MS-A0211 / Period II 2017

Final Exam, 12.12.2017 time 16.30-19.30

Aalto University

No calculators or notes of any kind are allowed

This exam consists of 6 problems, each of equal weight.

Notation for vectors: $\langle a, b, c\rangle = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$.

Question 1: Here are three unrelated questions

(a) Evaluate the following limit or show it does not exist. $\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2 + y^8}$

(b) Sketch the domain of the function $f(x, y) = \sqrt{x} + \sqrt{4 - x^2 - y^2}$.

(c) Find the equation of the plane passing through the points $(2, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 3)$.

Question 2: Suppose that we do not have an equation for the function $f(x, y)$, but we know that $f(3, 1) = 2$ and the two curves $\mathbf{r}_1(t) = \langle t - t^3 + 3, 1 - t + 2t^2, 2 + t\rangle$ and $\mathbf{r}_2(s) = \langle s^3 - 2s + 4, s, 2\rangle$ both lie on the surface S given by the graph of $z = f(x, y)$.

(a) Find the tangent plane to the surface at the point $(3, 1, 2)$.

(b) Find an approximate value of $f(3.3, 1.1)$ using linear approximation.

Question 3: Here are two independent questions on extreme values.

(a) Find a value of c such that the function

$$f(x, y) = \frac{x^3}{3} + \frac{cy^2}{2} + xy$$

has a local maximum somewhere. Justify your answer using the second derivative test.

(b) Consider a metal plate which is a disk of radius $\sqrt{2}$ centered at $(4, 0)$ in the xy-plane. Note that the equation of the corresponding circle is $(x - 4)^2 + y^2 = 2$. The temperature of the disk is given by $T(x, y) = \ln(x + y)$. Find the absolute minimum and maximum temperature on the disk.

Question 4: Suppose that the temperature at a point (x, y) in the xy-plane is given by $T(x, y, z) = 100e^{-2x^2+y}$.

(a) Sketch the level curve passing through the point $(2, 1)$.

(b) Find a parametric equation for the curve in part (a).

(c) Use part (b) to find a tangent vector to the level curve at $(2, 1)$.

(d) Find the rate of change of temperature at the point $(2, 1)$ in the direction toward the point $(3, 3)$.

(e) In which direction does the temperature increase most rapidly at the point (2, 1)? Describe the direction using a unit vector.

(f) Find the rate of change of temperature at the point $(2, 1)$ in the direction of the vector you found in part (c). Is the answer what you expected. Explain.

Question 5: Here are two independent questions on double integrals.

(a) Compute the following integral by reversing the order of integration.

$$\int_0^2 \int_{x^2}^4 \frac{1}{1 + y^{3/2}}\, dy\, dx$$

(b) Write a double integral in **polar coordinates** that equals the surface area of the portion of $x^2 + y^2 + z^2 = 9$ that lies between $z = 1$ and $z = 2$. You do NOT have to evaluate the integral.

Question 6: Here are two independent questions on triple integrals.

(a) Use spherical coordinates to find the volume of the region that lies below $z = \sqrt{x^2 + y^2}$ and inside $x^2 + y^2 + z^2 = 8$. First sketch the region.

(b) Write a triple integral that represent the volume of the region that lies above $z = \sqrt{x^2 + y^2}$, below $z = 2 + \sqrt{x^2 + y^2}$ and inside $z = x^2 + y^2$. You do NOT have to evaluate the integral.