Combinatorial Optimization

Dias, Schiewe, Pattanaki *Instructor: Pattanaki*

NP Problems and Polynomial Transformation

§ Week VII §

Problem 1: NP Problems and Aliens

Highly intelligent aliens land on Earth and tell us the following two things and then leave immediately.

1. The 3-Coloring problem (which is NP-complete) is solvable in worst-case $O(n^9)$ time, where n denotes the number of vertices in the graph.

2. There is no algorithm for 3-Coloring that runs faster than $\Omega(n^7)$ time in the worst case.

Assuming these two facts, for each of the following assertions, indicate whether it can be inferred from the information the aliens have given us. (In all cases, time complexities are understood to be worst-case running time.) Provide a short justification in each case.

- All NP-complete problems are solvable in polynomial time.
- All problems in NP, even those that are not NP-complete, are solvable in polynomial time.
- All NP-hard problems are solvable in polynomial time.
- All NP-complete problems are solvable in $O(n^9)$ time.
- No NP-complete problem can be solved faster than $\Omega(n^7)$

Problem 2: Hamiltonian Path

Given an undirected graph $G = (V, E)$, a Hamiltonian path is a simple path (not a cycle) that visits every vertex in the graph. The Hamiltonian Path problem (HP) is the problem of determining whether a given graph has a Hamiltonian path.

1. Show that HP is in NP.
2. Professor Gwen Stacy observes that if a graph has a Hamiltonian Cycle, then it also has a Hamiltonian Path. He suggests the following trivial reduction in order to prove that HP is NP-hard. Given a graph G for the Hamiltonian Cycle problem, simply output a copy of this graph. Explain why Professor Stacy's reduction is incorrect.
3. Give a (correct) proof that HP is NP-hard. (Hint: The reduction is from the Hamiltonian Cycle problem)

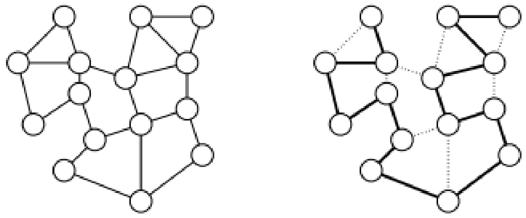

Figure 1: Hamiltonian path

Problem 3: NP-Complete

Prove that the following problems are NP-complete.

1. Given two undirected graphs G and H, is G isomorphic to a subgraph of H?

2. Given an undirected graph G, does G have a spanning tree in which every node has degree at most 17?

3. Given an undirected graph G, does G have a spanning tree with at most 42 leaves?