Week VII

Overview

- Polynomial Transformation;
- SAT;
- Knapsack Problem;
- Clique vs Independent Set;
- TSP vs Hamiltonian Cycle;
- Independent Set vs Vertex Cover;
- 3-SAT vs Clique

Polynomial Transformation

- Show it is in NP:
  $\longrightarrow$ Verify that if a candidate solution is valid in polynomial time;

- Show it is NP-Hard:
  $\longrightarrow$ Reduce to a known NP-Complete problem.

Definition

*SAT is the problem of deciding (requires a **yes**/**no** answer) if there is an assignment to the variables of a **Boolean** formula such that the formula is satisfied.*

One of the first known NP problems.

Clique vs Independent Set

Clique and Independent Set Reduction:

- For a graph $G = (V, E)$, build a complimentary graph G';
- For every $v \in V$, it creates another set of nodes $v \in V'$;
- Add an edge in G' for every edge not in G.

TSP vs Hamiltonian Cycle

TSP and Hamiltonian Cycle reduction:

- For a graph $G = (V, E)$, build a complimentary graph G';
- For every pair of nodes (u, v) without an edge in G, add an edge in G'.
- If edge (u, v) exist in G, set the weight to zero, otherwise assign weight equal to one.

Independent Set and Vertex Cover

If S is an independent set, there is no edge $(u, v) \in G$, such that both v and u are in S. Therefore, either v or u **has to be in** $V - S$.

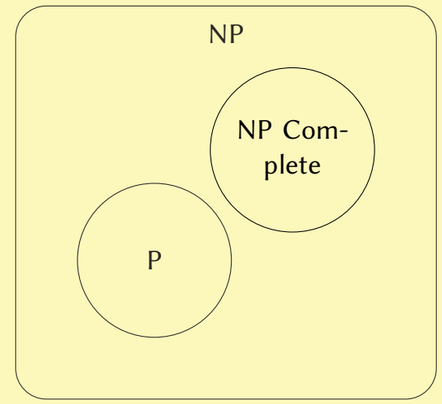
If $V - S$ is a vertex cover, between any pair of nodes $u, v \in S$, the edge connecting them **would not exist** in $V - S$, otherwise it violates the definition of such vertex cover. Hence, no pair in S can be reached by a single edge, creating an independent set.

Remark: Independent Set of size k corresponds to a Vertex Cover of size $V - |k|$.

2-SAT and Vertex Cover

A 3-SAT is composed from three-literal clauses. The goal is to reduce a clique of size k in a group of k clauses ϕ.

- Building a graph G of k clusters with a **maximum** of 3 nodes in each cluster;
- Each cluster corresponds to a **clause** in ϕ;
- Each node in a cluster is **labeled with a literal from the clause**;
- An edge is put between all pairs of nodes in different cluster **except for pairs of the form** $(x, \bar{x})$;
- **No edge is put between any pair of nodes in the same cluster**.



Fernando Dias
fernando.dias@aalto.fi
Combinatorial Optimization

Aalto University