MS-A0211 2017 Final Exam: Answers and Hints

February 2024

1 Question 1.

a) The limit does not exist. Prove by showing different limits along different paths to $(0, 0)$. Consider for example the cases when $x = 0$ and when $x = y^4$.
b) The domain is given by the conditions $x \geq 0$ and $x^2 + y^2 \leq 4$, which are necessary so that the square roots stay real.
c) Find two vectors that exist on the plane that we want to find. Their cross product gives a vector which is perpendicular to the plane. Answer: $3x + 6y + 2z = 6$

2 Question 2.

a) Get the gradient of the curves r_1, r_2 at the point $(3, 1, 2)$. The cross product of the gradient vectors is normal to the tangent plane. Answer: $3(x - 3) + (y - 1) - 2(z - 2) = 0$.
b) Substitute $x = 3.3, y = 1.1$ into the tangent equation, $z_0 = 2$ based on the point $(3, 1, 2)$. Answer: $z = 2.5$.

3 Question 3.

a) Find the critical points of the function. Determine the nature of the critical points using second derivative and choose a suitable value for c. Answer: Local maximum at $(\frac{1}{c}, -\frac{1}{c^2})$ when $c < 0$.
b) Check the critical points of the function inside the disk. If there are no critical points inside the disk the extreme values must be on the boundary. Lagrangian multipliers can be used to find the extreme values. Answer: The maximum is $T(5, 1) = \ln(6)$ and the minimum is $T(3, -1) = \ln(2)$.

4 Question 4.

a) The level curve passing through the point $(2, 1)$ consists of all the points (x, y) such that $T(x, y) = T(2, 1) = 100e^{-7}$. The curve is given by $y = 2x^2 - 7$.
b) The curve can be parametrised as $r(t) = (t, 2t^2 - 7)$.
c) The tangent vector can be gotten from the derivative of the parametric equation. Answer: $r'(2) = (1, 8)$.
d) Consider the gradient from the point $(2, 1)$ to $(3, 3)$. Answer: $\frac{600e^{-7}}{\sqrt{5}}$.
e) The greatest change in temperature is along the gradient. Answer: $\frac{1}{\sqrt{65}}(-8, 1)$.
f) A functions value is constant along its level curve. Answer: 0.

5 Question 5.

a) Consider how the reversing affects the integration bounds. Answer: $\frac{4}{3}\ln(3)$.
b) Consider the constraints on the radius r. Answer: $\int_0^{2\pi} \int_{\sqrt{5}}^{\sqrt{8}} \sqrt{\frac{9}{9-r^2}} r\, dr\, d\theta$.

6 Question 6.

a) $z = \sqrt{x^2 + y^2}$ defines a cone. Answer: $\int_0^{2\pi} \int_{\pi/4}^{\pi} \int_0^{\sqrt{8}} \rho^2 \sin\phi\, dp\, d\phi\, d\theta$.
b) Consider using cylindrical coordinates. Answer: $\int_0^{2\pi} \int_0^1 \int_r^{2+r} r\, dz\, dr\, d\theta + \int_0^{2\pi} \int_1^2 \int_{r^2}^{2+r} r\, dz\, dr\, d\theta$.