percent, the average of past market returns.

Unfortunately, this is *not* the way to do it: r_m is not likely to be stable over time. Remember that it is the sum of the risk-free interest rate r_f and a premium for risk. We know that r_f varies. For example, as we finish this chapter in mid-1998, Treasury bills yield about 5 percent, more than 1 percentage point above the 3.8 percent average return of Treasury bills.

What if you were called upon to estimate r_m in 1998? Would you have said 13 percent? That would have squeezed the risk premium by 1.3 percentage points. A more sensible procedure takes the current interest rate on Treasury bills plus 9.2 percent, the average *risk premium* shown in Table 7.1. With a rate of 5 percent for Treasury bills, that gives

$$r_m\,(1998) = r_f\,(1998) + \text{normal risk premium}$$

$$= .05 + .092 = .142, \text{ or } 14.2\%$$

The crucial assumption here is that there is a normal, stable risk premium on the market portfolio, so that the expected *future* risk premium can be measured by the average past risk premium.

Even with 72 years of data, we can't estimate the market risk premium exactly; nor can we be sure that investors today are demanding the same reward for risk that they were in the 1940s or 1960s. All this leaves plenty of room for argument about what the risk premium *really* is. Many financial managers and economists believe that long-run historical returns are the best measure available. Others have a gut instinct that investors don't need such a large risk premium to persuade them to hold common stocks.[5]

In a recent survey of financial economists, more than a quarter of those polled believed that the expected risk premium was about 8 percent, but most of the

[5]There is some theory behind this gut instinct. The high risk premiums earned in the market seem to imply that investors are extremely risk-averse. If that is true, investors ought to cut back their consumption when stock prices fall and wealth decreases. But the evidence suggests that, when stock prices fall, investors spend at nearly the same rate. This is difficult to reconcile with high risk aversion and a high market risk premium. See R. Mehra and E. Prescott "The Equity Premium: a Puzzle," *Journal of Monetary Economics* 15 (1985), pp. 145–161.

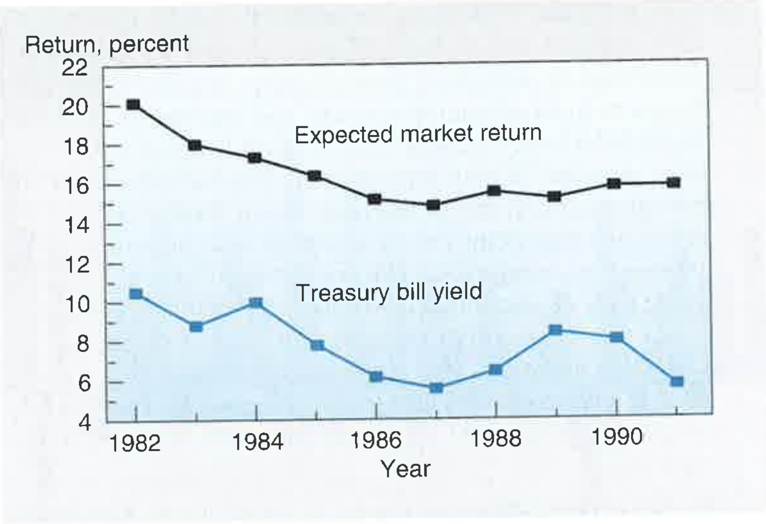


Figure 7.3

Expected market returns estimated using the constant-growth DCF formula. The spread between these estimates and Treasury bill yields varies, but it is consistent with the long-run average risk premium shown in Table 7.1.

Source: R. S. Harris and F. C. Marston, "Estimating Shareholder Risk Premia Using Analysts' Growth Forecasts," *Financial Management* 21 (Summer 1992), pp. 63–70.

remainder opted for a figure between 4 and 7 percent. The average estimate was just over 6 percent.[6]

The financial managers who rely on history generally use a market risk premium between 8 and 9 percent. The historical average stayed close to 8.5 percent from the 1980s to the mid-1990s. Moreover, the historical average does not change materially if returns from the highly volatile markets of the 1920s, 1930s, and early 1940s are excluded. For example, the average risk premium for the 50 years from 1948 to 1997 is 9.0 percent, only slightly below the average reported in Table 7.1.

Risk premiums between 8 and 9 percent seem consistent with some other evidence. For example, Harris and Marston have used security analysts' earnings–growth forecasts and the constant-growth DCF formula to estimate expected rates of return on a large sample of common stocks. Their findings are summarized in Figure 7.3. Over the period 1982 to 1991, analysts appeared to forecast a market return about 8 ½ percent over the risk-free rate.[7]

Of course 1991 may seem like ancient history, particularly given the spectacular performance of the stock market in the last half of the 1990s. (The returns in

[6] I. Welch, "Views of Financial Economists on the Equity Premium and Other Issues," Anderson Graduate School of Management, UCLA, April 16, 1998. Some of those questioned may have thought that they were being asked for the difference between the compound annual return on bonds and stocks. This is lower than the expected (arithmetic average) risk premium over Treasury bills.

[7] Kaplan and Ruback, in an analysis of valuations in 51 takeovers between 1983 and 1989, found that acquiring companies appeared to base discount rates on a market risk premium of about 7.5 percent over average returns on long-term Treasury bonds. The risk premium over Treasury bills would have been about a percentage point higher. This is also consistent with a market risk premium between 8 and 9 percent. See S. Kaplan and R. S. Ruback, "The Valuation of Cash Flow Forecasts: An Empirical Analysis," *Journal of Finance* 50 (September 1995), pp. 1059–1093.

1995, 1996, and 1997 were, respectively, 37.4, 23.1, and 33.4 percent![8]) Could these high rates of return be explained in part by a *decline* in the risk premium demanded by investors? Such a decline would reduce the discount rate used by investors to value common stocks and increase stock prices. Thus the immediate impact of a lower market risk premium is rising stock prices and *higher* observed rates of return. It may seem paradoxical, but advocates for 5 or 6 percent market risk premiums point to the high recent returns as support for their case. These advocates also point out that average risk premiums have been lower since the 1960s—the average is 6.6 percent from 1963 through 1997. Of course shorter averaging periods also introduce more random noise into the estimate.

If you believe that the expected market risk premium is a lot less than the historical averages, you probably also believe that history was unexpectedly kind to investors, and that investors' good luck is unlikely to be repeated. Here are two reasons why history *may* overstate the risk premium that investors now demand:

- Part of the capital appreciation in United States equities over the past 70 years has come from a fall in the dividend yield (from 5.4 percent in 1926 to 2.4 percent in 1997). It's hard to believe that the dividend yield will continue to fall. If the annual capital appreciation had simply matched the growth in dividends, the realized risk premium would have been about 1 percentage point lower.
- The United States has been among the world's most consistently prosperous countries. Other economies have languished or been wracked by war or civil unrest. By focusing on the equity returns in the United States, we may obtain a biased view of what investors expected.[9] Perhaps the historical averages miss the possibility that the United States could have turned out to be one of those less fortunate countries.

Brealey and Myers have no official position on the exact market risk premium, but we believe a range of 6 to 8.5 percent is reasonable for the United States. We are most comfortable with figures towards the upper end of the range.

7.2 MEASURING PORTFOLIO RISK

You now have a couple of benchmarks. You know the discount rate for safe projects, and you know the rate for average-risk projects. But you *don't* know yet how to estimate discount rates for assets that do not fit these simple cases. To do that, you have to learn (1) how to measure risk and (2) the relationship between risks borne and risk premiums demanded.

Figure 7.4 shows the 72 annual rates of return calculated by Ibbotson Associates for Standard and Poor's Composite Index. The fluctuations in year-to-year returns are remarkably wide. The highest annual return was 54.0 percent in 1933—a partial rebound from the stock market crash of 1929–1932. However, there were losses exceeding 25 percent in four years, the worst being the −43.3 percent return in 1931.

[8]We write this at the end of 1998, another excellent year for the U.S. stock market. Let's hope the market doesn't crash before the millennium.

[9]W. Goetzmann and P. Jorion, "A Century of Global Stockmarkets," NBER Working Paper, January 1997.

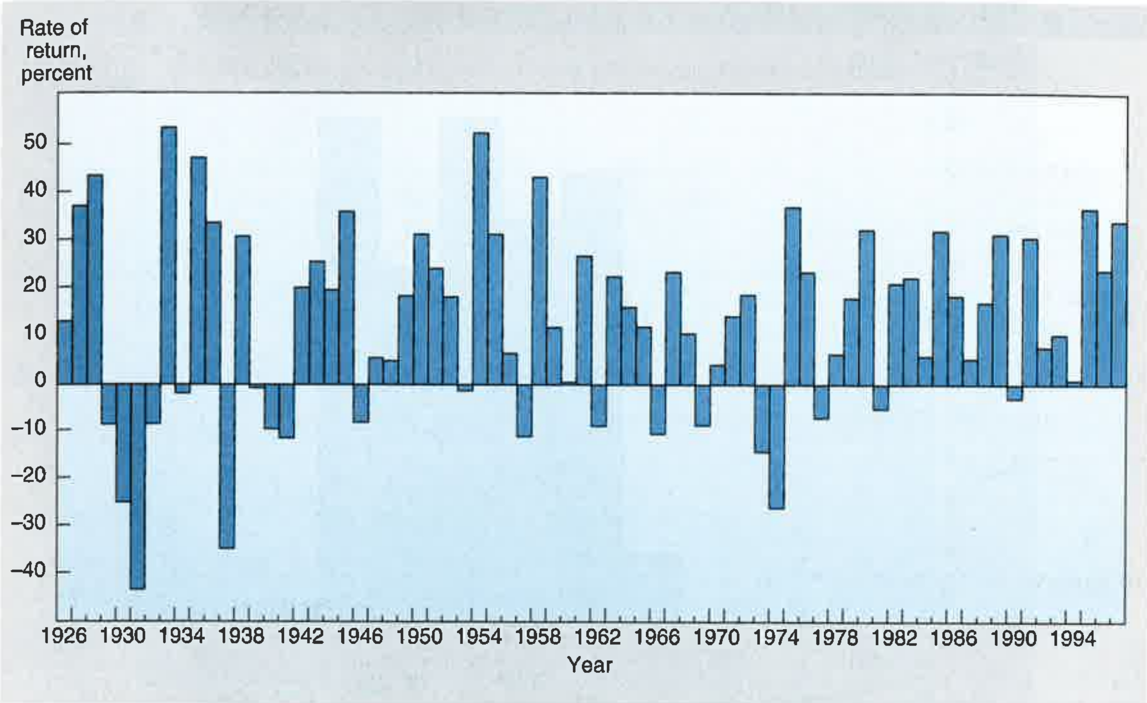


Figure 7.4

The stock market has been a profitable but extremely variable investment.
Source: Ibbotson Associates, Inc., *1998 Yearbook.* © 1998, Ibbotson Associates, Inc.

Another way of presenting these data is by a histogram or frequency distribution. This is done in Figure 7.5, where the variability of year-to-year returns shows up in the wide "spread" of outcomes.

Variance and Standard Deviation

The standard statistical measures of spread are **variance** and **standard deviation.** The variance of the market return is the expected squared deviation from the expected return. In other words,

$$\text{Variance}(\tilde{r}_m) = \text{the expected value of } (\tilde{r}_m - r_m)^2$$

where $\tilde{r}_m$ is the actual return and r_m is the expected return.[10] The standard deviation is simply the square root of the variance:

[10]One more technical point: When variance is estimated from a sample of *observed* returns, we add the squared deviations and divide by $N - 1$, where N is the number of observations. We divide by $N - 1$ rather than N to correct for what is called *the loss of a degree of freedom.* The formula is

$$\text{Variance}(\tilde{r}_m) = \frac{1}{N-1}\sum_{t=1}^{N}(\tilde{r}_{mt} - r_m)^2$$

where $\tilde{r}_{mt}$ = market return in period t,

r_m = mean of the values of $\tilde{r}_{mt}$.

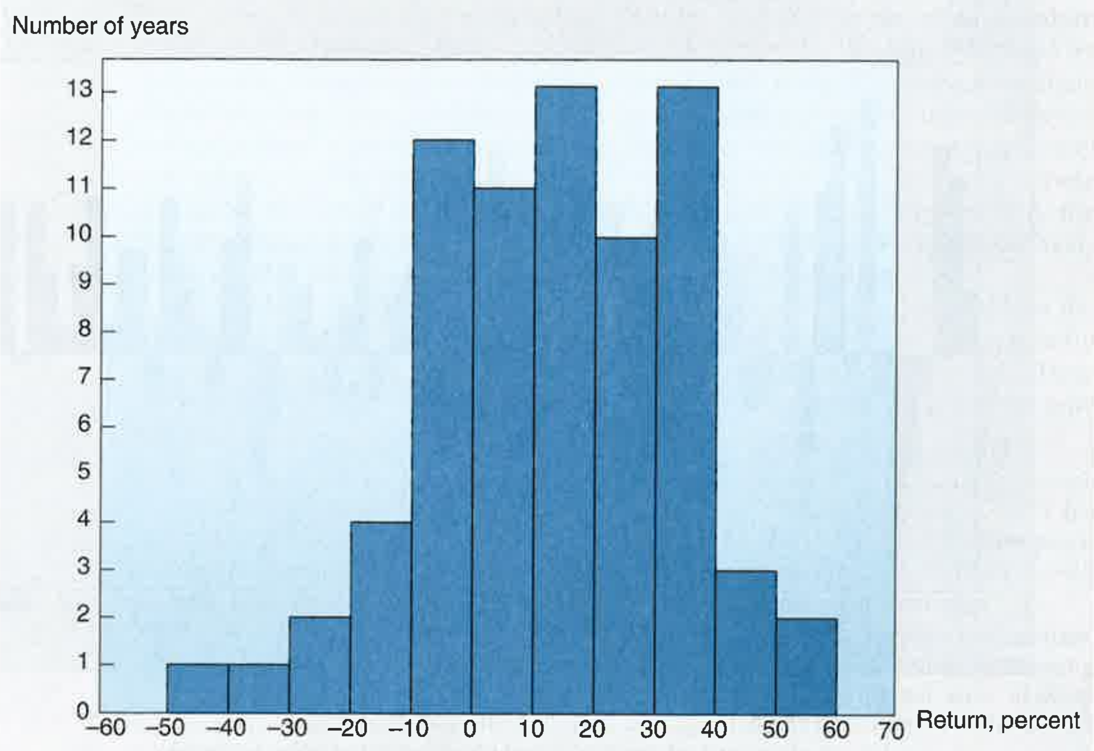


Figure 7.5

Histogram of the annual rates of return from the stock market in the United States, 1926–1997, showing the wide spread of returns from investment in common stocks.
Source: Ibbotson Associates, Inc., *1998 Yearbook.*

$$\text{Standard deviation of } \tilde{r}_m = \sqrt{\text{variance } (\tilde{r}_m)}$$

Standard deviation is often denoted by σ and variance is often denoted by σ^2.

Here is a very simple example showing how variance and standard deviation are calculated. Suppose that you are offered the chance to play the following game. You start by investing $100. Then two coins are flipped. For each head that comes up you get back your starting balance *plus* 20 percent, and for each tail that comes up you get back your starting balance *less* 10 percent. Clearly there are four equally likely outcomes:

- Head + head: You gain 40 percent.
- Head + tail: You gain 10 percent.
- Tail + head: You gain 10 percent.
- Tail + tail: You lose 20 percent.

There is a chance of 1 in 4, or .25, that you will make 40 percent; a chance of 2 in 4, or .5, that you will make 10 percent; and a chance of 1 in 4, or .25, that you will

TABLE 7.2

The coin-tossing game: calculating variance and standard deviation

(1) PERCENT RATE OF RETURN ($\tilde{r}$)	(2) DEVIATION FROM EXPECTED RETURN ($\tilde{r}-r$)	(3) SQUARED DEVIATION $[(\tilde{r}-r)^2]$	(4) PROBABILITY	(5) PROBABILITY × SQUARED DEVIATION
+40	+30	900	.25	225
+10	0	0	.5	0
−20	−30	900	.25	225
			Variance = expected value of $(\tilde{r} - r)^2$ =	450
			Standard deviation = $\sqrt{\text{variance}} = \sqrt{450}$ =	21

lose 20 percent. The game's expected return is, therefore, a weighted average of the possible outcomes:

$$\text{Expected return} = (.25 \times 40) + (.5 \times 10) + (.25 \times -20) = +10\%$$

Table 7.2 shows that the variance of the percentage returns is 450. Standard deviation is the square root of 450, or 21. This figure is in the same units as the rate of return, so we can say that the game's variability is 21 percent.

One way of defining uncertainty is to say that more things can happen than will happen. The risk of an asset can be completely expressed, as we did for the coin-tossing game, by writing all possible outcomes and the probability of each. For real assets this is cumbersome and often impossible. Therefore we use variance or standard deviation to summarize the spread of possible outcomes.[11]

These measures are natural indexes of risk.[12] If the outcome of the coin-tossing game had been certain, the standard deviation would have been zero. The actual standard deviation is positive because we *don't* know what will happen.

Or think of a second game, the same as the first except that each head means a 35 percent gain and each tail means a 25 percent loss. Again, there are four equally likely outcomes:

- Head + head: You gain 70 percent.
- Head + tail: You gain 10 percent.
- Tail + head: You gain 10 percent.
- Tail + tail: You lose 50 percent.

[11]Which of the two we use is solely a matter of convenience. Since standard deviation is in the same units as the rate of return, it is generally more convenient to use standard deviation. However, when we are talking about the *proportion* of risk that is due to some factor, it is usually less confusing to work in terms of the variance.

[12]As we explain in Chapter 8, standard deviation and variance are the correct measures of risk if the returns are normally distributed.

For this game the expected return is 10 percent, the same as that of the first game. But its standard deviation is double that of the first game, 42 versus 21 percent. By this measure the second game is twice as risky as the first.

Measuring Variability

In principle, you could estimate the variability of any portfolio of stocks or bonds by the procedure just described. You would identify the possible outcomes, assign a probability to each outcome, and grind through the calculations. But where do the probabilities come from? You can't look them up in the newspaper; newspapers seem to go out of their way to avoid definite statements about prospects for securities. We once saw an article headlined "Bond Prices Possibly Set to Move Sharply Either Way." Stockbrokers are much the same. Yours may respond to your query about possible market outcomes with a statement like this:

> *The market currently appears to be undergoing a period of consolidation. For the intermediate term, we would take a constructive view, provided economic recovery continues. The market could be up 20 percent a year from now, perhaps more if inflation moderates. On the other hand, . . .*

The Delphic oracle gave advice, but no probabilities.

Most financial analysts start by observing past variability. Of course, there is no risk in hindsight, but it is reasonable to assume that portfolios with histories of high variability also have the least predictable future performance.

The annual standard deviations and variances observed for our five portfolios over the period 1926–1997 were:[13]

PORTFOLIO	STANDARD DEVIATION (σ)	VARIANCE (σ^2)
Treasury bills	3.2	10.2
Long-term government bonds	9.2	84.6
Corporate bonds	8.7	75.7
Common stocks (S&P 500)	20.3	412.1
Small-firm common stocks	33.9	1149.2

As expected, Treasury bills were the least variable security, and small-firm stocks were the most variable. Government and corporate bonds hold the middle ground.[14]

[13]Ibbotson Associates, Inc., *1998 Yearbook.* In discussing the riskiness of *bonds,* be careful to specify the time period and whether you are speaking in real or nominal terms. The *nominal* return on a long-term government bond is absolutely certain to an investor who holds on until maturity; in other words, it is risk-free if you forget about inflation. After all, the government can always print money to pay off its debts. However, the real return on Treasury securities is uncertain because no one knows how much each future dollar will buy.

The bond returns reported by Ibbotson Associates were measured annually. The returns reflect year-to-year changes in bond prices as well as interest received. The *one-year* returns on long-term bonds are risky in *both* real and nominal terms.

[14]You may have noticed that corporate bonds come in just ahead of government bonds in terms of low variability. You shouldn't get excited about this. The problem is that it is difficult to get two sets of bonds that are alike in all other respects. For example, most corporate bonds are *callable* (i.e., the company has an option to repurchase them for their face value). Government bonds are not callable. Also interest payments are higher on corporate bonds. Therefore, investors in corporate bonds get their money sooner. As we will see in Chapter 26, this also reduces the bond's variability.

You may find it interesting to compare the coin-tossing game and the stock market as alternative investments. The stock market generated an average annual return of 13.0 percent with a standard deviation of 20.3 percent. The game offers 10 and 21 percent, respectively—slightly lower return and about the same variability. Your gambling friends may have come up with a crude representation of the stock market.

Of course, there is no reason to believe that the market's variability should stay the same over more than 70 years. For example, it is clearly less now than in the Great Depression of the 1930s. Here are standard deviations of the returns on the S&P index for successive periods starting in 1926:[15]

PERIOD	MARKET STANDARD DEVIATION (σ_M)
1926–1929	23.9%
1930–1939	41.6
1940–1949	17.5
1950–1959	14.1
1960–1969	13.1
1970–1979	17.1
1980–1989	19.4
1990–1997	14.3

These figures do not support the widespread impression of especially volatile stock prices during the 1980s and early 1990s. These years were below average on the volatility front.

However, there were brief episodes of extremely high volatility. On Black Monday, October 19, 1987, the market index fell by 23 percent *on a single day.* The standard deviation of the index for the week surrounding Black Monday was equivalent to 89 percent per year. Fortunately, volatility dropped back to normal levels within a few weeks after the crash.

How Diversification Reduces Risk

We can calculate our measures of variability equally well for individual securities and portfolios of securities. Of course, the level of variability over more than 70 years is less interesting for specific companies than for the market portfolio—it is a rare company that faces the same business risks today as it did in 1926.

Table 7.3 presents estimated standard deviations for 10 well-known common stocks for a recent five-year period.[16] Do these standard deviations look high to you? They should. Remember that the market portfolio's standard deviation was

[15]These estimates are derived from *monthly* rates of return. Annual observations are insufficient for estimating variability decade by decade. The monthly variance is converted to an annual variance by multiplying by 12. That is, the variance of the monthly return is one-twelfth of the annual variance. The longer you hold a security or portfolio, the more risk you have to bear.

This conversion assumes that successive monthly returns are statistically independent. This is, in fact, a good assumption, as we will show in Chapter 13.

Because variance is approximately proportional to the length of time interval over which a security or portfolio return is measured, *standard deviation* is proportional to the square root of the interval.

[16]These standard deviations are also calculated from monthly data.

TABLE 7.3

Standard deviations for selected U.S. common stocks, August 1993–July 1998 (figures in percent per year)

STOCK	STANDARD DEVIATION (σ)	STOCK	STANDARD DEVIATION (σ)
AT&T	22.6	General Electric	18.8
Bristol-Myers Squibb	17.1	McDonald's	20.8
Coca-Cola	19.7	Microsoft	29.4
Compaq	42.0	Reebok	35.4
Exxon	13.7	Xerox	24.3

about 14 percent from 1990 to 1997. Of our individual stocks only Exxon had a standard deviation less than 14 percent. Most stocks are substantially more variable than the market portfolio; only a handful are less variable.

Take a look also at Table 7.4, which shows the standard deviations of some well-known stocks from different countries and of the markets in which they trade. Some of these stocks are much more variable than others, but you can see that once again the individual stocks are more variable than the market indexes.

This raises an important question: The market portfolio is made up of individual stocks, so why doesn't its variability reflect the average variability of its components? The answer is that *diversification reduces variability.*

TABLE 7.4

Standard deviation for selected foreign stocks and market indexes, May 1993–April 1998 (figures in percent per year)

STOCK	STANDARD DEVIATION (σ)	MARKET	STANDARD DEVIATION (σ)	STOCK	STANDARD DEVIATION (σ)	MARKET	STANDARD DEVIATION (σ)
BP	16.3	UK	12.2	LVMH[b]	25.8	France	16.6
Deutsche Bank	23.2	Germany	11.3	Nestlé	18.9	Switzerland	14.6
Fiat[a]	35.2	Italy[a]	24.5	Sony	27.5	Japan	17.4
Hudson Bay	26.3	Canada	11.7	Telefonica de Argentina[c]	52.2	Argentina	28.6
KLM	30.1	Netherlands	14.2				

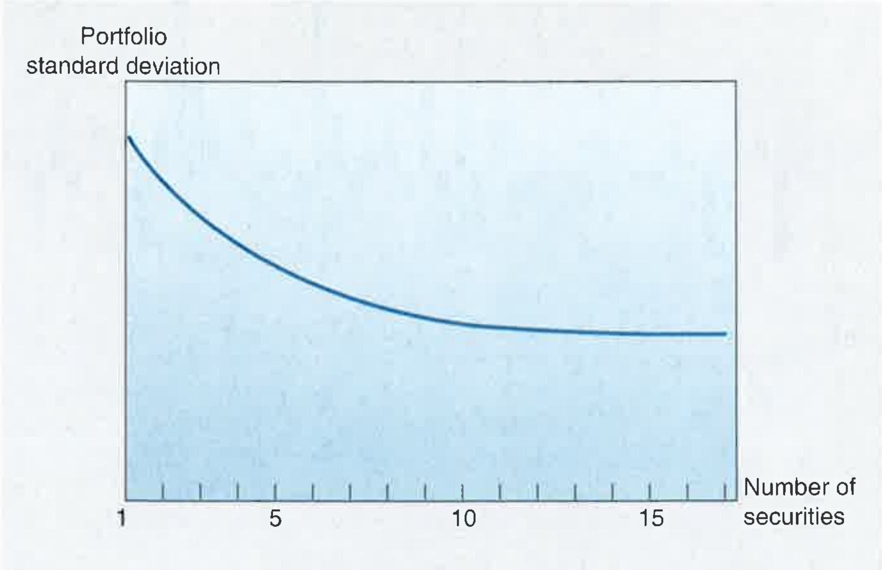
[a]September 1992–August 1997

[b]LVMH is LVMH Moet Hennessy Louis Vuitton, producer of a range of luxury goods.

[c]May 1994–April 1998

Figure 7.6

Diversification reduces risk (standard deviation) rapidly at first, then more slowly.



Even a little diversification can provide a substantial reduction in variability. Suppose you calculate and compare the standard deviations of randomly chosen one-stock portfolios, two-stock portfolios, five-stock portfolios, etc. You can see from Figure 7.6 that diversification can cut the variability of returns about in half. But you can get most of this benefit with relatively few stocks: The improvement is slight when the number of securities is increased beyond, say, 20 or 30.

Diversification works because prices of different stocks do not move exactly together. Statisticians make the same point when they say that stock price changes are less than perfectly correlated. Look, for example, at Figure 7.7. The top panel shows returns for Biotechnology General Corporation. We chose Biotech General because its stock has been unusually volatile. The middle panel shows returns for Compaq stock, which has also had its ups and downs. But on many occasions a decline in the value of one stock was offset by a rise in the price of the other.[17] Therefore there was an opportunity to reduce your risk by diversification. Figure 7.7 shows that if you had divided your funds evenly between the two stocks, the variability of your portfolio would have been substantially less than the average variability of the two stocks.[18]

The risk that potentially can be eliminated by diversification is called **unique risk.**[19] Unique risk stems from the fact that many of the perils that surround an individual company are peculiar to that company and perhaps its immediate competitors. But there is also some risk that you can't avoid, regardless of how much you diversify. This risk is generally known as **market risk.**[20] Market risk

[17]Over this period the correlation between the returns on the two stocks was approximately zero.

[18]The standard deviations of Biotech General and Compaq were 59.6 and 42.0 percent, respectively. The standard deviation of a portfolio with half invested in each was 35.7 percent.

[19]Unique risk may be called *unsystematic risk, residual risk, specific risk,* or *diversifiable risk.*

[20]Market risk may be called *systematic risk* or *undiversifiable risk.*

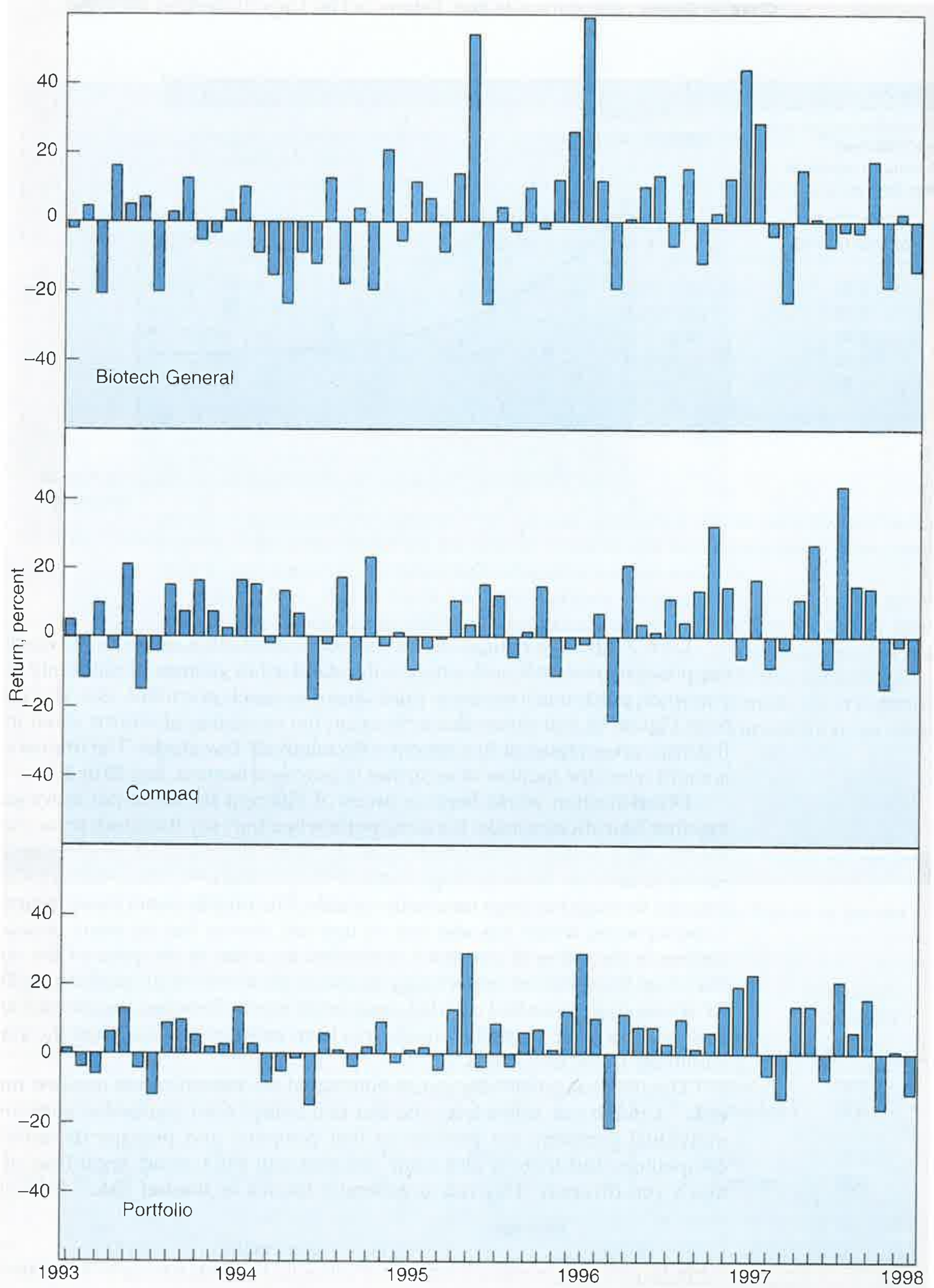


Figure 7.7

The variability of a portfolio with equal holdings in Compaq and Biotech General would have been less than the average variability of the individual stocks. These returns run from August 1993 to July 1998.

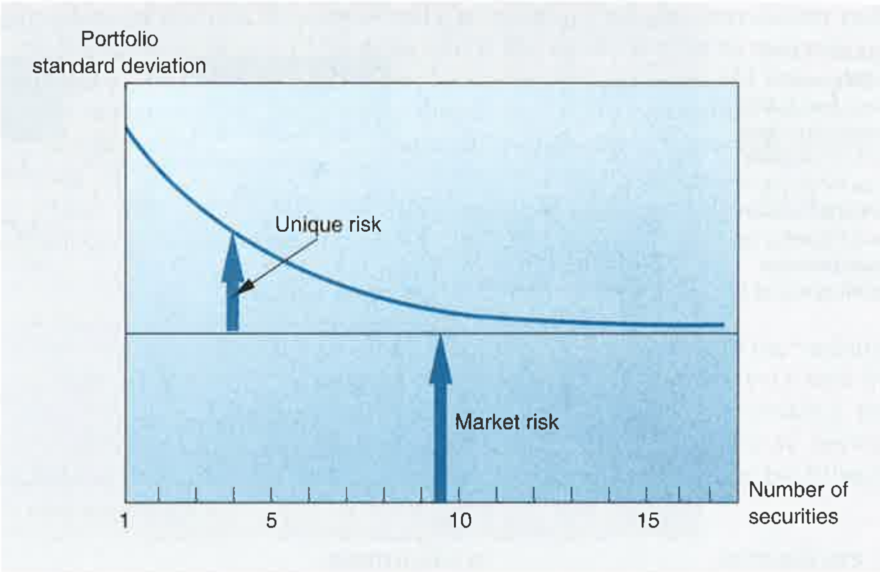


Figure 7.8

Diversification eliminates unique risk. But there is some risk that diversification *cannot* eliminate. This is called *market risk.*

stems from the fact that there are other economywide perils which threaten all businesses. That is why stocks have a tendency to move together. And that is why investors are exposed to market uncertainties, no matter how many stocks they hold.

In Figure 7.8 we have divided the risk into its two parts—unique risk and market risk. If you have only a single stock, unique risk is very important; but once you have a portfolio of 20 or more stocks, diversification has done the bulk of its work. For a reasonably well-diversified portfolio, only market risk matters. Therefore, the predominant source of uncertainty for a diversified investor is that the market will rise or plummet, carrying the investor's portfolio with it.

7.3 CALCULATING PORTFOLIO RISK

We have given you an intuitive idea of how diversification reduces risk, but to understand fully the effect of diversification, you need to know how the risk of a portfolio depends on the risk of the individual shares.

Suppose that 55 percent of your portfolio is invested in the shares of Bristol-Myers Squibb and the remainder is invested in McDonald's. You expect that over the coming year Bristol-Myers will give a return of 10 percent and McDonald's, 20 percent. The expected return on your portfolio is simply a weighted average of the expected returns on the individual stocks:[21]

$$\text{Expected portfolio return} = (0.55 \times 10) + (0.45 \times 20) = 14.5\%$$

[21] Let's check this. Suppose you invest $55 in Bristol-Myers and $45 in McDonald's. The expected dollar return on your Bristol-Myers is .10 × 55 = $5.50, and on McDonald's it is .20 × 45 = $9.00. The expected dollar return on your portfolio is 5.50 + 9.00 = $14.50. The portfolio *rate* of return is 14.50/100 = 0.145, or 14.5 percent.

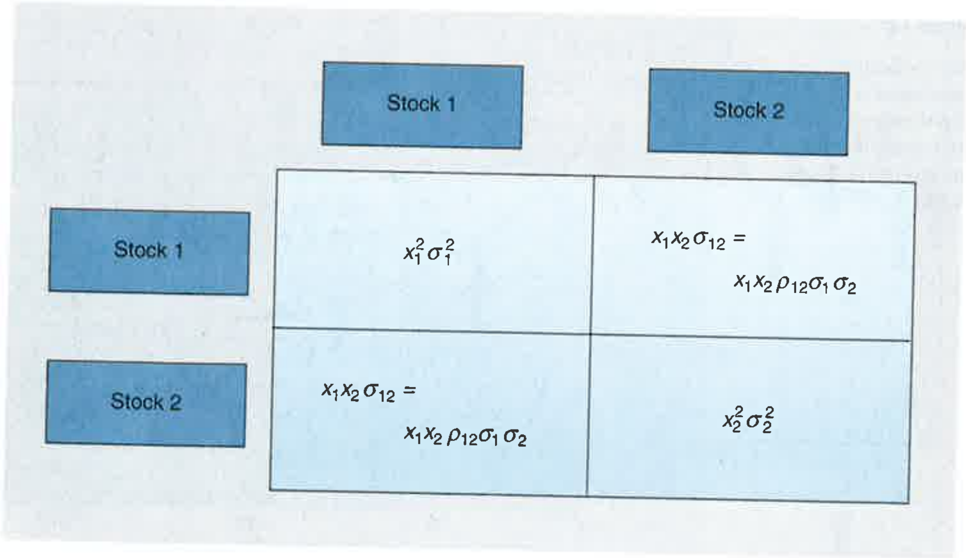


Figure 7.9

The variance of a two-stock portfolio is the sum of these four boxes. x_i = proportion invested in stock i; σ_i^2 = variance of return on stock i; σ_{ij} = covariance of returns on stocks i and j ($\rho_{ij}\sigma_i\sigma_j$); ρ_{ij} = correlation between returns on stocks i and j.

Calculating the expected portfolio return is easy. The hard part is to work out the risk of your portfolio. In the past the standard deviation of returns was 17.1 percent for Bristol-Myers and 20.8 percent for McDonald's. You believe that these figures are a good forecast of the spread of possible *future* outcomes. At first you may be inclined to assume that the standard deviation of your portfolio is a weighted average of the standard deviations of the two stocks, that is $(.55 \times 17.1) + (.45 \times 20.8) = 18.8$ percent. That would be correct *only* if the prices of the two stocks moved in perfect lockstep. In any other case, diversification reduces the risk below this figure.

The exact procedure for calculating the risk of a two-stock portfolio is given in Figure 7.9. You need to fill in four boxes. To complete the top left box, you weight the variance of the returns on stock 1 (σ_1^2) by the *square* of the proportion invested in it (x_1^2). Similarly, to complete the bottom right box, you weight the variance of the returns on stock 2 (σ_2^2) by the *square* of the proportion invested in stock 2 (x_2^2).

The entries in these diagonal boxes depend on the variances of stocks 1 and 2; the entries in the other two boxes depend on their **covariance.** As you might guess, the covariance is a measure of the degree to which the two stocks "covary." The covariance can be expressed as the product of the correlation coefficient ρ_{12} and the two standard deviations:[22]

$$\text{Covariance between stocks 1 and 2} = \sigma_{12} = \rho_{12}\sigma_1\sigma_2$$

For the most part stocks tend to move together. In this case the correlation coefficient ρ_{12} is positive, and therefore the covariance σ_{12} is also positive. If the

[22] Another way to define the covariance is as follows:

$$\text{Covariance between stocks 1 and 2} = \sigma_{12} = \text{expected value of } (\tilde{r}_1 - r_1) \times (\tilde{r}_2 - r_2)$$

Note that any security's covariance with itself is just its variance:

$$\sigma_{11} = \text{expected value of } (\tilde{r}_1 - r_1) \times (\tilde{r}_1 - r_1)$$
$$= \text{expected value of } (\tilde{r}_1 - r_1)^2 = \text{variance of stock 1} = \sigma_1^2 .$$

prospects of the stocks were wholly unrelated, both the correlation coefficient and the covariance would be zero; and if the stocks tended to move in opposite directions, the correlation coefficient and the covariance would be negative. Just as you weighted the variances by the square of the proportion invested, so you must weight the covariance by the *product* of the two proportionate holdings x_1 and x_2.

Once you have completed these four boxes, you simply add the entries to obtain the portfolio variance:

$$\text{Portfolio variance} = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2(x_1x_2\rho_{12}\sigma_1\sigma_2)$$

The portfolio standard deviation is, of course, the square root of the variance.

Now you can try putting in some figures for Bristol-Myers and McDonald's. We said earlier that if the two stocks were perfectly correlated, the standard deviation of the portfolio would lie 45 percent of the way between the standard deviations of the two stocks. Let us check this out by filling in the boxes with $\rho_{12} = +1$.

	BRISTOL-MYERS	MCDONALD'S
Bristol-Myers	$x_1^2\sigma_1^2 = (.55)^2 \times (17.1)^2$	$x_1x_2\rho_{12}\sigma_1\sigma_2$ $= .55 \times .45 \times 1 \times 17.1 \times 20.8$
McDonald's	$x_1x_2\rho_{12}\sigma_1\sigma_2$ $= .55 \times .45 \times 1 \times 17.1 \times 20.8$	$x_2^2\sigma_2^2 = (.45)^2 \times (20.8)^2$

The variance of your portfolio is the sum of these entries:

$$\text{Portfolio variance} = [(.55)^2 \times (17.1)^2] + [(.45)^2 \times (20.8)^2] + 2(.55 \times .45 \times 1 \times 17.1 \times 20.8) = 352.1$$

The standard deviation is $\sqrt{352.1} = 18.8$ percent or 45 percent of the way between 17.1 and 20.8.

Bristol-Myers and McDonald's do not move in perfect lockstep. If past experience is any guide, the correlation between the two stocks is about .15. If we go through the same exercise again with $\rho_{12} = +.15$, we find

$$\text{Portfolio variance} = [(.55)^2 \times (17.1)^2] + [(.45)^2 \times (20.8)^2] + 2(.55 \times .45 \times .15 \times 17.1 \times 20.8) = 202.5$$

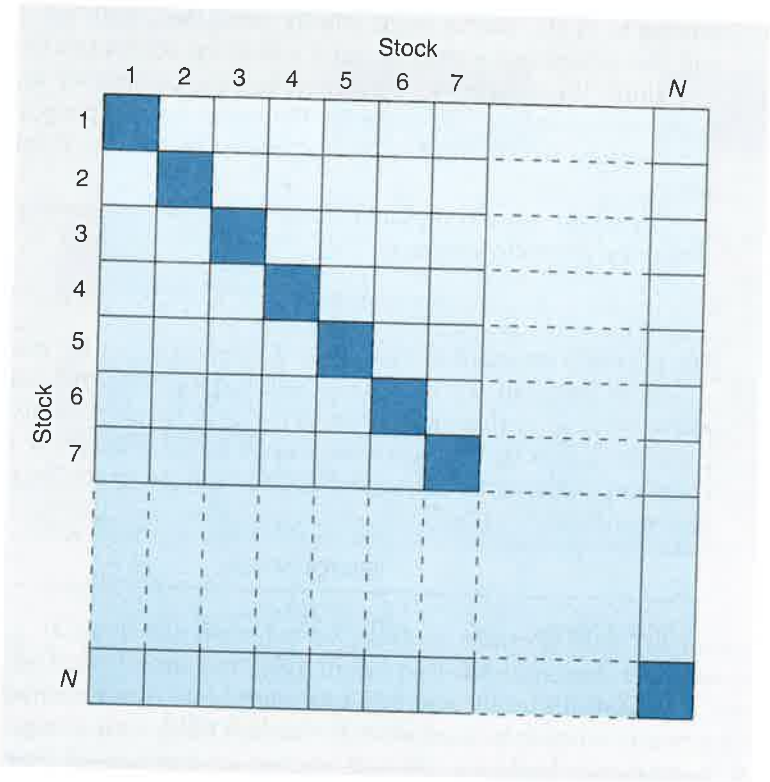
The standard deviation is $\sqrt{202.5} = 14.2$ percent. The risk is now *less* than 45 percent of the way between 17.1 and 20.8; in fact, it is less than the risk of investing in Bristol-Myers alone.

The greatest payoff to diversification comes when the two stocks are negatively correlated. Unfortunately, this almost never occurs with real stocks, but just for illustration, let us assume it for Bristol-Myers and McDonald's. And as long as we are being unrealistic, we might as well go whole hog and assume perfect negative correlation ($\rho_{12} = -1$). In this case,

$$\text{Portfolio variance} = [(.55)^2 \times (17.1)^2] + [(.45)^2 \times (20.8)^2] + 2[.55 \times .45 \times (-1) \times 17.1 \times 20.8] = 0$$

Figure 7.10

To find the variance of an N-stock portfolio, we must add the entries in a matrix like this. The diagonal cells contain variance terms ($x_i^2\sigma_i^2$), and the off-diagonal cells contain covariance terms ($x_i x_j \sigma_{ij}$).



When there is perfect negative correlation, there is always a portfolio strategy (represented by a particular set of portfolio weights) which will completely eliminate risk.[23] It's too bad perfect negative correlation doesn't really occur between common stocks.

General Formula for Computing Portfolio Risk

The method for calculating portfolio risk can easily be extended to portfolios of three or more securities. We just have to fill in a larger number of boxes. Each of those down the diagonal—the shaded boxes in Figure 7.10—contains the variance weighted by the square of the proportion invested. Each of the other boxes contains the covariance between that pair of securities, weighted by the product of the proportions invested.[24]

[23]Since the standard deviation of McDonald's is 1.2 times that of Bristol-Myers, you need to invest 1.2 times as much in Bristol-Myers to eliminate risk in this two-stock portfolio.

[24]The formal equivalent to "add up all the boxes" is

$$\text{Portfolio variance} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

Notice that when $i = j$, σ_{ij} is just the variance of stock i.

Limits to Diversification

Did you notice in Figure 7.10 how much more important the covariances become as we add more securities to the portfolio? When there are just two securities, there are equal numbers of variance boxes and of covariance boxes. When there are many securities, the number of covariances is much larger than the number of variances. Thus the variability of a well-diversified portfolio reflects mainly the covariances.

Suppose we are dealing with portfolios in which equal investments are made in each of N stocks. The proportion invested in each stock is, therefore, $1/N$. So in each variance box we have $(1/N)^2$ times the variance, and in each covariance box we have $(1/N)^2$ times the covariance. There are N variance boxes and $N^2 - N$ covariance boxes. Therefore,

$$\text{Portfolio variance} = N\left(\frac{1}{N}\right)^2 \times \text{average variance}$$
$$+ (N^2 - N)\left(\frac{1}{N}\right)^2 \times \text{average covariance}$$
$$= \frac{1}{N} \times \text{average variance} + \left(1 - \frac{1}{N}\right) \times \text{average covariance}$$

Notice that as N increases, the portfolio variance steadily approaches the average covariance. If the average covariance were zero, it would be possible to eliminate *all* risk by holding a sufficient number of securities. Unfortunately common stocks move together, not independently. Thus most of the stocks that the investor can actually buy are tied together in a web of positive covariances which set the limit to the benefits of diversification. Now we can understand the precise meaning of the market risk portrayed in Figure 7.8. It is the average covariance which constitutes the bedrock of risk remaining after diversification has done its work.

7.4 HOW INDIVIDUAL SECURITIES AFFECT PORTFOLIO RISK

We presented earlier some data on the variability of 10 individual U.S. securities. Compaq had the highest standard deviation and Exxon had the lowest. If you had held Compaq on its own, the spread of possible returns would have been three times greater than if you had held Exxon on its own. But that is not a very interesting fact. Wise investors don't put all their eggs into just one basket: They reduce their risk by diversification. They are therefore interested in the effect that each stock will have on the risk of their portfolio.

This brings us to one of the principal themes of this chapter: *The risk of a well-diversified portfolio depends on the market risk of the securities included in the portfolio.* Tattoo that statement on your forehead if you can't remember it any other way. It is one of the most important ideas in this book.

Market Risk Is Measured by Beta

If you want to know the contribution of an individual security to the risk of a well-diversified portfolio, it is no good thinking about how risky that security is if held in isolation—you need to measure its *market* risk, and that boils down to measuring how sensitive it is to market movements. This sensitivity is called **beta** (β).

TABLE 7.5

Betas for selected U.S. common stocks, August 1993–July 1998

STOCK	BETA	STOCK	BETA
AT&T	.65	General Electric	1.29
Bristol-Myers Squibb	.95	McDonald's	.95
Coca-Cola	.98	Microsoft	1.26
Compaq	1.13	Reebok	.87
Exxon	.73	Xerox	1.05

Stocks with betas greater than 1.0 tend to amplify the overall movements of the market. Stocks with betas between 0 and 1.0 tend to move in the same direction as the market, but not as far. Of course, the market is the portfolio of all stocks, so the "average" stock has a beta of 1.0. Table 7.5 reports betas for the 10 well-known common stocks we referred to earlier.

Over the five years from mid-1993 to mid-1998, General Electric had a beta of 1.29. If the future resembles the past, this means that *on average* when the market rises an extra 1 percent, GE's stock price will rise by an extra 1.29 percent. When the market falls an extra 2 percent, GE's stock prices will fall an extra $2 \times 1.29 = 2.58$ percent. Thus a line fitted to a plot of GE's returns versus market returns has a slope of 1.29. See Figure 7.11.

Of course GE's stock returns are not perfectly correlated with market returns. The company is also subject to unique risk, so the actual returns will be scattered

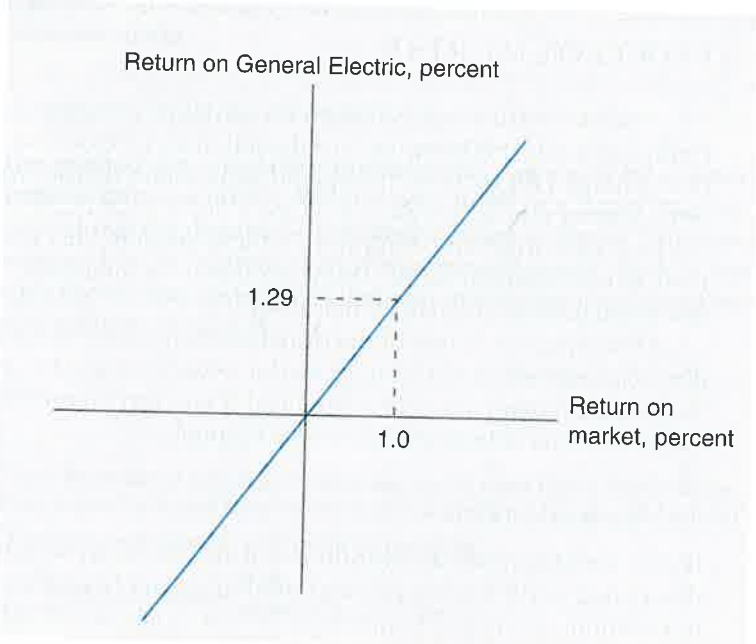


Figure 7.11

The return on General Electric stock changes on average by 1.29 percent for each additional 1 percent change in the market return. Beta is therefore 1.29.

about the line in Figure 7.11. Sometimes GE will head south while the market goes north, and vice versa.

Of the 10 stocks in Table 7.5 General Electric has the highest beta. AT&T is at the other extreme. A line fitted to a plot of AT&T's returns versus market returns would be less steep: Its slope would be only .65.

Just as we can measure how the returns of a U.S. stock are affected by fluctuations in the U.S. market, so we can measure how stocks in other countries are affected by movements in *their* markets. Table 7.6 shows the betas for the sample of foreign stocks.

Why Security Betas Determine Portfolio Risk

Let's review the two crucial points about security risk and portfolio risk:

- Market risk accounts for most of the risk of a well-diversified portfolio.
- The beta of an individual security measures its sensitivity to market movements.

It's easy to see where we are headed: In a portfolio context, a security's risk is measured by beta. Perhaps we could just jump to that conclusion, but we'd rather explain it. In fact, we'll offer two explanations.

Explanation 1: Where's Bedrock? Look back to Figure 7.8, which shows how the standard deviation of portfolio return depends on the number of securities in the portfolio. With more securities, and therefore better diversification, portfolio risk declines until all unique risk is eliminated and only the bedrock of market risk remains.

Where's bedrock? It depends on the average beta of the securities selected.

Suppose we constructed a portfolio containing a large number of stocks—500, say—drawn randomly from the whole market. What would we get? The market itself, or a portfolio *very* close to it. The portfolio beta would be 1.0, and the correlation with the market would be 1.0. If the standard deviation of the market were 20 percent (roughly its average for 1926–1997), then the portfolio standard deviation would also be 20 percent.

TABLE 7.6

Betas for selected foreign stocks, May 1993–April 1998 (betas are measured relative to the stock's home market)

STOCK	BETA	STOCK	BETA
BP (UK)	.74	LVMH (France)	1.00
Deutsche Bank (Germany)	1.05	Nestlé (Switzerland)	1.01
Fiat (Italy)[a]	1.11	Sony (Japan)	1.03
Hudson Bay (Canada)	.51	Telefonica de Argentina[b]	1.31
KLM (Netherlands)	1.13		

[a]September 1992–August 1997.

[b]May 1994–April 1998.

But suppose we constructed the portfolio from a large group of stocks with an average beta of 1.5. Again we would end up with a 500-stock portfolio with virtually no unique risk—a portfolio that moves almost in lockstep with the market. However, *this* portfolio's standard deviation would be 30 percent, 1.5 times that of the market.[25] A well-diversified portfolio with a beta of 1.5 will amplify every market move by 50 percent and end up with 150 percent of the market's risk.

Of course, we could repeat the same experiment with stocks with a beta of .5 and end up with a well-diversified portfolio half as risky as the market. Figure 7.12 shows these three cases.

The general point is this: The risk of a well-diversified portfolio is proportional to the portfolio beta, which equals the average beta of the securities included in the portfolio. This shows you how portfolio risk is driven by security betas.

Explanation 2: Betas and Covariances. A statistician would define the beta of stock *i* as

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$$

where σ_{im} is the covariance between stock *i*'s return and the market return, and σ_m^2 is the variance of the market return.

It turns out that this ratio of covariance to variance measures a stock's contribution to portfolio risk. You can see this by looking back at our calculations for the risk of the portfolio of Bristol-Myers Squibb and McDonald's.

Remember that the risk of this portfolio was the sum of the following cells:

	BRISTOL-MYERS	MCDONALD'S
Bristol-Myers	$(.55)^2 \times (17.1)^2$	$.55 \times .45 \times .15 \times 17.1 \times 20.8$
McDonald's	$.55 \times .45 \times .15 \times 17.1 \times 20.8$	$(.45)^2 \times (20.8)^2$

If we add each *row* of cells, we can see how much of the portfolio's risk comes from Bristol-Myers and how much comes from McDonald's:

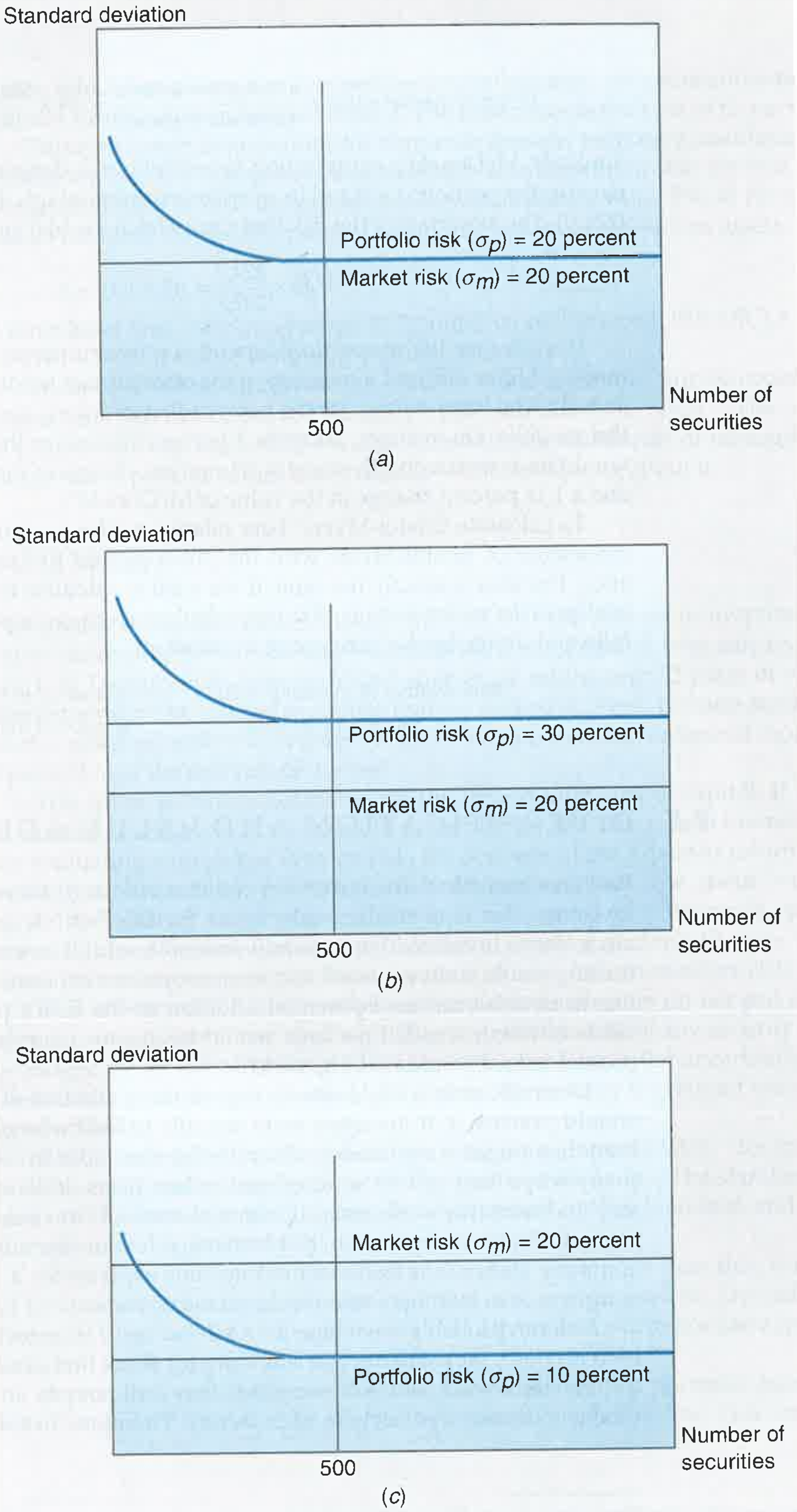
STOCK	CONTRIBUTION TO RISK
Bristol-Myers	$.55 \times \{[.55 \times (17.1)^2] + (.45 \times .15 \times 17.1 \times 20.8)\} = .55 \times 184.8$
McDonald's	$.45 \times \{[.55 \times .15 \times 17.1 \times 20.8] + [.45 \times (20.8)^2]\} = .45 \times 224.0$
Total portfolio	202.5

Bristol-Myers' contribution to portfolio risk depends on its relative importance in the portfolio (.55) and its average covariance with the stocks in the portfolio (184.8). (Notice that the average covariance of Bristol-Myers with the portfolio includes its covariance with itself, i.e., its variance.) The *proportion* of the risk that comes from the Bristol-Myers holding is

[25] A 500-stock portfolio with $\beta = 1.5$ would still have some unique risk because it would be unduly concentrated in high-beta industries. Its actual standard deviation would be a bit higher than 30 percent. If that worries you, relax; we will show you in Chapter 8 how you can construct a fully diversified portfolio with a beta of 1.5 by borrowing and investing in the market portfolio.

Figure 7.12

(*a*) A randomly selected 500-stock portfolio ends up with β = 1 and a standard deviation equal to the market's—in this case 20 percent. (*b*) A 500-stock portfolio constructed with stocks with average β = 1.5 has a standard deviation of about 30 percent—150 percent of the market's. (*c*) A 500-stock portfolio constructed with stocks with average β = .5 has a standard deviation of about 10 percent—half the market's.



$$\text{Relative market value} \times \frac{\text{average covariance}}{\text{portfolio variance}} = .55 \times \frac{184.8}{202.5} = .55 \times .91 = .5$$

Similarly, McDonald's contribution to portfolio risk depends on its relative importance in the portfolio (.45) and its average covariance with the stocks in the portfolio (224.0). The *proportion* of the risk that comes from the McDonald's holding is also .5:

$$.45 \times \frac{224.0}{202.5} = .45 \times 1.11 = .5$$

In each case the proportion depends on two numbers: the relative size of the holding (.55 or .45) and a measure of the effect of that holding on portfolio risk (.91 or 1.11). The latter values are the betas of Bristol-Myers and McDonald's *relative to that portfolio.* On average, an extra 1 percent change in the value of the portfolio would be associated with an extra .91 percent change in the value of Bristol-Myers and a 1.11 percent change in the value of McDonald's.

To calculate Bristol-Myers' beta relative to the portfolio, we simply take the covariance of Bristol-Myers with the portfolio and divide by the portfolio variance. The idea is exactly the same if we wish to calculate the beta of Bristol-Myers *relative to the market portfolio.* We just calculate its covariance with the market portfolio and divide by the variance of the market:

$$\begin{array}{c}\text{Beta relative to market portfolio} \\ \text{(or, more simply, beta)}\end{array} = \frac{\text{covariance with market}}{\text{variance of market}} = \frac{\sigma_{im}}{\sigma_m^2}$$

7.5 DIVERSIFICATION AND VALUE ADDITIVITY

We have seen that diversification reduces risk and, therefore, makes sense for investors. But does it also make sense for the firm? Is a diversified firm more attractive to investors than an undiversified one? If it is, we have an *extremely* disturbing result. If diversification is an appropriate corporate objective, each project has to be analyzed as a potential addition to the firm's portfolio of assets. The value of the diversified package would be greater than the sum of the parts. So present values would no longer add.

Diversification is undoubtedly a good thing, but that does not mean that firms should practice it. If investors were *not* able to hold a large number of securities, then they might want firms to diversify for them. But investors *can* diversify.[26] In many ways they can do so more easily than firms. Individuals can invest in the steel industry this week and pull out next week. A firm cannot do that. To be sure, the individual would have to pay brokerage fees on the purchase and sale of steel company shares, but think of the time and expense for a firm to acquire a steel company or to start up a new steel-making operation.

You can probably see where we are heading. If investors can diversify on their own account, they will not pay any *extra* for firms that diversify. And if they have a sufficiently wide choice of securities, they will not pay any *less* because they are unable to invest separately in each factory. Therefore, in countries like the United

[26]One of the simplest ways for an individual to diversify is to buy shares in a mutual fund which holds a diversified portfolio.

States, which have large and competitive capital markets, diversification does not add to a firm's value or subtract from it. The total value is the sum of its parts.

This conclusion is important for corporate finance, because it justifies adding present values. The concept of *value additivity* is so important that we will give a formal definition of it. If the capital market establishes a value PV(A) for asset A and PV(B) for B, the market value of a firm that holds only these two assets is

$$PV(AB) = PV(A) + PV(B)$$

A three-asset firm combining assets A, B, and C would be worth PV(ABC) = PV(A) + PV(B) + PV(C), and so on for any number of assets.

We have relied on intuitive arguments for value additivity. But the concept is a general one that can be proved formally by several different routes.[27] The concept of value additivity seems to be widely accepted, for thousands of managers add thousands of present values daily, usually without thinking about it.

7.6 SUMMARY

Our review of capital market history showed that the returns to investors have varied according to the risks they have borne. At one extreme, very safe securities like U.S. Treasury bills have provided an average return over 72 years of only 3.8 percent a year. The riskiest securities that we looked at were common stocks. The stock market provided an average return of 13.0 percent, a premium of more than 9 percent over the safe rate of interest.

This gives us two benchmarks for the opportunity cost of capital. If we are evaluating a safe project, we discount at the current risk-free rate of interest. If we are evaluating a project of average risk, we discount at the expected return on the average common stock. Historical evidence suggests that it is about 9 percent above the risk-free rate, but many financial managers and economists opt for a lower figure. That still leaves us with a lot of assets that don't fit these simple cases. Before we can deal with them, we need to learn how to measure risk.

Risk is best judged in a portfolio context. Most investors do not put all their eggs into one basket: They diversify. Thus the effective risk of any security cannot be judged by an examination of that security alone. Part of the uncertainty about the security's return is diversified away when the security is grouped with others in a portfolio.

Risk in investment means that future returns are unpredictable. This spread of possible outcomes is usually measured by standard deviation. The standard deviation of the *market portfolio*, generally represented by the Standard and Poor's Composite Index, is around 20 percent a year.

Most individual stocks have higher standard deviations than this, but much of their variability represents *unique* risk that can be eliminated by diversification. Diversification cannot eliminate *market* risk. Diversified portfolios are exposed to variation in the general level of the market.

A security's contribution to the risk of a well-diversified portfolio depends on how the security is liable to be affected by a general market decline. This sensitivity

[27]You may wish to refer to the Appendix to Chapter 33, which discusses diversification and value additivity in the context of mergers.

to market movements is known as *beta* (β). Beta measures the amount that investors expect the stock price to change for each additional 1 percent change in the market. The average beta of all stocks is 1.0. A stock with a beta greater than 1 is unusually sensitive to market movements; a stock with a beta below 1 is unusually insensitive to market movements. The standard deviation of a well-diversified portfolio is proportional to its beta. Thus a diversified portfolio invested in stocks with a beta of 2.0 will have twice the risk of a diversified portfolio with a beta of 1.0.

One theme of this chapter is that diversification is a good thing *for the investor.* This does not imply that *firms* should diversify. Corporate diversification is redundant if investors can diversify on their own account. Since diversification does not affect the firm value, present values add even when risk is explicitly considered. Thanks to *value additivity,* the net present value rule for capital budgeting works even under uncertainty.

Further Reading

A very valuable record of the performance of United States securities since 1926 is:

Ibbotson Associates, Inc.: *Stocks, Bonds, Bills, and Inflation, 1998 Yearbook,* Ibbotson Associates, Chicago, 1998.

Merton discusses the problems encountered in measuring average returns from historical data:

R. C. Merton: "On Estimating the Expected Return on the Market: An Exploratory Investigation," *Journal of Financial Economics,* 8:323–361 (December 1980).

The classic analysis of the degree to which stocks move together is:

B. F. King: "Market and Industry Factors in Stock Price Behavior," *Journal of Business, Security Prices: A Supplement,* 39:179–190 (January 1966).

There have been several studies of the way that standard deviation is reduced by diversification, including:

M. Statman: "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis,* 22:353–364 (September 1987).

Formal proofs of the value additivity principle can be found in:

S. C. Myers: "Procedures for Capital Budgeting under Uncertainty," *Industrial Management Review,* 9:1–20 (Spring 1968).

L. D. Schall: "Asset Valuation, Firm Investment and Firm Diversification," *Journal of Business,* 45:11–28 (January 1972).

Quiz

1. (a) What was the average annual return on United States common stocks from 1926 to 1997 (approximately)?
   (b) What was the average difference between this return and the return on Treasury bills?
   (c) What was the average return on Treasury bills in real terms?
   (d) What was the standard deviation of returns on the market index?
   (e) Was this standard deviation more or less than on most individual stocks?
2. A game of chance offers the following odds and payoffs. Each play of the game costs $100, so the net profit per play is the payoff less $100.

PROBABILITY	PAYOFF	NET PROFIT
.10	$500	$400
.50	100	0
.40	0	–100

What are the expected cash payoff and expected rate of return? Calculate the variance and standard deviation of this rate of return.

3. The following table shows the nominal returns on the Mexican stock market and the Mexican rate of inflation.
   (a) What was the standard deviation of the market returns?
   (b) Calculate the average real return.

YEAR	NOMINAL RETURN (%)	INFLATION (%)
1993	45.7	8.0
1994	–4.9	7.1
1995	16.5	52.0
1996	21.9	27.7
1997	53.4	15.7

4. Fill in the missing words:
   Risk is usually measured by the variance of returns or the ______, which is simply the square root of the variance. As long as the stock price changes are not perfectly ______, the risk of a diversified portfolio is ______ than the average risk of the individual stocks. The risk that can be eliminated by diversification is known as ______ risk. But diversification cannot remove all risk; the risk that it cannot eliminate is known as ______ risk.
5. Lawrence Interchange, ace mutual fund manager, produced the following percentage rates of return from 1993 to 1997. Rates of return on the S&P 500 are given for comparison.

	1993	1994	1995	1996	1997
Mr. Interchange	14.4	–4.0	+40.0	+16.1	+28.4
S&P 500	+10.0	+1.3	+37.4	+23.1	+33.4

   Calculate the average return and standard deviation of Mr. Interchange's mutual fund. Did he do better or worse than the S&P by these measures?
6. True or false?
   (a) Investors prefer diversified companies because they are less risky.
   (b) If stocks were perfectly positively correlated, diversification would not reduce risk.
   (c) The contribution of a stock to the risk of a well-diversified portfolio depends on its market risk.
   (d) A well-diversified portfolio with a beta of 2.0 is twice as risky as the market portfolio.
   (e) An undiversified portfolio with a beta of 2.0 is less than twice as risky as the market portfolio.
7. In which of the following situations would you get the largest reduction in risk by spreading your investment across two stocks?
   (a) The two shares are perfectly correlated.
   (b) There is no correlation.
   (c) There is modest negative correlation.
   (d) There is perfect negative correlation.
8. To calculate the variance of a three-stock portfolio, you need to add nine boxes:

TABLE 7.7

See Quiz question 11

STOCK	EXPECTED STOCK RETURN IF MARKET RETURN IS: −10%	+10%
A	0	+20
B	−20	+20
C	−30	0
D	+15	+15
E	+10	−10

Use the same symbols that we used in this chapter; for example, x_1 = proportion invested in stock 1 and σ_{12} = covariance between stocks 1 and 2. Now complete the nine boxes.

9. Suppose the standard deviation of the market return is 20 percent.
   (a) What is the standard deviation of returns on a well-diversified portfolio with a beta of 1.3?
   (b) What is the standard deviation of returns on a well-diversified portfolio with a beta of 0?
   (c) A well-diversified portfolio has a standard deviation of 15 percent. What is its beta?
   (d) A poorly diversified portfolio has a standard deviation of 20 percent. What can you say about its beta?
10. A portfolio contains equal investments in 10 stocks. Five have a beta of 1.2; the remainder have a beta of 1.4. What is the portfolio beta?
    (a) 1.3.
    (b) Greater than 1.3 because the portfolio is not completely diversified.
    (c) Less than 1.3 because diversification reduces beta.
11. What is the beta of each of the stocks shown in Table 7.7?
12. True or false? Why? "Diversification reduces risk. Therefore corporations ought to favor capital investments with low correlations with their existing lines of business."

Practice Questions

1. Here are inflation rates and stock market and Treasury bill returns between 1993 and 1997:

YEAR	INFLATION	S&P 500 RETURN	T-BILL RETURN
1993	+2.8%	+10.0%	+2.9%
1994	+2.7	+1.3	+3.9
1995	+2.5	+37.4	+5.6
1996	+3.3	+23.1	+5.2
1997	+1.7	+33.4	+5.3

(a) What was the real return on the S&P 500 in each year?
(b) What was the average real return?
(c) What was the risk premium in each year?
(d) What was the average risk premium?
(e) What was the standard deviation of the risk premium?

2. You toss a die. If the number on the die is less than three, you receive $10. If it is greater than four, you pay $20. Otherwise you call it quits. What is the expected payoff? What is the standard deviation?

3. Each of the following statements is dangerous or misleading. Explain why.
   (a) A long-term United States government bond is always absolutely safe.
   (b) All investors should prefer stocks to bonds because stocks offer higher long-run rates of return.
   (c) The best practical forecast of future rates of return on the stock market is a 5- or 10-year average of historical returns.

4. "There's upside risk and downside risk. Standard deviation doesn't distinguish between them." Do you think the speaker has a fair point?

5. Hippique s.a., which owns a stable of racehorses, has just invested in a mysterious black stallion with great form but disputed bloodlines. Some experts in horseflesh predict the horse will win the coveted Prix de Bidet; others argue that it should be put out to grass. Is this a risky investment for Hippique shareholders? Explain.

6. Lonesome Gulch Mines has a standard deviation of 42 percent per year and a beta of +.10. Amalgamated Copper has a standard deviation of 31 percent a year and a beta of +.66. Explain why Lonesome Gulch is the safer investment for a diversified investor.

7. Respond to the following comments:
   (a) "Risk is not variability. If I know a stock is going to fluctuate between $10 and $20, I can make myself a bundle."
   (b) "There are all sorts of risk in addition to beta risk. There's the risk that we'll have a downturn in demand, there's the risk that my best plant manager will drop dead, there's the risk of a hike in steel prices. You've got to take all these things into consideration."
   (c) "Risk to me is the probability of loss."
   (d) "Those guys who suggest beta is a measure of risk make the big assumption that betas don't change."

8. Lambeth Walk invests 60 percent of his funds in stock I and the balance in stock J. The standard deviation of returns on I is 10 percent, and on J it is 20 percent. Calculate the variance of portfolio returns, assuming
   (a) The correlation between the returns is 1.0.
   (b) The correlation is .5.
   (c) The correlation is 0.

9. (a) How many variance terms and how many covariance terms do you need to calculate the risk of a 100-share portfolio?
   (b) Suppose all stocks had a standard deviation of 30 percent and a correlation with each other of .4. What is the standard deviation of the returns on a portfolio that has equal holdings in 50 stocks?
   (c) What is the standard deviation of a fully diversified portfolio of such stocks?

10. Suppose that the standard deviation of returns from a typical share is about .40 (or 40 percent) a year. The correlation between the returns of each pair of shares is about .3.
   (a) Calculate the variance and standard deviation of the returns on a portfolio that has equal investments in two shares, three shares, and so on, up to 10 shares.

(b) Use your estimates to draw two graphs like Figure 7.8 (one for variance, the other for standard deviation). How large is the underlying market risk that cannot be diversified away?

(c) Now repeat the problem, assuming that the correlation between each pair of stocks is zero.

11. You believe that there is a 40 percent chance that stock A will decline by 10 percent and a 60 percent chance that it will rise by 20 percent. Correspondingly, there is a 30 percent chance that stock B will decline by 10 percent and a 70 percent chance that it will rise by 20 percent. The correlation coefficient between the two stocks is .7. Calculate the expected return, the variance, and the standard deviation for each stock. Then calculate the covariance between their returns.

12. Table 7.8 shows standard deviations and correlation coefficients for seven stocks from different countries. Calculate the variance of a portfolio 40 percent invested in BP, 40 percent invested in KLM, and 20 percent invested in Nestlé.

13. Look back at your calculations for question 12. Calculate each stock's beta relative to the three-stock portfolio.

14. Your eccentric Aunt Claudia has left you $50,000 in Hudson Bay shares plus $50,000 cash. Unfortunately her will requires that the Hudson Bay stock not be sold for one year and the $50,000 cash must be entirely invested in one of the stocks shown in Table 7.8. What is the safest attainable portfolio under these restrictions?

15. There are few, if any, real companies with negative betas. But suppose you found one with $\beta = -.25$.

(a) How would you expect this stock's rate of return to change if the overall market rose by an extra 5 percent? What if the market fell by an extra 5 percent?

(b) You have $1 million invested in a well-diversified portfolio of stocks. Now you receive an additional $20,000 bequest. Which of the following actions will yield the safest overall portfolio return?

(i) Invest $20,000 in Treasury bills (which have $\beta = 0$).
(ii) Invest $20,000 in stocks with $\beta = 1$.
(iii) Invest $20,000 in the stock with $\beta = -.25$.
Explain your answer.

TABLE 7.8

Standard deviations and correlation coefficients for a sample of seven stocks

	CORRELATION COEFFICIENTS							
	BP	DEUTSCHE BANK	HUDSON BAY	KLM	LVMH	NESTLÉ	SONY	STANDARD DEVIATION
BP	1	.31	–.08	.14	.18	.25	.42	16.3%
Deutsche Bank		1	.11	.51	.22	.49	.41	23.2
Hudson Bay			1	–.06	–.16	.03	–.02	26.3
KLM				1	.32	.20	.20	30.1
LVMH					1	.26	.23	25.8
Nestlé						1	.78	18.9
Sony							1	27.5

Note: Correlations and standard deviations are calculated using returns measured in each country's own currency. In other words, they assume that the investor is hedged against exchange risk.

16. The market portfolio has a standard deviation of 20 percent, and the covariance between the returns on the market and those on stock Z is 800.
    (a) What is the beta of stock Z?
    (b) What is the standard deviation of a fully diversified portfolio of such stocks?
    (c) What is the average beta of all stocks?
    (d) If the market portfolio gave an extra return of 5 percent, how much extra return can you expect from stock Z?
17. Diversification has enormous value to investors, yet opportunities for diversification should not sway capital investment decisions by corporations. How would you explain this apparent paradox?

Challenge Questions

1. Here are some historical data on the risk characteristics of Boeing and Polaroid:

	BOEING	POLAROID
β (beta)	.78	1.01
Yearly standard deviation of return (%)	21	26

Assume the standard deviation of the return on the market was 15 percent.
   (a) The correlation coefficient of Boeing's return versus Polaroid's is .37. What is the standard deviation of a portfolio invested half in Boeing and half in Polaroid?
   (b) What is the standard deviation of a portfolio invested one-third in Boeing, one-third in Polaroid, and one-third in Treasury bills?
   (c) What is the standard deviation if the portfolio is split evenly between Boeing and Polaroid and is financed at 50 percent margin, i.e., the investor puts up only 50 percent of the total amount and borrows the balance from the broker?
   (d) What is the *approximate* standard deviation of a portfolio composed of 100 stocks with betas of .78 like Boeing? How about 100 stocks like Polaroid? *Hint:* Part (*d*) should not require anything but the simplest arithmetic to answer.
2. Suppose that Treasury bills offer a return of about 6 percent and the expected market risk premium is 8.5 percent. The standard deviation of Treasury-bill returns is zero and the standard deviation of market returns is 20 percent. Use the formula for portfolio risk to calculate the standard deviation of portfolios with different proportions in Treasury bills and the market. (Note that the covariance of two rates of return must be zero when the standard deviation of one return is zero.) Graph the expected returns and standard deviations.
3. It is often useful to know how well your portfolio is diversified. Two measures have been suggested:
   (a) The variance of the returns on a fully diversified portfolio as a proportion of the variance of returns on *your* portfolio.
   (b) The number of shares in a portfolio that (*i*) have the same risk as yours, (*ii*) are invested in "typical" shares, and (*iii*) have equal amounts invested in each share.

   Suppose that you hold eight stocks. All are fairly typical; they have a standard deviation of 40 percent a year and the correlation between each pair is .3. Of your fund, 20 percent is invested in one stock, 20 percent is invested in a second stock, and the remaining 60 percent is spread evenly over a further six stocks. Calculate each measure of portfolio diversification. What are the advantages and disadvantages of each?

Chapter Eight

RISK AND RETURN

In Chapter 7 we began to come to grips with the problem of measuring risk. Here is the story so far.

The stock market is risky because there is a spread of possible outcomes. The usual measure of this spread is the standard deviation or variance. The risk of any stock can be broken down into two parts. There is the *unique risk* that is peculiar to that stock, and there is the *market risk* that is associated with marketwide variations. Investors can eliminate unique risk by holding a well-diversified portfolio, but they cannot eliminate market risk. *All* the risk of a fully diversified portfolio is market risk.

A stock's contribution to the risk of a fully diversified portfolio depends on its sensitivity to market changes. This sensitivity is generally known as *beta*. A security with a beta of 1.0 has average market risk—a well-diversified portfolio of such securities has the same standard deviation as the market index. A security with a beta of .5 has below-average market risk—a well-diversified portfolio of these securities tends to move half as far as the market moves and has half the market's standard deviation.

In this chapter we build on this newfound knowledge. We present leading theories linking risk and return in a competitive economy, and we show how these theories can be used to estimate the returns required by investors in different stock market investments. We start with the most widely used theory, the capital asset pricing model, which builds directly on the ideas developed in the last chapter. We will also look at another class of models, known as arbitrage pricing or factor models. Then in Chapter 9 we show how these ideas can help the financial manager cope with risk in practical capital budgeting situations.

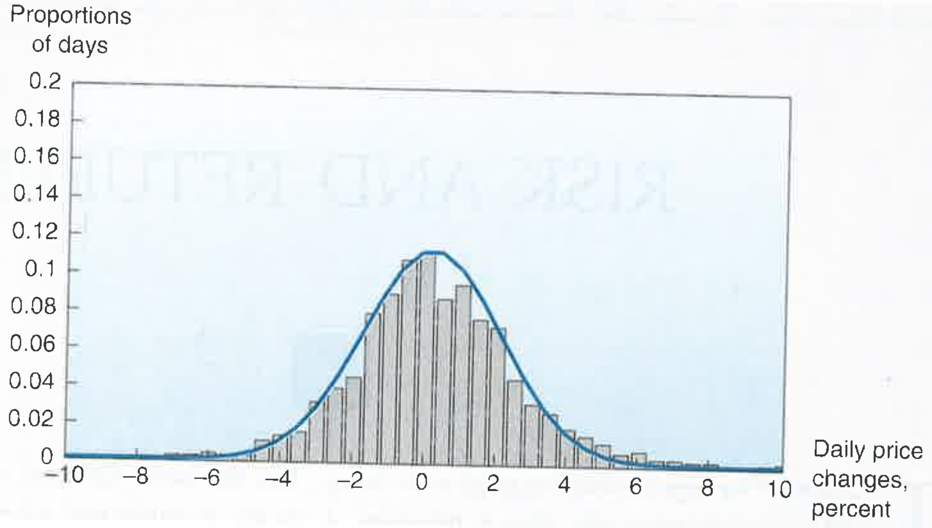
8.1 HARRY MARKOWITZ AND THE BIRTH OF PORTFOLIO THEORY

Most of the ideas in Chapter 7 date back to an article written in 1952 by Harry Markowitz.[1] Markowitz drew attention to the common practice of portfolio diversification and showed exactly how an investor can reduce the standard deviation of portfolio returns by choosing stocks that do not move exactly together. But

[1]H. M. Markowitz, "Portfolio Selection," *Journal of Finance* 7 (March 1952), pp. 77–91.

Figure 8.1

Daily price changes for Microsoft are approximately normally distributed. This plot spans 1986 to 1997.



Markowitz did not stop there; he went on to work out the basic principles of portfolio construction. These principles are the foundation for much of what has been written about the relationship between risk and return.

We begin with Figure 8.1, which shows a histogram of the daily returns on Microsoft stock from 1986 to 1997. On this histogram we have superimposed a bell-shaped normal distribution. The result is typical: When measured over some fairly short interval, the past rates of return on any stock conform closely to a normal distribution.[2]

Normal distributions can be completely defined by two numbers. One is the average or expected return; the other is the variance or standard deviation. Now you can see why in Chapter 7 we discussed the calculation of expected return and standard deviation. They are not just arbitrary measures: If returns are normally distributed, they are the *only* two measures that an investor need consider.

Figure 8.2 pictures the distribution of possible returns from two investments. Both offer an expected return of 10 percent, but A has much the wider spread of possible outcomes. Its standard deviation is 15 percent; the standard deviation of B is 7.5 percent. Most investors dislike uncertainty and would therefore prefer B to A.

Figure 8.3 pictures the distribution of returns from two other investments. This time both have the *same* standard deviation, but the expected return is 20 percent from stock C and only 10 percent from stock D. Most investors like high expected return and would therefore prefer C to D.

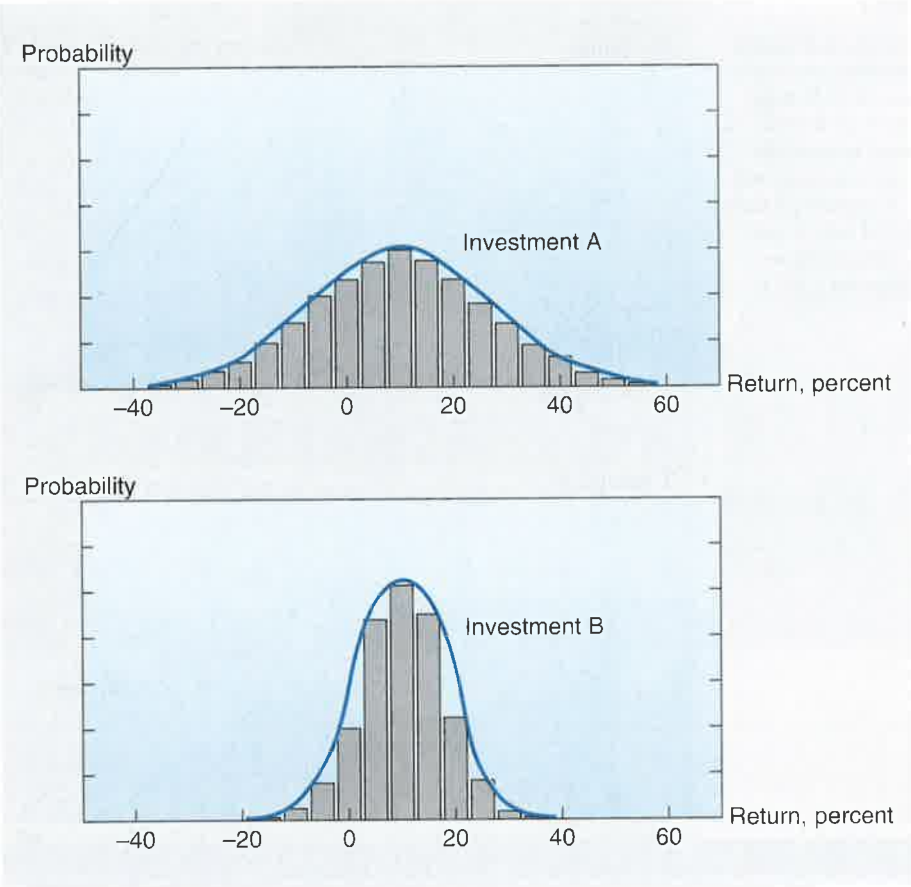
Combining Stocks into Portfolios

Suppose that you are wondering whether to invest in shares of Bristol-Myers Squibb or McDonald's. You decide that McDonald's offers an expected return of 20 percent and Bristol-Myers offers an expected return of 10 percent. After looking

[2]If you were to measure returns over *long* intervals, the distribution would be skewed. For example, you would encounter returns greater than 100 percent but none less than −100 percent. The distribution of returns over periods of, say, one year would be better approximated by a *lognormal* distribution. The lognormal distribution, like the normal, is completely specified by its mean and standard deviation.

Figure 8.2

These two investments both have an *expected* return of 10 percent but because investment A has the greater spread of *possible* returns, it is more risky than B. We can measure this spread by the standard deviation. Investment A has a standard deviation of 15 percent; B, 7.5 percent. Most investors would prefer B to A.



back at the past variability of the two stocks, you also decide that the standard deviation of returns is 17.1 percent for Bristol-Myers and 20.8 percent for McDonald's. McDonald's offers the higher expected return, but it is considerably more risky.

Now there is no reason to restrict yourself to holding only one stock. For example, in Section 7.3 we analyzed what would happen if you invested 55 percent of your money in Bristol-Myers and 45 percent in McDonald's. The expected return on this portfolio is 14.5 percent, which is simply a weighted average of the expected returns on the two holdings. What about the risk of such a portfolio? We know that thanks to diversification the portfolio risk is less than the average of the risks of the separate stocks. In fact, on the basis of past experience the standard deviation of this portfolio is 14.2 percent.[3]

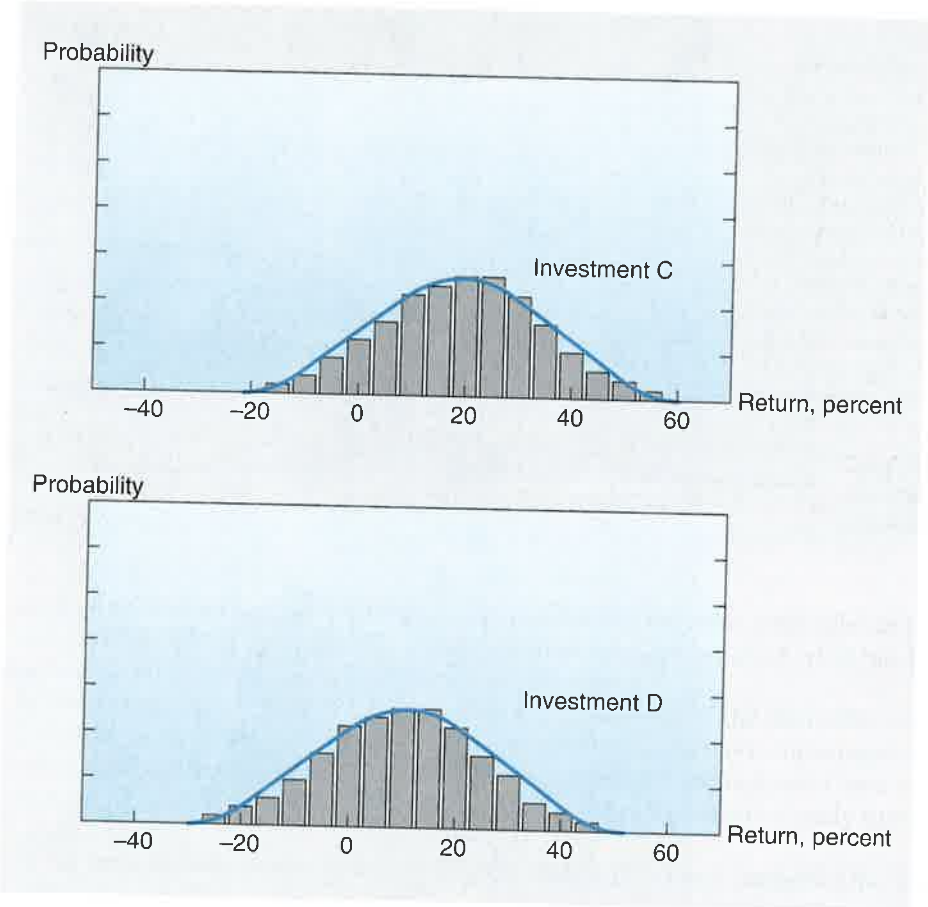
[3]We pointed out in Section 7.3 that the correlation between the returns of Bristol-Myers and McDonald's has been about .15. The variance of a portfolio which is invested 55 percent in Bristol-Myers and 45 percent in McDonald's is

$$\begin{aligned}\text{Variance} &= x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2\\ &= [(.55)^2 \times (17.1)^2] + [(.45)^2 \times (20.8)^2] + 2(.55 \times .45 \times .15 \times 17.1 \times 20.8)\\ &= 202.5\end{aligned}$$

The portfolio standard deviation is $\sqrt{202.5} = 14.2$ percent.

Figure 8.3

The standard deviation of possible returns is 15 percent for both these investments, but the expected return from C is 20 percent compared with an expected return from D of only 10 percent. Most investors would prefer C to D.



In Figure 8.4 we have plotted the expected return and risk that you could achieve by different combinations of the two stocks. Which of these combinations is best? That depends on your stomach. If you want to stake all on getting rich quickly, you will do best to put all your money in McDonald's. If you want a more peaceful life, you should invest most of your money in Bristol-Myers; to minimize risk you should keep a small investment in McDonald's.[4]

In practice, you are not limited to investing in only two stocks. Our next task, therefore, is to find a way to identify the best portfolios of 10, 100, or 1,000 stocks.

We'll start with 10. Suppose that you can choose a portfolio from any of the stocks listed in the first column of Table 8.1. After analyzing the prospects for each firm, you come up with the return forecasts shown in the second column of the table. You use data for the past five years to estimate the risk of each stock (column 3) and the correlation between the returns on each pair of stocks.[5]

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[4]The portfolio with the minimum risk has 38.7 percent in McDonald's. We assume in Figure 8.4 that you may not take negative positions in any stock, i.e., we rule out short sales.

[5]There are 90 correlation coefficients, so we have not listed them in Table 8.1.

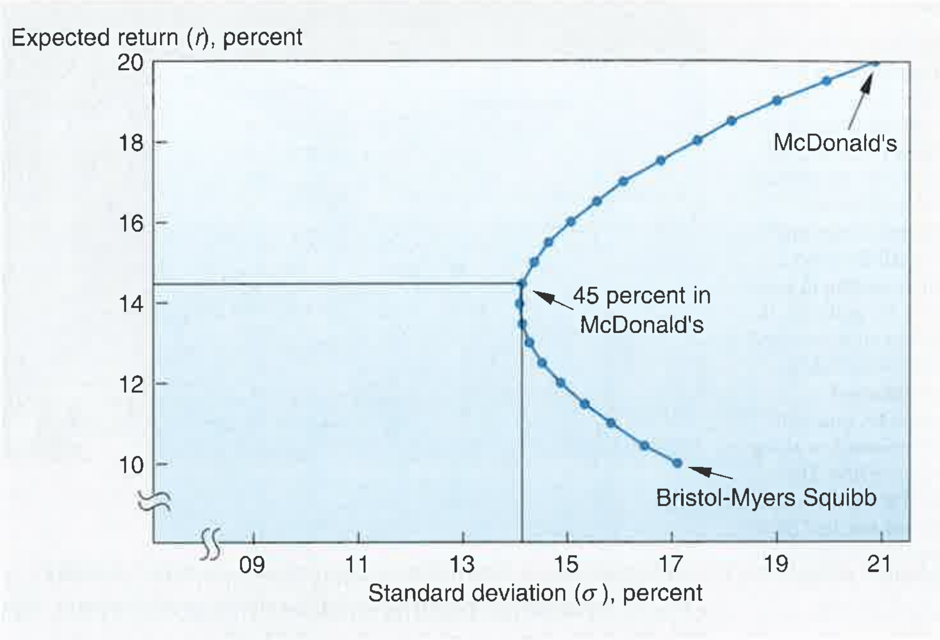


Figure 8.4

The curved line illustrates how expected return and standard deviation change as you hold different combinations of two stocks. For example, if you invest 45 percent of your money in McDonald's and the remainder in Bristol-Myers, your expected return is 14.5 percent, which is 45 percent of the way between the expected returns on the two stocks. The standard deviation is 14.2 percent, which is *less* than 45 percent of the way between the standard deviations on the two stocks. This is because diversification reduces risk.

TABLE 8.1

Examples of efficient portfolios chosen from 10 stocks (all figures are percentages)

	EXPECTED RETURN	STANDARD DEVIATION	EFFICIENT PORTFOLIOS—PERCENTAGES ALLOCATED TO EACH STOCK			
			A	B	C	D
AT&T	13.7	22.6	-	2.0	18.5	18.2
Bristol-Myers Squibb	10.0	17.1	-	-	-	18.7
Coca-Cola	16.6	19.7	-	-	14.7	10.2
Compaq	24.2	42.0	100	10.5	4.5	1.6
Exxon	15.4	13.7	-	-	33.3	40.7
General Electric	17.9	18.8	-	15.5	2.0	-
McDonald's	20.0	20.8	-	34.9	14.8	8.0
Microsoft	21.6	29.4	-	11.6	4.0	-
Reebok	21.5	35.4	-	11.7	4.5	2.5
Xerox	18.6	24.3	-	13.8	3.7	-
Expected portfolio return			24.2	20.2	17.0	14.8
Portfolio standard deviation			42.0	15.5	11.4	10.7

Note: Standard deviations and the correlations between stock returns were estimated from monthly stock returns, May 1993–April 1998. Efficient portfolios are calculated assuming that short sales are prohibited.

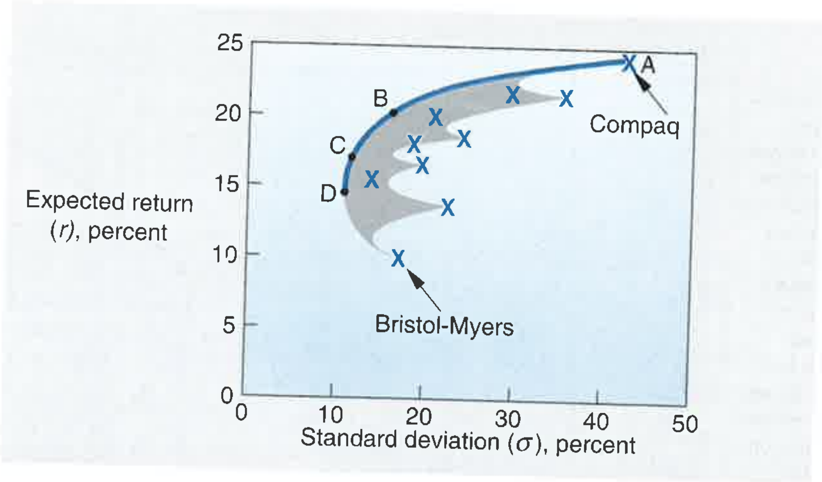


Figure 8.5

Each cross shows the expected return and standard deviation of one of the 10 stocks in Table 8.1. The shaded area shows the possible combinations of expected return and standard deviation from investing in a *mixture* of these stocks. If you like high expected returns and dislike high standard deviations, you will prefer portfolios along the heavy line. These are *efficient* portfolios. We have marked the four efficient portfolios described in Table 8.1 (A, B, C, and D).

Now turn to Figure 8.5. Each cross marks the combination of risk and return offered by a different individual security. For example, Compaq has the highest standard deviation; it also offers the highest expected return. It is represented by the cross at the upper right of Figure 8.5.

By mixing investment in individual securities, you can obtain an even wider selection of risk and return: in fact, *anywhere* in the shaded area in Figure 8.5. But where in the shaded area is best? Well, what is your goal? Which direction do you want to go? The answer should be obvious: You want to go up (to increase expected return) and to the left (to reduce risk). Go as far as you can, and you will end up with one of the portfolios that lies along the heavy solid line. Markowitz called them **efficient portfolios.** These portfolios are clearly better than any in the interior of the shaded area.

We will not calculate this set of efficient portfolios here, but you may be interested in how to do it. Think back to the capital rationing problem in Section 5.5. There we wanted to deploy a limited amount of capital investment in a mixture of projects to give the highest total NPV. Here we want to deploy an investor's funds to give the highest expected return for a given standard deviation. In principle, both problems can be solved by hunting and pecking—but only in principle. To solve the capital rationing problem, we can employ linear programming; to solve the portfolio problem, we would turn to a variant of linear programming known as *quadratic programming.* Given the expected return and standard deviation for each stock, as well as the correlation between each pair of stocks, we could give a computer a standard quadratic program and tell it to calculate the set of efficient portfolios.

Four of these efficient portfolios are marked in Figure 8.5. Their compositions are summarized in Table 8.1. Portfolio A offers the highest expected return; A is entirely invested in one stock, Compaq. Portfolio D promises minimum risk; you can see from Table 8.1 that it is broadly diversified. Notice that D includes a small investment in Compaq and Reebok, but not in GE, Microsoft, or Xerox, although the latter stocks are individually less risky. The reason? On past evidence the fortunes of GE, Microsoft, and Xerox are more highly correlated with those of the other stocks, so they provide less effective diversification.

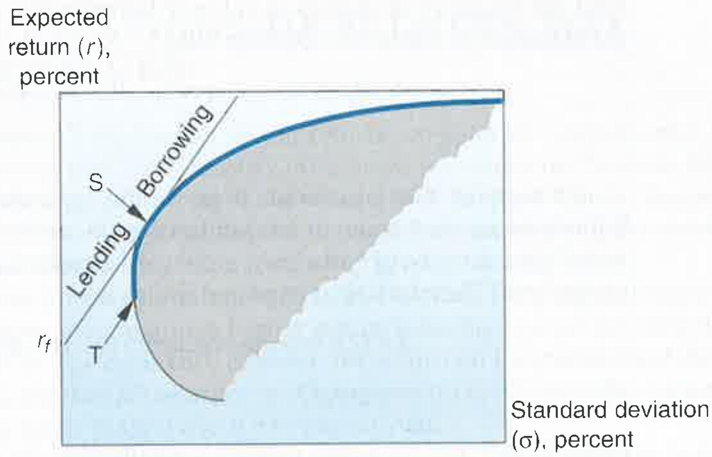


Figure 8.6

Lending and borrowing extend the range of investment possibilities. If you invest in portfolio S and lend or borrow at the risk-free interest rate, r_f, you can achieve any point along the straight line from r_f through S. This gives you a higher expected return for any level of risk than if you just invest in common stocks.

Table 8.1 also shows the compositions of two other efficient portfolios B and C with intermediate levels of risk and expected return.

We Introduce Borrowing and Lending

Of course, large investment funds can choose from thousands of stocks and thereby achieve a wider choice of risk and return. This choice is represented in Figure 8.6 by the shaded, broken-egg-shaped area. The set of efficient portfolios is again marked by the heavy curved line.

Now we introduce yet another possibility. Suppose that you can also lend and borrow money at some risk-free rate of interest r_f. If you invest some of your money in Treasury bills (i.e., lend money) and place the remainder in common stock portfolio S, you can obtain any combination of expected return and risk along the straight line joining r_f and S in Figure 8.6.[6] Since borrowing is merely negative lending, you can extend the range of possibilities to the right of S by borrowing funds at an interest rate of r_f and investing them as well as your own money in portfolio S.

Let us put some numbers on this. Suppose that portfolio S has an expected return of 15 percent and a standard deviation of 16 percent. Treasury bills offer an interest rate (r_f) of 5 percent and are risk-free (i.e., their standard deviation is zero). If you invest half your money in portfolio S and lend the remainder at 5 percent, the expected return on your investment is halfway between the expected return on S and the interest rate on Treasury bills:

$$r = (\tfrac{1}{2} \times \text{expected return on S}) + (\tfrac{1}{2} \times \text{interest rate})$$

$$= 10\%$$

[6]If you want to check this, write down the formula for the standard deviation of a two-stock portfolio:

$$\text{Standard deviation} = \sqrt{x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2}$$

Now see what happens when security 2 is riskless, i.e., when $\sigma_2 = 0$.

And the standard deviation is halfway between the standard deviation of S and the standard deviation of Treasury bills:

$$\sigma = (\tfrac{1}{2} \times \text{standard deviation of S}) + (\tfrac{1}{2} \times \text{standard deviation of bills})$$
$$= 8\%$$

Or suppose that you decide to go for the big time: You borrow at the Treasury bill rate an amount equal to your initial wealth, and you invest everything in portfolio S. You have twice your own money invested in S, but you have to *pay* interest on the loan. Therefore your expected return is

$$r = (2 \times \text{expected return on S}) - (1 \times \text{interest rate})$$
$$= 25\%$$

And the standard deviation of your investment is

$$\sigma = (2 \times \text{standard deviation of S}) - (1 \times \text{standard deviation of bills})$$
$$= 32\%$$

You can see from Figure 8.6 that when you lend a portion of your money, you end up partway between r_f and S; if you can borrow money at the risk-free rate, you can extend your possibilities beyond S. You can also see that regardless of the level of risk you choose, you can get the highest expected return by a mixture of portfolio S and borrowing or lending. S is the *best* efficient portfolio. There is no reason ever to hold, say, portfolio T.

If you have a graph of efficient portfolios, as in Figure 8.6, finding this best efficient portfolio is easy. Start on the vertical axis at r_f and draw the steepest line you can to the curved heavy line of efficient portfolios. That line will be tangent to the heavy line. The efficient portfolio at the tangency point is better than all the others. Notice that it offers the highest *ratio* of risk premium to standard deviation.

This means that we can separate the investor's job into two stages. First, the best portfolio of common stocks must be selected—S in our example.[7] Second, this portfolio must be blended with borrowing or lending to obtain an exposure to risk that suits the particular investor's taste. Each investor, therefore, should put money into just two benchmark investments—a risky portfolio S and a risk-free loan (borrowing or lending).[8]

What does portfolio S look like? If you have better information than your rivals, you will want the portfolio to include relatively large investments in the stocks you think are undervalued. But in a competitive market you are unlikely to have a monopoly of good ideas. In that case there is no reason to hold a different portfolio of common stocks from anybody else. In other words, you might just as well hold the market portfolio. That is why many professional investors invest in a market-index portfolio and why most others hold well-diversified portfolios.

[7] Portfolio S is the point of tangency to the set of efficient portfolios. It offers the highest expected risk premium $(r - r_f)$ per unit of standard deviation (σ).

[8] This *separation theorem* was first pointed out by J. Tobin in "Liquidity Preference as Behavior toward Risk," *Review of Economic Studies* 25 (February 1958), pp. 65–86.

8.2 THE RELATIONSHIP BETWEEN RISK AND RETURN

In Chapter 7 we looked at the returns on selected investments. The least risky investment was U.S. Treasury bills. Since the return on Treasury bills is fixed, it is unaffected by what happens to the market. In other words, Treasury bills have a beta of 0. We also considered a much riskier investment, the market portfolio of common stocks. This has average market risk: Its beta is 1.0.

Wise investors don't take risks just for fun. They are playing with real money. Therefore, they require a higher return from the market portfolio than from Treasury bills. The difference between the return on the market and the interest rate is termed the *market risk premium.* Over a period of 72 years the market risk premium ($r_m - r_f$) has averaged about 9 percent a year.

In Figure 8.7 we have plotted the risk and expected return from Treasury bills and the market portfolio. You can see that Treasury bills have a beta of 0 and a risk premium of 0.[9] The market portfolio has a beta of 1.0 and a risk premium of $r_m - r_f$. This gives us two benchmarks for the expected risk premium. But what is the expected risk premium when beta is not 0 or 1?

In the mid-1960s three economists—William Sharpe, John Lintner, and Jack Treynor—produced an answer to this question.[10] Their answer is known as the **capital asset pricing model,** or **CAPM.** The model's message is both startling and simple. In a competitive market, the expected risk premium varies in direct proportion to beta. This means that in Figure 8.7 all investments must plot along the sloping line, known as the **security market line.** The expected risk premium on an investment with a beta of .5 is, therefore, *half* the expected risk premium on the market; the expected risk premium on an investment with a beta of 2.0 is *twice* the expected risk premium on the market. We can write this relationship as

Expected risk premium on stock = beta × expected risk premium on market

$$r - r_f = \beta(r_m - r_f)$$

Some Estimates of Expected Returns

Before we tell you where the formula comes from, let us use it to figure out what returns investors are looking for from particular stocks. To do this, we need three numbers: β, r_f, and $r_m - r_f$. We gave you estimates of the betas of 10 stocks in Table 7.5. In July 1998 the interest rate on Treasury bills was about 5.5 percent.

How about the market risk premium? As we pointed out in the last chapter, we can't measure $r_m - r_f$ with precision. From past evidence it appears to be 8 to

[9] Remember that the risk premium is the difference between the investment's expected return and the risk-free rate. For Treasury bills, the difference is zero.

[10] W. F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance* 19 (September 1964), pp. 425–442 and J. Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics* 47 (February 1965), pp. 13–37. Treynor's article has not been published.

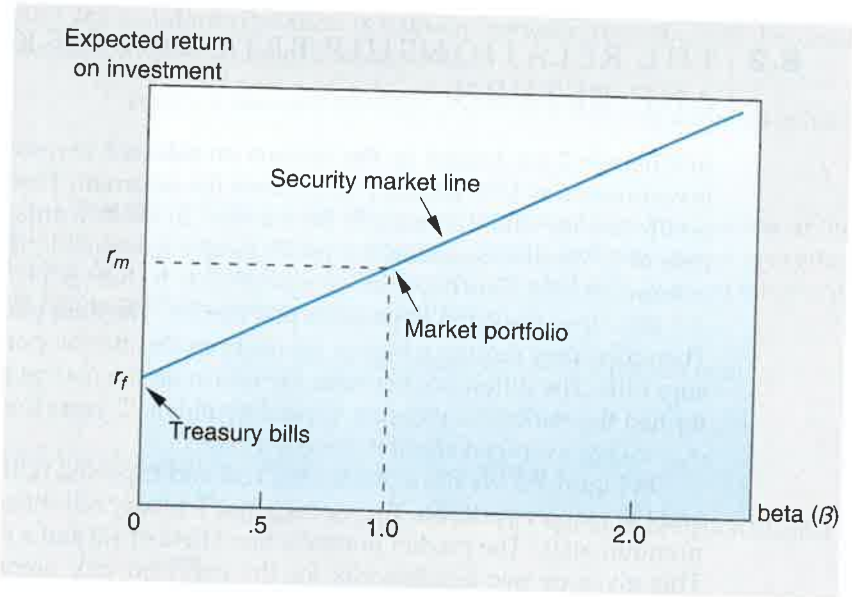


Figure 8.7

The capital asset pricing model states that the expected risk premium on each investment is proportional to its beta. This means that each investment should lie on the sloping security market line connecting Treasury bills and the market portfolio.

9 percent, although many economists and financial managers would forecast a lower figure. Let's use 8 percent in this example.

Table 8.2 puts these numbers together to give an estimate of the expected return on each stock. The stock with the lowest beta in our sample is AT&T. Our estimate of the expected return from AT&T is 10.7 percent. The stock with the highest beta is General Electric. Our estimate of its expected return is 15.8 percent, 10.3 percent more than the interest rate on Treasury bills.

TABLE 8.2

These estimates of the returns *expected* by investors in July 1998 were based on the capital asset pricing model. We assumed 5.5 percent for the interest rate r_f and 8 percent for the expected market premium $r_m - r_f$.

STOCK	BETA (β)	EXPECTED RETURN $r_f + \beta(r_m - r_f)$
AT&T	.65	10.7%
Bristol-Myers Squibb	.95	13.1
Coca-Cola	.98	13.3
Compaq	1.13	14.5
Exxon	.73	11.3
General Electric	1.29	15.8
McDonald's	.95	13.1
Microsoft	1.26	15.6
Reebok	.87	12.5
Xerox	1.05	13.9

You can also use the capital asset pricing model to find the discount rate for a new capital investment. For example, suppose that you are analyzing a proposal by Compaq to expand its capacity. At what rate should you discount the forecast cash flows? According to Table 8.2, investors are looking for a return of 14.5 percent from businesses with the risk of Compaq. So the cost of capital for a further investment in the same business is 14.5 percent.[11]

In practice, choosing a discount rate is seldom so easy. (After all, you can't expect to be paid a fat salary just for plugging numbers into a formula.) For example, you must learn how to adjust for the extra risk caused by company borrowing and how to estimate the discount rate for projects that do not have the same risk as the company's existing business. There are also tax issues. But these refinements can wait until later.[12]

Review of the Capital Asset Pricing Model

Let's review four basic principles of portfolio selection:

1. Investors like high expected return and low standard deviation. Common stock portfolios that offer the highest expected return for a given standard deviation are known as *efficient portfolios.*
2. If the investor can lend or borrow at the risk-free rate of interest, one efficient portfolio is better than all the others: the portfolio that offers the highest ratio of risk premium to standard deviation (that is, portfolio S in Figure 8.6). A risk-averse investor will put part of his money in this efficient portfolio and part in the risk-free asset. A risk-tolerant investor may put all her money in this portfolio or she may borrow and put in even more.
3. The composition of this best efficient portfolio depends on the investor's assessments of expected returns, standard deviations, and correlations. But suppose everybody has the same information and the same assessments. If there is no superior information, each investor should hold the same portfolio as everybody else; in other words, everyone should hold the market portfolio.

Now let's go back to the risk of individual stocks:

4. Don't look at the risk of a stock in isolation but at its contribution to portfolio risk. This contribution depends on the stock's sensitivity to changes in the value of the portfolio.
5. A stock's sensitivity to changes in the value of the *market* portfolio is known as *beta.* Beta, therefore, measures the marginal contribution of a stock to the risk of the market portfolio.

Now if everyone holds the market portfolio, and if beta measures each security's contribution to the market portfolio risk, then it's no surprise that the risk premium demanded by investors is proportional to beta. That's what the CAPM says.

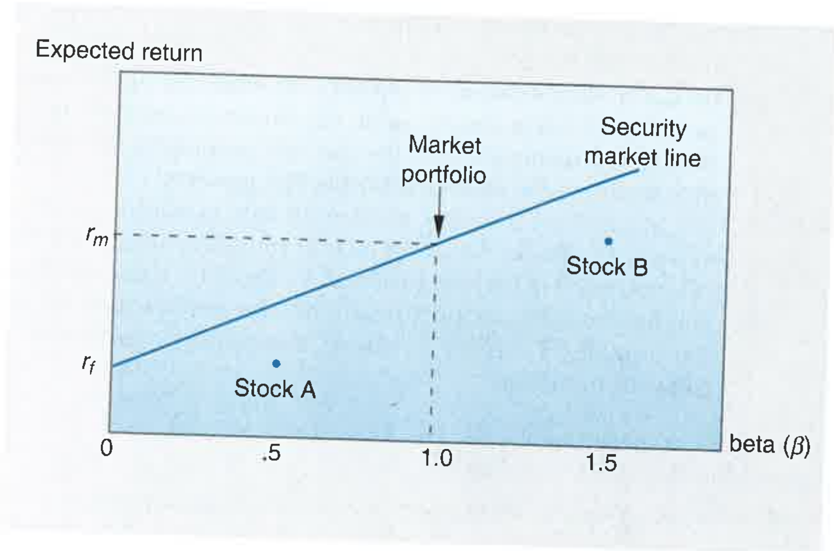
[11]Remember that instead of investing in plant and machinery, the firm could return the money to the shareholders. The opportunity cost of investing is the return that shareholders could expect to earn by buying financial assets. This expected return depends on the market risk of the assets.

[12]Tax issues arise because a corporation must pay tax on income from an investment in Treasury bills or other interest-paying securities. It turns out that the correct discount rate for risk-free investments is the *after-tax* Treasury bill rate. We come back to this point in Chapters 19 and 25.

Various other points on the practical use of betas and the capital asset pricing model are covered in Chapter 9.

Figure 8.8

In equilibrium no stock can lie below the security market line. For example, instead of buying stock A, investors would prefer to lend part of their money and put the balance in the market portfolio. And instead of buying stock B, they would prefer to borrow and invest in the market portfolio.



What if a Stock Did Not *Lie on the Security Market Line?*

Imagine that you encounter stock A in Figure 8.8. Would you buy it? We hope not[13]—if you want an investment with a beta of .5, you could get a higher expected return by investing half your money in Treasury bills and half in the market portfolio. If everybody shares your view of the stock's prospects, the price of A will have to fall until the expected return matches what you could get elsewhere.

What about stock B in Figure 8.8? Would you be tempted by its high return? You wouldn't if you were smart. You could get a higher expected return for the same beta by borrowing 50 cents for every dollar of your own money and investing in the market portfolio. Again, if everybody agrees with your assessment, the price of stock B cannot hold. It will have to fall until the expected return on B is equal to the expected return on the combination of borrowing and investment in the market portfolio.

We have made our point. An investor can always obtain an expected risk premium of $\beta(r_m - r_f)$ by holding a mixture of the market portfolio and a risk-free loan. So in well-functioning markets nobody will hold a stock that offers an expected risk premium of *less* than $\beta(r_m - r_f)$. But what about the other possibility? Are there stocks that offer a higher expected risk premium? In other words, are there any that lie above the security market line in Figure 8.8? If we take all stocks together, we have the market portfolio. Therefore, we know that stocks *on average* lie on the line. Since none lies *below* the line, then there also can't be any that lie *above* the line. Thus each and every stock must lie on the security market line and offer an expected risk premium of

$$r - r_f = \beta(r_m - r_f)$$

[13]Unless, of course, we were trying to sell it.

8.3 VALIDITY AND ROLE OF THE CAPITAL ASSET PRICING MODEL

Any economic model is a simplified statement of reality. We need to simplify in order to interpret what is going on around us. But we also need to know how much faith we can place in our model.

Let us begin with some matters about which there is broad agreement. First, few people quarrel with the idea that investors require some extra return for taking on risk. That is why common stocks have given on average a higher return than U.S. Treasury bills. Who would want to invest in risky common stocks if they offered only the *same* expected return as bills? We wouldn't, and we suspect you wouldn't either.

Second, investors do appear to be concerned principally with those risks that they cannot eliminate by diversification. If this were not so, we should find that stock prices increase whenever two companies merge to spread their risks. And we should find that investment companies which invest in the shares of other firms are more highly valued than the shares they hold. But we don't observe either phenomenon. Mergers undertaken just to spread risk don't increase stock prices, and investment companies are no more highly valued than the stocks they hold.

The capital asset pricing model captures these ideas in a simple way. That is why many financial managers find it the most convenient tool for coming to grips with the slippery notion of risk. And it is why economists often use the capital asset pricing model to demonstrate important ideas in finance even when there are other ways to prove these ideas. But that doesn't mean that the capital asset pricing model is ultimate truth. We will see later that it has several unsatisfactory features, and we will look at some alternative theories. Nobody knows whether one of these alternative theories is eventually going to come out on top or whether there are other, better models of risk and return that have not yet seen the light of day.

Tests of the Capital Asset Pricing Model

Imagine that in 1931 ten investors gathered together in a Wall Street bar to discuss their portfolios. Each agreed to follow a different investment strategy. Investor 1 opted to buy the 10 percent of New York Stock Exchange stocks with the lowest estimated betas; investor 2 chose the 10 percent with the next-lowest betas; and so on, up to investor 10, who agreed to buy the stocks with the highest betas. They also undertook that at the end of every year they would reestimate the betas of all NYSE stocks and reconstitute their portfolios.[14] Finally, they promised that they would return 60 years later to compare results, and so they parted with much cordiality and good wishes.

In 1991 the same 10 investors, now much older and wealthier, met again in the same bar. Figure 8.9 shows how they had fared. Investor 1's portfolio turned out to be much less risky than the market; its beta was only .49. However, investor 1 also realized the lowest return, 9 percent above the risk-free rate of interest. At the

[14]Betas were estimated using returns over the previous 60 months.

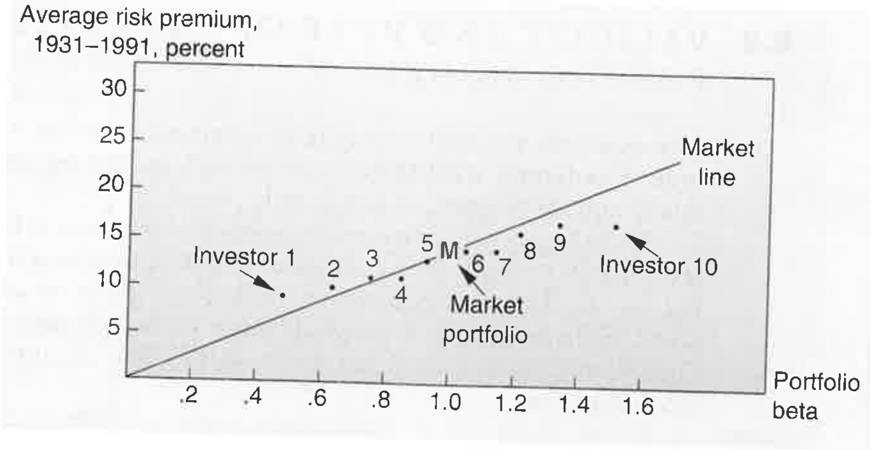


Figure 8.9

The capital asset pricing model states that the expected risk premium from any investment should lie on the market line. The dots show the actual average risk premiums from portfolios with different betas. The high-beta portfolios generated higher average returns, just as predicted by the CAPM. But the high-beta portfolios plotted below the market line, and four of the five low-beta portfolios plotted above. A line fitted to the 10 portfolio returns would be "flatter" than the market line.

Source: F. Black, "Beta and Return," *Journal of Portfolio Management* 20 (Fall 1993), pp. 8–18.

other extreme, the beta of investor 10's portfolio was 1.52, about three times that of investor 1's portfolio. But investor 10 was rewarded with the highest return, averaging 17 percent a year above the interest rate. So over this 60-year period returns did indeed increase with beta.

As you can see from Figure 8.9, the market portfolio over the same 60-year period provided an average return of 14 percent above the interest rate[15] and (of course) had a beta of 1.0. The CAPM predicts that the risk premium should increase in proportion to beta, so that the returns of each portfolio should lie on the upward-sloping security market line in Figure 8.9. Since the market provided a risk premium of 14 percent, investor 1's portfolio, with a beta of .49, should have provided a risk premium of a shade under 7 percent and investor 10's portfolio, with a beta of 1.52, should have given a premium of a shade over 21 percent. You can see that, while high-beta stocks performed better than low-beta stocks, the difference was not as great as the CAPM predicts.

Figure 8.9 provides broad support for the CAPM, though it suggests that the line relating return to beta has been "too flat." But the model has come under fire on two fronts. First, the slope of the line has been particularly flat in recent years. For example, Figure 8.10 shows how our 10 investors fared between 1966 and 1991. Now it's less clear who is buying the drinks: The portfolios of investors 1 and 10 had very different betas but both earned the same average return over these 25 years. Of course, the line was correspondingly steeper before 1966. This is also shown in Figure 8.10.

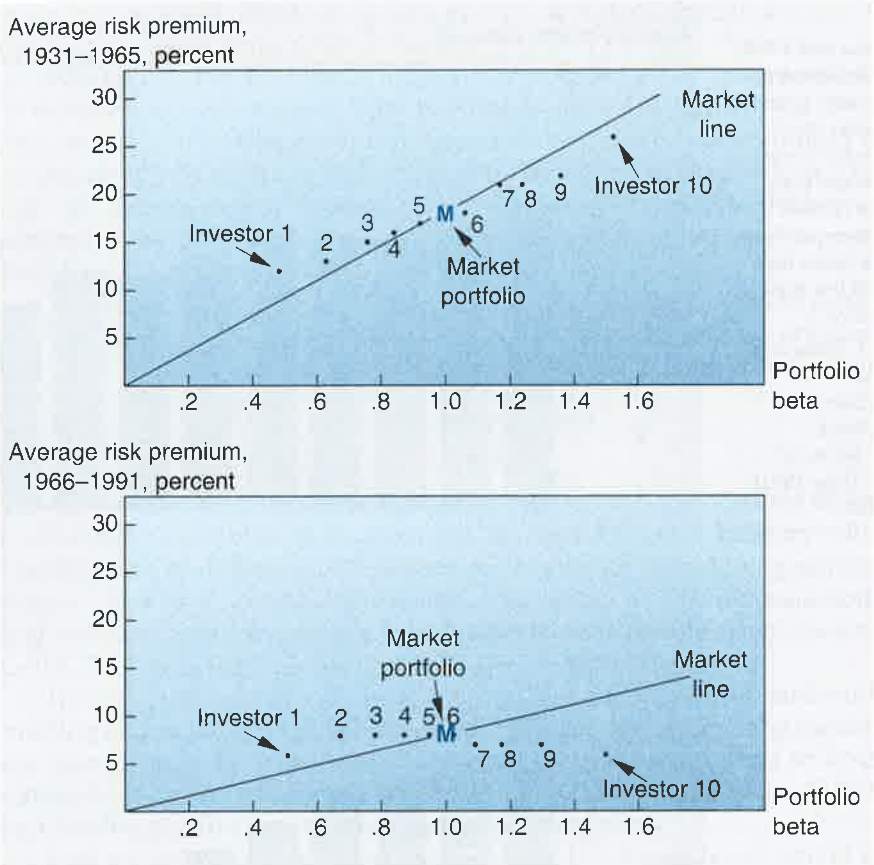
Second, critics of the CAPM point out that while return has not risen with beta in recent years, it has been related to other measures. For example, Figure 8.11 shows that from 1963 to 1990 small-company stocks performed substantially bet-

[15]In Figure 8.9 the stocks in the "market portfolio" are weighted equally. Since the stocks of small firms have provided higher average returns than those of large firms, the risk premium on an equally weighted index is higher than on a value-weighted index. This is one reason for the difference between the 14 percent market risk premium in Figure 8.9 and the 8.4 percent premium reported in Table 7.1.

Figure 8.10

The relationship between beta and actual average return has been much weaker since the mid-1960s. Compare Figure 8.9.

Source: F. Black, "Beta and Return," *Journal of Portfolio Management* 20 (Fall 1993), pp. 8–18.



ter than large-company stocks[16] and that stocks with high ratios of book to market value performed much better than stocks with a low ratio of book to market.[17] Apparently, stocks of small companies and companies with high book-to-market ratios were exposed to risks not captured in the CAPM; this could account for their higher returns.

But the CAPM predicts that beta is the *only* reason that expected returns differ. If investors *expected* the returns to depend on firm size or book-to-market ratio, then the simple version of the CAPM cannot be the whole truth. Such findings have prompted headlines like "Is Beta Dead?" in the business press.[18]

What's going on here? It is hard to say. Defenders of the capital asset pricing model emphasize that it is concerned with *expected* returns, whereas we can observe only *actual* returns. Actual stock returns reflect expectations, but they also

[16]We pointed out in Section 7.1 that since the mid-1960s the stocks of small firms have provided higher average returns than those of large firms.

[17]Small-firm stocks have higher betas, but the difference in betas is not sufficient to explain the difference in returns. There is no simple relationship between book-to-market ratios and beta.

[18]A. Wallace, "Is Beta Dead?" *Institutional Investor* 14 (July 1980), pp. 22–30. Similar obituaries have been circulating for many years. Perhaps this goes to the CAPM's credit: Only a strong theory can survive several funerals.

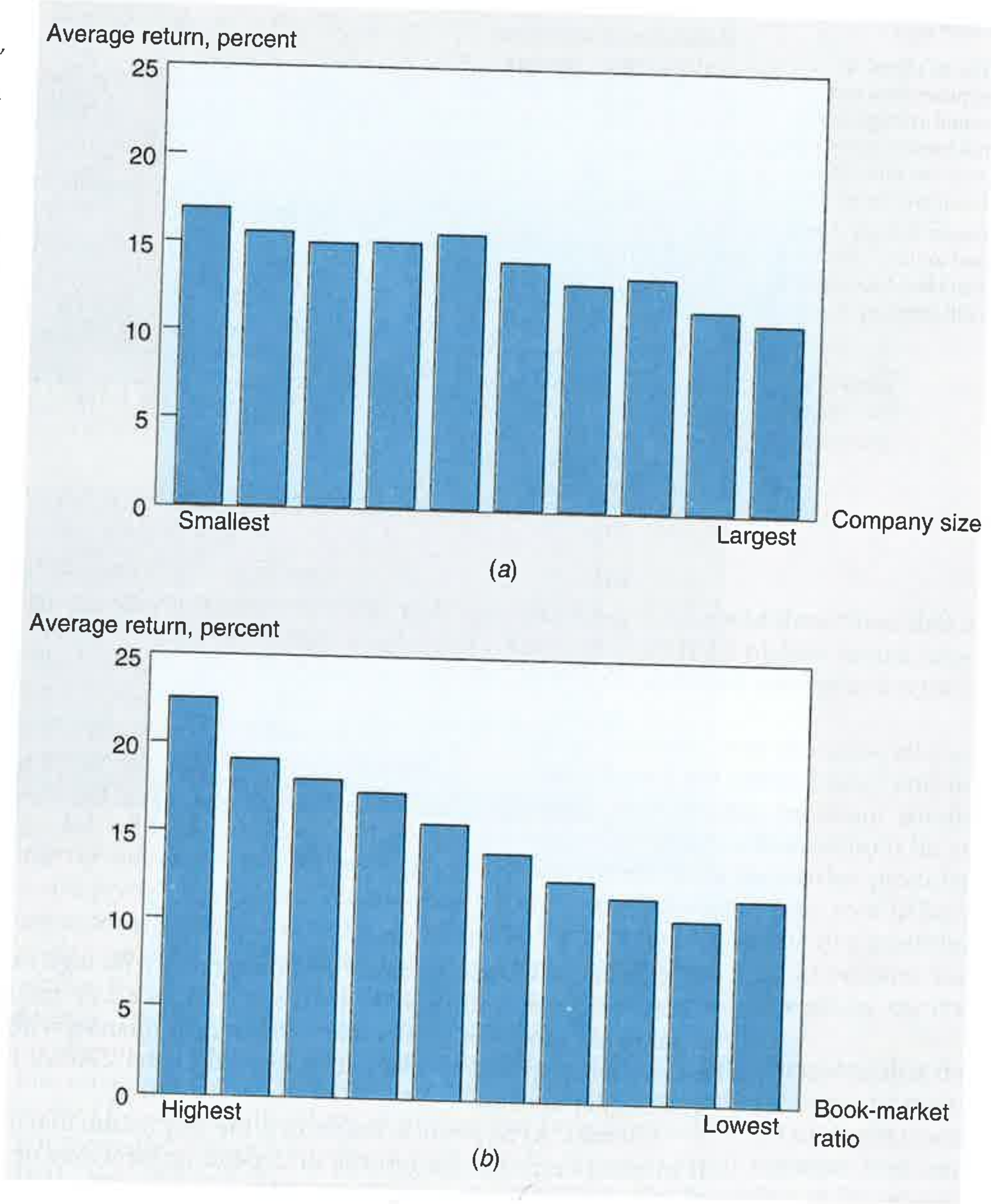


Figure 8.11

(a) Since the mid-1960s, stocks of small companies have done systematically better than stocks of large companies. (b) Stocks with high ratios of book value to price per share have done better than stocks with low book-to-price ratios.

Source: E. F. Fama and K. R. French, "The Cross-Section of Expected Stock Returns," *Journal of Finance* 47 (June 1992), pp. 427–465.

embody lots of "noise"—the steady flow of surprises that conceal whether on average investors have received the returns that they expected. This noise may make it impossible to judge whether the model holds better in one period than another.[19] Perhaps the best that we can do is to focus on the longest period for which there is reasonable data. This would take us back to Figure 8.9, which sug-

[19] A second problem with testing the model is that the market portfolio should contain all risky investments, including stocks, bonds, commodities, real estate—even human capital. Most market indexes contain only a sample of common stocks. See, for example, R. Roll, "A Critique of the Asset Pricing Theory's Tests; Part 1: On Past and Potential Testability of the Theory," *Journal of Financial Economics* 4 (March 1977), pp. 129–176.

gests that expected returns do indeed increase with beta, though less rapidly than the simple version of the CAPM predicts.[20]

What about the anomalous relationship between stock returns and firm size or the book-to-market ratio? Both have been well documented, yet if you look long and hard at past stock returns, you are bound to find some strategy that just by chance would have worked in the past. This practice is known as "data mining" or "data snooping." Maybe the size and book-to-market effects are simply chance results, the effect of data snooping. If so, they should vanish now that they have been discovered.[21]

One thing is for sure: It will be very hard to reject the CAPM beyond all reasonable doubt. Data and statistics will probably not give final answers soon, so the plausibility of the CAPM *theory* will have to be weighed along with the "facts."

Assumptions behind the Capital Asset Pricing Model

The capital asset pricing model rests on several assumptions that we did not fully spell out. For example, we assumed that investment in U.S. Treasury bills is risk-free. It is true that there is little chance of default, but they don't guarantee a *real* return. There is still some uncertainty about inflation. Another assumption was that investors can *borrow* money at the same rate of interest at which they can lend. Generally borrowing rates are higher than lending rates.

It turns out that many of these assumptions are not crucial, and with a little pushing and pulling it is possible to modify the capital asset pricing model to handle them. The really important idea is that investors are content to invest their money in a limited number of benchmark portfolios. (In the basic CAPM these benchmarks are Treasury bills and the market portfolio.)

In these modified CAPMs expected return still depends on market risk, but the definition of market risk depends on the nature of the benchmark portfolios.[22] In practice, none of these alternative capital asset pricing models is as widely used as the standard version.

8.4 SOME ALTERNATIVE THEORIES

Consumption Betas versus Market Betas

The capital asset pricing model pictures investors as solely concerned with the level and uncertainty of their future wealth. But for most people wealth is not an end in itself. What good is wealth if you can't spend it? People invest now to provide

[20]We say "simple version" because Fischer Black has shown that if there are borrowing restrictions, there should still exist a positive relationship between expected return and beta, but the security market line would be less steep as a result. See F. Black, "Capital Market Equilibrium with Restricted Borrowing," *Journal of Business* 45 (July 1972), pp. 444–455.

[21]For example, there is some evidence that the size effect has become less important since it was first discovered by Rolf Banz in 1981. See R. Banz, "The Relationship between Return and Market Values of Common Stock," *Journal of Financial Economics* 9 (March 1981), pp. 3–18.

[22]For example, see M. C. Jensen (ed.), *Studies in the Theory of Capital Markets*, Frederick A. Praeger, Inc., New York, 1972. In the introduction Jensen provides a very useful summary of some of these variations on the capital asset pricing model.

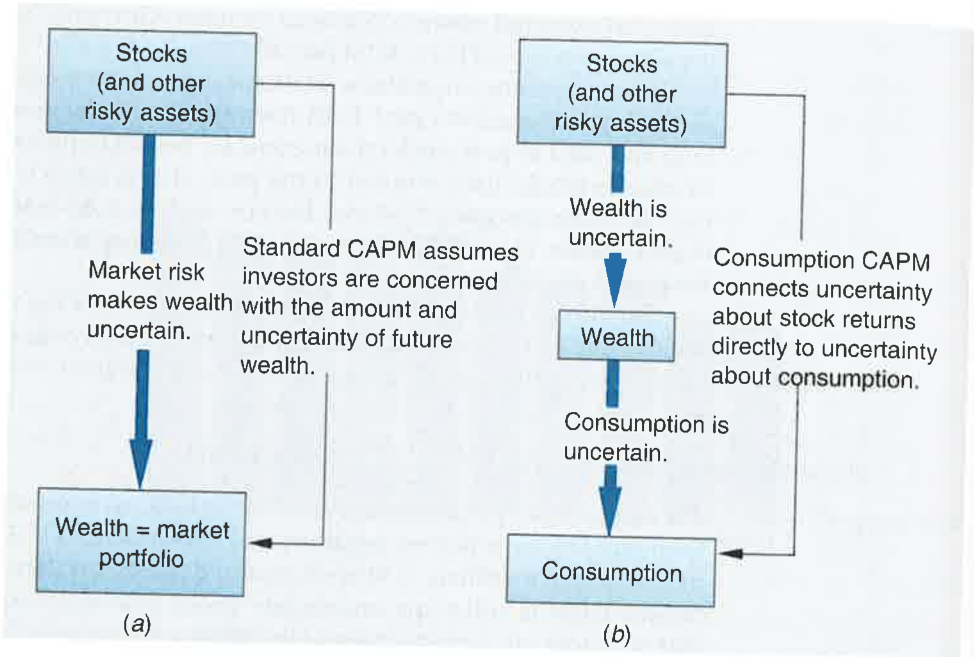


Figure 8.12

(a) The standard CAPM concentrates on how stocks contribute to the level and uncertainty of investor's wealth. Consumption is outside the model. (b) The consumption CAPM defines risk as a stock's contribution to uncertainty about consumption. Wealth (the intermediate step between stock returns and consumption) drops out of the model.

future consumption for themselves or for their families and heirs. The most important risks are those which might force a cutback of future consumption.

Douglas Breeden has developed a model in which a security's risk is measured by its sensitivity to changes in investors' consumption. If he is right, a stock's expected return should move in line with its *consumption beta* rather than its market beta. Figure 8.12 summarizes the chief differences between the standard and consumption CAPMs. In the standard model investors are concerned exclusively with the amount and uncertainty of their future wealth. Each investor's wealth ends up perfectly correlated with the return on the market portfolio; the demand for stocks and other risky assets is thus determined by their market risk. The deeper motive for investing—to provide for consumption—is outside the model.

In the consumption CAPM, uncertainty about stock returns is connected directly to uncertainty about consumption. Of course, consumption depends on wealth (portfolio value), but wealth does not appear explicitly in the model.

The consumption CAPM has several appealing features. For example, you don't have to identify the market or any other benchmark portfolio. You don't have to worry that Standard and Poor's Composite Index doesn't track returns on bonds, commodities, and real estate.

However, you do have to be able to measure consumption. *Quick:* How much did you consume last month? It's easy to count the hamburgers and movie tickets, but what about the depreciation on your car or washing machine or the daily cost of your homeowner's insurance policy? We suspect that your estimate of total consumption will rest on rough or arbitrary allocations and assumptions. And if it's hard for you to put a dollar value on your total consumption, think of the task facing a government statistician asked to estimate month-by-month consumption for all of us.

Compared to stock prices, estimated aggregate consumption changes smoothly and gradually over time. Changes in consumption often seem to be out of phase with the stock market. Individual stocks seem to have low or erratic consumption betas. Moreover, the volatility of consumption appears too low to explain the past average rates of return on common stocks unless one assumes unreasonably high investor risk aversion.[23] These problems may reflect our poor measures of consumption or perhaps poor models of how individuals distribute consumption over time. It seems too early for the consumption CAPM to see practical use.

Arbitrage Pricing Theory

The capital asset pricing theory begins with an analysis of how investors construct efficient portfolios. Stephen Ross's **arbitrage pricing theory,** or **APT,** comes from a different family entirely. It does not ask which portfolios are efficient. Instead, it starts by *assuming* that each stock's return depends partly on pervasive macroeconomic influences or "factors" and partly on "noise"—events that are unique to that company. Moreover, the return is assumed to obey the following simple relationship:

$$\text{Return} = a + b_1(r_{\text{factor 1}}) + b_2(r_{\text{factor 2}}) + b_3(r_{\text{factor 3}}) + \cdots + \text{noise}$$

The theory doesn't say what the factors are: There could be an oil price factor, an interest-rate factor, and so on. The return on the market portfolio *might* serve as one factor, but then again it might not.

Some stocks will be more sensitive to a particular factor than other stocks. Exxon would be more sensitive to an oil factor than, say, Coca-Cola. If factor 1 picks up unexpected changes in oil prices, b_1 will be higher for Exxon.

For any individual stock there are two sources of risk. First is the risk that stems from the pervasive macroeconomic factors which cannot be eliminated by diversification. Second is the risk arising from possible events that are unique to the company. Diversification *does* eliminate unique risk, and diversified investors can therefore ignore it when deciding whether to buy or sell a stock. The expected risk premium on a stock is affected by factor or macroeconomic risk; it is *not* affected by unique risk.

Arbitrage pricing theory states that the expected risk premium on a stock should depend on the expected risk premium associated with each factor and the stock's sensitivity to each of the factors (b_1, b_2, b_3, etc.). Thus the formula is[24]

$$\begin{aligned}\text{Expected risk premium on investment} &= r - r_f \\ &= b_1(r_{\text{factor 1}} - r_f) + b_2(r_{\text{factor 2}} - r_f) + \cdots\end{aligned}$$

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[23]See R. Mehra and E. C. Prescott, "The Equity Risk Premium: A Puzzle," *Journal of Monetary Economics* 15 (1985), pp. 145–161.

[24]There may be some macroeconomic factors that investors are simply not worried about. For example, some macroeconomists believe that money supply doesn't matter and therefore investors are not worried about inflation. Such factors would not command a risk premium. They would drop out of the APT formula for expected return.

Notice that this formula makes two statements:

1. If you plug in a value of zero for each of the *b*'s in the formula, the expected risk premium is zero. A diversified portfolio that is constructed to have zero sensitivity to each macroeconomic factor is essentially risk-free and therefore must be priced to offer the risk-free rate of interest. If the portfolio offered a higher return, investors could make a risk-free (or "arbitrage") profit by borrowing to buy the portfolio. If it offered a lower return, you could make an arbitrage profit by running the strategy in reverse; in other words, you would *sell* the diversified zero-sensitivity portfolio and *invest* the proceeds in U.S. Treasury bills.
2. A diversified portfolio that is constructed to have exposure to, say, factor 1, will offer a risk premium, which will vary in direct proportion to the portfolio's sensitivity to that factor. For example, imagine that you construct two portfolios, A and B, which are affected only by factor 1. If portfolio A is twice as sensitive to factor 1 as portfolio B, portfolio A must offer twice the risk premium. Therefore, if you divided your money equally between U.S. Treasury bills and portfolio A, your combined portfolio would have exactly the same sensitivity to factor 1 as portfolio B and would offer the same risk premium.

   Suppose that the arbitrage pricing formula did *not* hold. For example, suppose that the combination of Treasury bills and portfolio A offered a higher return. In that case investors could make an arbitrage profit by selling portfolio B and investing the proceeds in the mixture of bills and portfolio A.

The arbitrage that we have described applies to well-diversified portfolios, where the unique risk has been diversified away. But if the arbitrage pricing relationship holds for all diversified portfolios, it must generally hold for the individual stocks. Each stock must offer an expected return commensurate with its contribution to portfolio risk. In the APT, this contribution depends on the sensitivity of the stock's return to unexpected changes in the macroeconomic factors.

A Comparison of the Capital Asset Pricing Model and Arbitrage Pricing Theory

Like the capital asset pricing model, arbitrage pricing theory stresses that expected return depends on the risk stemming from economywide influences and is not affected by unique risk. You can think of the factors in arbitrage pricing as representing special portfolios of stocks that tend to be subject to a common influence. If the expected risk premium on each of these portfolios is proportional to the portfolio's market beta, then the arbitrage pricing theory and the capital asset pricing model will give the same answer. In any other case they won't.

How do the two theories stack up? Arbitrage pricing has some attractive features. For example, the market portfolio that plays such a central role in the capital asset pricing model does not feature in arbitrage pricing theory.[25] So we don't have to worry about the problem of measuring the market portfolio, and in principle we can test the arbitrage pricing theory even if we have data on only a sample of risky assets.

[25] Of course, the market portfolio *may* turn out to be one of the factors, but that is not a necessary implication of arbitrage pricing theory.

Unfortunately you win some and lose some. Arbitrage pricing theory doesn't tell us what the underlying factors are—unlike the capital asset pricing model, which collapses *all* macroeconomic risks into a well-defined *single* factor, the return on the market portfolio.

APT Example

Arbitrage pricing theory will provide a good handle on expected returns only if we can (1) identify a reasonably short list of macroeconomic factors,[26] (2) measure the expected risk premium on each of these factors, and (3) measure the sensitivity of each stock to these factors. Let us look briefly at how Elton, Gruber, and Mei tackled each of these issues and estimated the cost of equity for a group of nine New York utilities.[27]

Step 1: Identify the Macroeconomic Factors Although APT doesn't tell us what the underlying economic factors are, Elton, Gruber, and Mei identified five principal factors that could affect either the cash flows themselves or the rate at which they are discounted. These factors are

FACTOR	MEASURED BY
Yield spread	Return on long government bond *less* return on 30-day Treasury bills
Interest rate	Change in Treasury bill return
Exchange rate	Change in value of dollar relative to basket of currencies
Real GNP	Change in forecasts of real GNP
Inflation	Change in forecasts of inflation

To capture any remaining pervasive influences, Elton, Gruber, and Mei also included a sixth factor, the portion of the market return that could not be explained by the first five.

Step 2: Estimate the Risk Premium for Each Factor Some stocks are more exposed than others to a particular factor. So we can estimate the sensitivity of a sample of stocks to each factor and then measure how much extra return investors would have received in the past for taking on factor risk. The results are shown in Table 8.3.

For example, stocks with positive sensitivity to real GNP tended to have higher returns when real GNP increased. A stock with an average sensitivity gave

[26]Some researchers have argued that there are four or five principal pervasive influences on stock prices, but others are not so sure. They point out that the more stocks you look at, the more factors you need to take into account. See, for example, P. J. Dhrymes, I. Friend, and N. B. Gultekin, "A Critical Reexamination of the Empirical Evidence on the Arbitrage Pricing Theory," *Journal of Finance* 39 (June 1984), pp. 323–346.

[27]See E. J. Elton, M. J. Gruber, and J. Mei, "Cost of Capital Using Arbitrage Pricing Theory: A Case Study of Nine New York Utilities," *Financial Markets, Institutions, and Instruments* 3 (August 1994), pp. 46–73. The study was prepared for the New York State Public Utility Commission. We described a parallel study in Chapter 4 which used the discounted-cash-flow model to estimate the cost of equity capital for the same group of firms.

TABLE 8.3

Estimated risk premiums for taking on factor risks, 1978–1990

FACTOR	ESTIMATED RISK PREMIUM $(r_{\text{factor}} - r_f)$*
Yield spread	5.10%
Interest rate	−.61
Exchange rate	−.59
Real GNP	.49
Inflation	−.83
Market	6.36

*The risk premiums have been scaled to represent the annual premiums for the average industrial stock in the Elton–Gruber–Mei sample.

Source: E. J. Elton, M. J. Gruber, and J. Mei, "Cost of Capital Using Arbitrage Pricing Theory: A Case Study of Nine New York Utilities," *Financial Markets, Institutions, and Instruments* 3 (August 1994), pp. 46–73.

investors an additional return of .49 percent a year compared with a stock that was completely unaffected by changes in real GNP. In other words, investors appeared to dislike "cyclical" stocks, whose returns were sensitive to economic activity, and demanded a higher return from these stocks.

By contrast, Table 8.3 shows that a stock with average exposure to *inflation* gave investors .83 percent a year *less* return than a stock with no exposure to inflation. Thus investors seemed to prefer stocks that protected them against inflation (stocks that did well when inflation accelerated), and they were willing to accept a lower expected return from such stocks.

Step 3: Estimate the Factor Sensitivities The estimates of the premiums for taking on factor risk can now be used to estimate the cost of equity for the group of New York State utilities. Remember, APT states that the risk premium for any asset depends on its sensitivities to factor risks (b) and the expected risk premium for each factor ($r_{\text{factor}} - r_f$). In this case there are six factors, so

$$r - r_f = b_1(r_{\text{factor 1}} - r_f) + b_2(r_{\text{factor 2}} - r_f) + \ldots + b_6(r_{\text{factor 6}} - r_f)$$

The first column of Table 8.4 shows the factor risks for the portfolio of utilities, and the second column shows the required risk premium for each factor (taken from Table 8.3). The third column is simply the product of these two numbers. It shows how much return investors demanded for taking on each factor risk. To find the expected risk premium, just add the figures in the final column:

$$\text{Expected risk premium} = r - r_f = 8.53\%$$

TABLE 8.4

Using APT to estimate the expected risk premium for a portfolio of nine New York State utility stocks

FACTOR	FACTOR RISK (b)	EXPECTED RISK PREMIUM ($r_{factor} - r_f$)*	× FACTOR RISK PREMIUM [$b(r_{factor} - r_f)$]
Yield spread	1.04	5.10%	5.30%
Interest rate	–2.25	–.61	1.37
Exchange rate	.70	–.59	–.41
GNP	.17	.49	.08
Inflation	–.18	–.83	.15
Market	.32	6.36	2.04
Total			8.53%

*Risk premiums have been restated as approximate annual rates.

Source: E. J. Elton, M. J. Gruber, and J. Mei, "Cost of Capital Using Arbitrage Pricing Theory: A Case Study of Nine New York Utilities," *Financial Markets, Institutions, and Instruments* 3 (August 1994), tables 3 and 4.

The one-year Treasury bill rate in December 1990, the end of the Elton–Gruber–Mei sample period, was about 7 percent, so the APT estimate of the expected return on New York State utility stocks was[28]

$$\begin{aligned}\text{Expected return} &= \text{risk-free interest rate} + \text{expected risk premium}\\ &= 7 + 8.53\\ &= 15.53\text{, or about } 15.5\%\end{aligned}$$

The Three-Factor Model

We noted earlier the research by Fama and French showing that stocks of small firms and those with a high book-to-market ratio have provided above-average returns. This could simply be a coincidence. But there is also evidence that these factors are related to company profitability and therefore may be picking up risk factors that are left out of the simple CAPM.[29]

[28]This estimate rests on risk premiums actually earned from 1978 to 1990, an unusually rewarding period for common stock investors. Estimates based on long-run market risk premiums would be lower. See E. J. Elton, M. J. Gruber, and J. Mei, "Cost of Capital Using Arbitrage Pricing Theory: A Case Study of Nine New York Utilities," *Financial Markets, Institutions, and Instruments* 3 (August 1994), pp. 46–73.

[29]E. F. Fama and K. R. French, "Size and Book-to-Market Factors in Earnings and Returns," *Journal of Finance* 50 (1995), pp. 131–155.

If investors do demand an extra return for taking on exposure to these factors, then we have a measure of the expected return that looks very much like arbitrage pricing theory:

$$r - r_f = b_{\text{market}}\,(r_{\text{market factor}}) + b_{\text{size}}\,(r_{\text{size factor}}) + b_{\text{book-to-market}}\,(r_{\text{book-to-market factor}})$$

This is commonly known as the Fama–French three-factor model. Using it to estimate expected returns is exactly the same as applying the arbitrage pricing theory. Here's an example.[30]

Step 1: Identify the Factors Fama and French have already identified the three factors that appear to determine expected returns. The returns on each of these factors are

FACTOR	MEASURED BY
Market factor	Return on market index *minus* risk-free interest rate
Size factor	Return on small firm stocks *less* return on large firm stocks
Book-to-market factor	Return on high book-to-market-ratio stocks *less* return on low book-to-market-ratio stocks

Step 2: Estimate the Risk Premium for Each Factor Here we need to rely on history. Fama and French find that between 1963 and 1994 the return on the market factor averaged about 5.2 percent per year, the difference between the return on small and large capitalization stocks was about 3.2 percent a year, while the difference between the annual return on stocks with high and low book-to-market ratios averaged 5.4 percent.[31]

Step 3: Estimate the Factor Sensitivities Some stocks are more sensitive than others to fluctuations in the returns on the three factors. Look, for example, at the first three columns of numbers in Table 8.5, which show some estimates by Fama and French of factor sensitivities for different industry groups. You can see, for example, that an increase of 1 percent in the return on the book-to-market factor *reduces* the return on computer stocks by .49 percent but *increases* the return on utility stocks by .38 percent.[32]

Once you have an estimate of the factor sensitivities, it is a simple matter to multiply each of them by the expected factor return and add up the results. For example, the fourth column of numbers shows that the expected risk premium on

[30]The example is taken from E. F. Fama and K. R. French, "Industry Costs of Equity," *Journal of Financial Economics* 43 (1997), pp. 153–193. Fama and French emphasize the imprecision involved in using either the CAPM or an APT-style model to estimate the returns that investors expect.

[31]Over the longer period 1929–1997, the average annual difference between the returns on small and large capitalization stocks was 2.4 percent a year. The difference between the return on stocks with high and low book-to-market ratios was 5.5 percent. See J. Davis, E. F. Fama, and K. R. French, "Characteristics, Covariances, and Average Returns: 1929–1997," Working paper, Center for Research in Security Prices, University of Chicago, April 1998.

[32]A 1 percent return on the book-to-market factor means that stocks with a high book-to-market ratio provide a 1 percent higher return than those with a low ratio.

TABLE 8.5

Estimates of industry risk premiums using the Fama–French three-factor model and the CAPM

	THREE-FACTOR MODEL				CAPM
	FACTOR SENSITIVITIES				
	b_{market}	b_{size}	$b_{book\text{-}to\text{-}market}$	EXPECTED RISK PREMIUM*	EXPECTED RISK PREMIUM (%)
Aircraft	1.15	.51	.00	7.54%	6.43%
Banks	1.13	.13	.35	8.08	5.55
Chemicals	1.13	–.03	.17	6.58	5.57
Computers	.90	.17	–.49	2.49	5.29
Construction	1.21	.21	–.09	6.42	6.52
Food	.88	–.07	–.03	4.09	4.44
Petroleum & gas	.96	–.35	.21	4.93	4.32
Pharmaceuticals	.84	–.25	–.63	.09	4.71
Tobacco	.86	–.04	.24	5.56	4.08
Utilities	.79	–.20	.38	5.41	3.39

*The expected risk premium equals the factor sensitivities multiplied by the factor risk premiums, that is, $(b_{market} \times 5.2) + (b_{size} \times 3.2) + (b_{book\text{-}to\text{-}market} \times 5.4)$.

Source: E. F. Fama and K. R. French, "Industry Costs of Equity," *Journal of Financial Economics* 43 (1997), pp. 153–193.

computer stocks is $r - r_f = (.90 \times 5.2) + (.17 \times 3.2) - (.49 \times 5.4) = 2.49$ percent. Compare this figure with the risk premium estimated using the capital asset pricing model (the final column of Table 8.5). The three-factor model provides a substantially lower estimate of the risk premium for computer stocks than the CAPM. Why? Largely because computer stocks have a low exposure (–.49) to the book-to-market factor.

8.5 SUMMARY

The basic principles of portfolio selection boil down to a commonsense statement that investors try to increase the expected return on their portfolios and to reduce the standard deviation of that return. A portfolio that gives the highest expected return for a given standard deviation, or the lowest standard deviation for a given expected return, is known as an *efficient portfolio*. To work out which portfolios are efficient, an investor must be able to state the expected return and standard deviation of each stock and the degree of correlation between each pair of stocks.

Investors who are restricted to holding common stocks should choose efficient portfolios that suit their attitudes to risk. But investors who can also borrow and lend at the risk-free rate of interest should choose the *best* common stock portfolio *regardless* of their attitudes to risk. Having done that, they can then set the risk of their overall portfolio by deciding what proportion of their money they are willing to invest in stocks. The best efficient portfolio offers the highest ratio of forecasted risk premium to portfolio standard deviation.

For an investor who has only the same opportunities and information as everybody else, the best stock portfolio is the same as the best stock portfolio for other investors. In other words, he or she should invest in a mixture of the market portfolio and a risk-free loan (i.e., borrowing or lending).

A stock's marginal contribution to portfolio risk is measured by its sensitivity to changes in the value of the portfolio. The marginal contribution of a stock to the risk of the *market portfolio* is measured by *beta*. That is the fundamental idea behind the capital asset pricing model, which concludes that each security's expected risk premium should increase in proportion to its beta:

$$\text{Expected risk premium} = \text{beta} \times \text{market risk premium}$$

$$r - r_f = \beta(r_m - r_f)$$

The capital asset pricing theory is the best-known model of risk and return. It is plausible and widely used but far from perfect. Actual returns are related to beta over the long run, but the relationship is not as strong as the CAPM predicts, and other factors seem to explain returns better since the mid-1960s. Stocks of small companies, and stocks with high book values relative to market prices, appear to have risks not captured by the CAPM.

The CAPM has also been criticized for its strong simplifying assumptions. A new theory called the *consumption* capital asset pricing model suggests that security risk reflects the sensitivity of returns to changes in investors' *consumption*. This theory calls for a consumption beta rather than a beta relative to the market portfolio.

The arbitrage pricing theory offers an alternative theory of risk and return. It states that the expected risk premium on a stock should depend on the stock's exposure to several pervasive macroeconomic factors that affect stock returns:

$$\text{Expected risk premium} = b_1(r_{\text{factor 1}} - r_f) + b_2(r_{\text{factor 2}} - r_f) + \cdots$$

Here b's represent the individual security's sensitivities to the factors, and $r_{\text{factor}} - r_f$ is the risk premium demanded by investors who are exposed to this factor.

Arbitrage pricing theory does not say what these factors are. It asks for economists to hunt for unknown game with their statistical tool kits. The hunters have returned with several candidates, including unanticipated changes in

- The level of industrial activity.
- The rate of inflation.
- The spread between short- and long-term interest rates.

Fama and French have suggested three different factors:

- The return on the market portfolio less the risk-free rate of interest.
- The difference between the return on small- and large-firm stocks.
- The difference between the return on stocks with high book-to-market ratios and stocks with low book-to-market ratios.

In the Fama–French three-factor model, the expected return on each stock depends on its exposure to these three factors.

Each of these different models of risk and return has its fan club. However, all financial economists agree on two basic ideas: (1) Investors require extra expected return for taking on risk, and (2) they appear to be concerned predominantly with the risk that they cannot eliminate by diversification.

Further Reading

The pioneering article on portfolio selection is:

H. M. Markowitz: "Portfolio Selection," *Journal of Finance,* 7:77–91 (March 1952).

There are a number of textbooks on portfolio selection which explain both Markowitz's original theory and some ingenious simplified versions. See, e.g.:

E. J. Elton and M. J. Gruber: *Modern Portfolio Theory and Investment Analysis,* 5th ed., John Wiley & Sons, New York, 1995.

Of the three pioneering articles on the capital asset pricing model, Jack Treynor's has never been published. The other two articles are:

W. F. Sharpe: "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance,* 19:425–442 (September 1964).

J. Lintner: "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics,* 47:13–37 (February 1965).

The subsequent literature on the capital asset pricing model is enormous. The following book provides a collection of some of the more important articles plus a very useful survey by Jensen:

M. C. Jensen (ed.): *Studies in the Theory of Capital Markets,* Frederick A. Praeger, Inc., New York, 1972.

The two most important early tests of the capital asset pricing model are:

E. F. Fama and J. D. MacBeth: "Risk, Return and Equilibrium: Empirical Tests," *Journal of Political Economy,* 81:607–636 (May 1973).

F. Black, M. C. Jensen, and M. Scholes: "The Capital Asset Pricing Model: Some Empirical Tests," in M. C. Jensen (ed.), *Studies in the Theory of Capital Markets,* Frederick A. Praeger, Inc., New York, 1972.

For a critique of empirical tests of the capital asset pricing model, see:

R. Roll: "A Critique of the Asset Pricing Theory's Tests; Part I: On Past and Potential Testability of the Theory," *Journal of Financial Economics,* 4:129–176 (March 1977).

Much of the recent controversy about the performance of the capital asset pricing model was prompted by Fama and French's paper. The paper by Black takes issue with Fama and French and updates the Black, Jensen, and Scholes test of the model:

E. F. Fama and K. R. French: "The Cross-Section of Expected Stock Returns," *Journal of Finance,* 47:427–465 (June 1992).

F. Black, "Beta and Return," *Journal of Portfolio Management,* 20:8–18 (Fall 1993).

Breeden's 1979 article describes the consumption asset pricing model, and the Breeden, Gibbons, and Litzenberger paper tests the model and compares it with the standard CAPM:

D. T. Breeden: "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics,* 7:265–296 (September 1979).

D. T. Breeden, M. R. Gibbons, and R. H. Litzenberger: "Empirical Tests of the Consumption-Oriented CAPM," *Journal of Finance,* 44:231–262 (June 1989).

Arbitrage pricing theory is described in Ross's 1976 paper.

S. A. Ross: "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory,* 13:341–360 (December 1976).

The most accessible recent implementation of APT is:

E. J. Elton, M. J. Gruber, and J. Mei, "Cost of Capital Using Arbitrage Pricing Theory: A Case Study of Nine New York Utilities," *Financial Markets, Institutions, and Instruments,* 3:46–73 (August 1994).

For an application of the Fama–French three-factor model, see:

E. F. Fama and K. R. French, "Industry Costs of Equity," *Journal of Financial Economics,* 43: 153–193 (February 1997).

Quiz

1. Here are returns and standard deviations for four investments.

	RETURN	STANDARD DEVIATION
Treasury bills	6%	0%
Stock P	10	14
Stock Q	14.5	28
Stock R	21.0	26

Calculate the standard deviations of the following portfolios.

(a) 50 percent in Treasury bills, 50 percent in stock P.

(b) 50 percent each in Q and R, assuming the shares have

- perfect positive correlation
- perfect negative correlation
- no correlation

(c) Plot a figure like Figure 8.4 for Q and R, assuming a correlation coefficient of .5.

(d) Stock Q has a lower return than R but a higher standard deviation. Does that mean that Q's price is too high, or that R's price is too low?

2. For each of the following pairs of investments, state which would always be preferred by a rational investor (assuming that these are the *only* investments available to the investor):

(a)	Portfolio A	r = 18 percent	σ = 20 percent
	Portfolio B	r = 14 percent	σ = 20 percent
(b)	Portfolio C	r = 15 percent	σ = 18 percent
	Portfolio D	r = 13 percent	σ = 8 percent
(c)	Portfolio E	r = 14 percent	σ = 16 percent
	Portfolio F	r = 14 percent	σ = 10 percent

3. Figures 8.13*a* and 8.13*b* purport to show the range of attainable combinations of expected return and standard deviation.

(a) Which diagram is incorrectly drawn and why?

(b) Which is the efficient set of portfolios?

(c) If r_f is the rate of interest, mark with an X the optimal stock portfolio.

4. (a) Plot the following risky portfolios on a graph:

	PORTFOLIO							
	A	B	C	D	E	F	G	H
Expected return (r), %	10	12.5	15	16	17	18	18	20
Standard deviation (σ), %	23	21	25	29	29	32	35	45

(b) Five of these portfolios are efficient, and three are not. Which are *in*efficient ones?

(c) Suppose you can also borrow and lend at an interest rate of 12 percent. Which of the above portfolios is best?

(d) Suppose you are prepared to tolerate a standard deviation of 25 percent. What is the maximum expected return that you can achieve if you cannot borrow or lend?

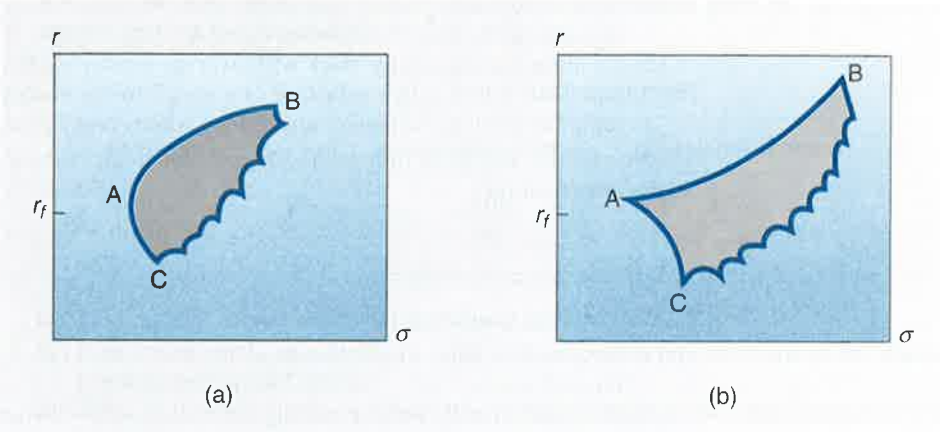
(e) What is your optimal strategy if you can borrow or lend at 12 percent and are prepared to tolerate a standard deviation of 25 percent? What is the maximum expected return that you can achieve?

Figure 8.13a

See Quiz question 3.

Figure 8.13b

See Quiz question 3.



5. How could an investor identify the *best* of a set of efficient portfolios of common stocks? What does "best" mean? Assume the investor can borrow or lend at the risk-free interest rate.
6. Suppose that the Treasury bill rate is 4 percent and the expected return on the market is 10 percent. Use the betas in Table 8.2.
   (a) Calculate the expected return from Microsoft.
   (b) Find the highest expected return that is offered by one of these stocks.
   (c) Find the lowest expected return that is offered by one of these stocks.
   (d) Would Compaq offer a higher or lower expected return if the interest rate was 6 rather than 4 percent? Assume that the expected return stays at 10 percent.
   (e) Would Exxon offer a higher or lower expected return if the interest rate was 6 percent?
7. True or false?
   (a) The CAPM implies that if you could find an investment with a negative beta, its expected return would be less than the interest rate.
   (b) The expected return on an investment with a beta of 2.0 is twice as high as the expected return on the market.
   (c) If a stock lies below the security market line, it is undervalued.
8. The CAPM has great theoretical, intuitive, and practical appeal. Nevertheless, many financial managers believe "beta is dead." Why?
9. Write out the APT equation for the expected rate of return on a risky stock.
10. Consider a three-factor APT model. The factors and associated risk premiums are

FACTOR	RISK PREMIUM
Change in GNP	5%
Change in energy prices	−1
Change in long-term interest rates	+2

Calculate expected rates of return on the following stocks. The risk-free interest rate is 7 percent.

(a) A stock whose return is uncorrelated with all three factors.
(b) A stock with average exposure to each factor (i.e., with $b = 1$ for each).

(c) A pure-play energy stock with high exposure to the energy factor (b = 2) but zero exposure to the other two factors.

(d) An aluminum company stock with average sensitivity to changes in interest rates and GNP, but negative exposure of $b = -1.5$ to the energy factor. (The aluminum company is energy-intensive and suffers when energy prices rise.)

11. Fama and French have proposed a three-factor model for expected returns. What are the three factors?

Practice Questions

1. True or false? Explain or qualify as necessary.
   (a) Investors demand higher expected rates of return on stocks with more variable rates of return.
   (b) The CAPM predicts that a security with a beta of 0 will offer a zero expected return.
   (c) An investor who puts $10,000 in Treasury bills and $20,000 in the market portfolio will have a beta of 2.0.
   (d) Investors demand higher expected rates of return from stocks with returns that are highly exposed to macroeconomic changes.
   (e) Investors demand higher expected rates of return from stocks with returns that are very sensitive to fluctuations in the stock market.
2. Look back at the calculation for Bristol-Myers and McDonald's in Section 8.1. Recalculate the expected portfolio return and standard deviation for different values of x_1 and x_2, assuming the correlation coefficient $\rho_{12} = 0$. Plot the range of possible combinations of expected return and standard deviation as in Figure 8.4. Repeat the problem for $\rho_{12} = +1$ and for $\rho_{12} = -1$.
3. Mark Harrywitz proposes to invest in two shares, X and Y. He expects a return of 12 percent from X and 8 percent from Y. The standard deviation of returns is 8 percent for X and 5 percent for Y. The correlation coefficient between the returns is .2.
   (a) Compute the expected return and standard deviation of the following portfolios:

PORTFOLIO	PERCENTAGE IN X	PERCENTAGE IN Y
1	50	50
2	25	75
3	75	25

   (b) Sketch the set of portfolios composed of X and Y.
   (c) Suppose that Mr. Harrywitz can also borrow or lend at an interest rate of 5 percent. Show on your sketch how this alters his opportunities. Given that he can borrow or lend, what proportions of the common stock portfolio should be invested in X and Y?
4. M. Grandet has invested 60 percent of his money in share A and the remainder in share B. He assesses their prospects as follows:

	A		B
Expected return (%)	15		20
Standard deviation (%)	20		22
Correlation between returns		.5	

   (a) What are the expected return and standard deviation of returns on his portfolio?
   (b) How would your answer change if the correlation coefficient was 0 or –.5?
   (c) Is M. Grandet's portfolio better or worse than one invested entirely in share A, or is it not possible to say?

5. Consider the returns forecast in two scenarios for the market portfolio, an aggressive stock A and a defensive stock D.

		RATE OF RETURN	
SCENARIO	MARKET	AGGRESSIVE STOCK A	DEFENSIVE STOCK D
Bust	–8%	–10%	–6%
Boom	32	38	24

   (a) Find the beta of each stock. In what way is stock D defensive?
   (b) If each scenario is equally likely, what is the expected rate of return on the market portfolio and on each stock?
   (c) If the T-bill rate is 4 percent, what rate of return on each stock is commensurate with its risk?
   (d) Which stock seems to be a better buy based on your answers to (a) through (c)?

6. The Treasury bill rate is 4 percent, and the expected return on the market portfolio is 12 percent. On the basis of the capital asset pricing model:
   (a) Draw a graph similar to Figure 8.7 showing how the expected return varies with beta.
   (b) What is the risk premium on the market?
   (c) What is the required return on an investment with a beta of 1.5?
   (d) If an investment with a beta of .8 offers an expected return of 9.8 percent, does it have a positive NPV?
   (e) If the market expects a return of 11.2 percent from stock X, what is its beta?

7. A company is deciding whether to issue stock to raise money for an investment project which has the same risk as the market and an expected return of 20 percent. If the risk-free rate is 10 percent and the expected return on the market is 15 percent, the company should go ahead
   (a) Unless the company's beta is greater than 2.0.
   (b) Unless the company's beta is less than 2.0.
   (c) Whatever the company's beta.

   Which answer is correct? Say briefly why.

8. The stock of United Merchants has a beta of 1.0 and very high unique risk. If the expected return on the market is 20 percent, the expected return on United Merchants will be
   (a) 10 percent if the interest rate is 10 percent.
   (b) 20 percent.
   (c) More than 20 percent because of the high unique risk.
   (d) Indeterminate unless you also know the interest rate.

   Which is the right answer? Explain *briefly* why.

9. The expected return on a stock is frequently written as $r = \alpha + \beta r_m$, where r_m is the expected return on the market. The capital asset pricing model says that in equilibrium
   (a) $\alpha = 0$.
   (b) $\alpha = r_f$ (the risk-free rate of interest).
   (c) $\alpha = (1 - \beta)r_f$.
   (d) $\alpha = (1 - r_f)$.

   Which is correct?

10. Percival Hygiene has $10 million invested in long-term corporate bonds. This bond portfolio's expected annual rate of return is 9 percent, and the annual standard deviation is 10 percent.

Amanda Reckonwith, Percival's financial adviser, recommends that Percival consider investing in an index fund which closely tracks the Standard and Poor's 500 index. The index has an expected return of 14 percent, and its standard deviation is 16 percent.

(a) Suppose Percival puts all his money in a combination of the index fund and Treasury bills. Can he thereby improve his expected rate of return without changing the risk of his portfolio? The Treasury bill yield is 6 percent.

(b) Could Percival do even better by investing equal amounts in the corporate bond portfolio and the index fund? The correlation between the bond portfolio and the index fund is +.1.

11. "There may be some truth in these CAPM and APT theories, but last year some stocks did much better than these theories predicted, and other stocks did much worse." Is this a valid criticism?

12. True or false?

(a) Stocks of small companies have done better than predicted by the CAPM.
(b) Stocks with high ratios of book value to market price have done better than predicted by the CAPM.
(c) On average, stock returns have been positively related to beta.

13. Some true or false questions about the APT:

(a) The APT factors cannot reflect diversifiable risks.
(b) The market rate of return cannot be an APT factor.
(c) Each APT factor must have a positive risk premium associated with it; otherwise the model is inconsistent.
(d) There is no theory that specifically identifies the APT factors.
(e) The APT model could be true but not very useful, for example, if the relevant factors change unpredictably.

14. Consider the following simplified APT model (compare Tables 8.3 and 8.4):

FACTOR	EXPECTED RISK PREMIUM
Market	6.4%
Interest rate	−.6
Yield spread	5.1

Calculate the expected return for the following stocks. Assume $r_f = 5$ percent.

	FACTOR RISK EXPOSURES		
	MARKET	INTEREST RATE	YIELD SPREAD
STOCK	(b_1)	(b_2)	(b_3)
P	1.0	−2.0	−.2
P^2	1.2	0	.3
P^3	.3	.5	1.0

15. Look again at practice question 14. Consider a portfolio with equal investments in stocks P, P^2, and P^3.

(a) What are the factor risk exposures for the portfolio?
(b) What is the portfolio's expected return?

16. The following table shows the sensitivity of four stocks to the three Fama–French factors in the five years to 1998. Estimate the expected return on each stock assuming that the interest rate is 5 percent, the expected risk premium on the market is 5.2 percent, the expected risk premium on the size factor is 3.2 percent, and the expected risk premium on the book-to-market factor is 5.4 percent. (These were the realized premia from 1963–1994.)

	FACTOR SENSITIVITIES			
FACTOR	AT&T	EXXON	GE	XEROX
Market	1.01	.29	.82	1.17
Size[a]	.39	.74	1.19	1.15
Book-to-market[b]	.51	−.22	−.63	.41

[a]Return on small firm stocks less return on large firm stocks.

[b]Return on high book-to-market-ratio stocks less return on low book-to-market-ratio stocks.

Challenge Questions

1. In footnote 4 we noted that the minimum-risk portfolio contained an investment of 61.3 percent in Bristol-Myers and 38.7 in McDonald's. Prove it. *Hint:* You need a little calculus to do so.
2. Look again at the set of efficient portfolios that we calculated in Section 8.1.
   (a) If the interest rate is 10 percent, which of the four efficient portfolios should you hold?
   (b) What is the beta of each holding relative to that portfolio? *Hint:* Remember that, if a portfolio is efficient, the expected risk premium on each holding must be proportional to the beta of the stock *relative to that portfolio.*
   (c) How would your answers to (a) and (b) change if the interest rate was 5 percent?
3. "Suppose you could forecast the behavior of APT factors, such as industrial production, interest rates, etc. You could then identify stocks' sensitivities to these factors, pick the right stocks, and make lots of money." Is this a good argument favoring the APT? Explain why or why not.
4. The following question illustrates the APT. Imagine that there are only two pervasive macroeconomic factors. Investments X, Y, and Z have the following sensitivities to these two factors:

INVESTMENT	b_1	b_2
X	1.75	.25
Y	−1.00	2.00
Z	2.00	1.00

We assume that the expected risk premium is 4 percent on factor 1 and 8 percent on factor 2. Treasury bills obviously offer zero risk premium.

(a) According to the APT, what is the risk premium on each of the three stocks?
(b) Suppose you buy $200 of X and $50 of Y and sell $150 of Z. What is the sensitivity of your portfolio to each of the two factors? What is the expected risk premium?

(c) Suppose you buy \$80 of X and \$60 of Y and sell \$40 of Z. What is the sensitivity of your portfolio to each of the two factors? What is the expected risk premium?

(d) Finally, suppose you buy \$160 of X and \$20 of Y and sell \$80 of Z. What is your portfolio's sensitivity now to each of the two factors? And what is the expected risk premium?

(e) Suggest two possible ways that you could construct a fund that has a sensitivity of .5 to factor 1 only. Now compare the risk premiums on each of these two investments.

(f) Suppose that the APT did *not* hold and that X offered a risk premium of 8 percent, Y offered a premium of 14 percent, and Z offered a premium of 16 percent. Devise an investment that has zero sensitivity to each factor and that has a positive risk premium.