CHAPTER 5

Risk Analysis in Capital Budgeting

The race is not to the swift, nor the battle to the strong, neither yet bread to the wise, nor yet riches to men of understanding, nor yet favour to men of skill; but time and chance happen to them all.

—ECCLESIASTES 9:11

The best laid schemes o' mice and men
Gang aft a-gley.

—ROBERT BURNS

Chapters 2 and 3 showed how to conduct a capital-budgeting analysis when a firm has detailed information about the timing and magnitude of the project's cash flows. This chapter shows how the analytical techniques developed for the case of complete certainty can be modified to deal with the more realistic situation in which project returns are highly uncertain. Risk analysis is increasingly important as the economic environment has become more uncertain—with developments halfway around the world affecting the survival of companies in their domestic markets—and the stakes higher—with multimillion- or billion-dollar investments in plant and equipment or research and development projects on the line.

Consider, for example, the following investment decision under uncertainty facing the new president of Beech Aircraft Corp.:[1] Should he gamble the company's future by spending $250 million to develop a brand-new executive airplane that promises a revolution in performance and economy? The eye-catching plane, built mostly of plastic materials reinforced with graphite fibers, is the Starship—an eight-passenger turboprop that looks more like a Hollywood vision of interstellar space travel than an earthbound airplane. (Because its stretched wings supposedly resemble those of a flying duck, its design type is known as a *canard*, the French word for duck.)

[1]Although the Starship is a real-world example, the financial data presented here are illustrative only.

The Starship will fly almost as fast as a jet and, at a $2.7 million projected price, would cost as much as some jets. But it is supposed to be 40% more fuel-efficient than are jets of comparable size and will provide an extremely comfortable ride. The first deliveries to customers are scheduled in three years.

As with all such bold ventures, there were a number of unknowns. Aside from the technical problems, of which there were several, the major uncertainties were in the area of marketing. These included both the size of the executive aviation market in three years and the Starship's likely share of that market. The latter depended on whether executives would entrust their lives to such a different type of airplane. Beech was gambling for extremely high stakes. If the plane succeeded, Beech could lead the industry for the next generation. If it failed, the firm would have a very turbulent future.

Although Beech's venture is riskier than most corporate projects, risk is present in all investments, especially, it seems, in those projects promising the highest returns. Thus, managers responsible for evaluating capital expenditure proposals should "pray for the best but prepare for the worst." This requires an ability to understand the different sources of uncertainty and how they might influence the value of proceeding with a given project.

Companies have responded to the added uncertainty they face by devoting more resources to the general area of risk analysis. The payoff from this effort is readily apparent: Companies are becoming more sophisticated in analyzing project risk, largely because of the increasingly powerful techniques made possible by a combination of new theoretical developments, high-powered computers, and the legions of business school graduates who understand the nature and application of these techniques.

The purpose of this chapter is to present and evaluate the techniques that firms use to incorporate risk in the capital-budgeting process. Section 5.1 reviews fundamental concepts of risk and discusses three possible ways of measuring project risk. Section 5.2 uses sensitivity analysis to assess the consequences for project profitability of variations in certain key parameters, and Section 5.3 shows how computer simulation can be used to determine the risk profile of a given investment. Section 5.4 shows how project risk can be accounted for within the basic capital-budgeting framework supplied by present value analysis. In Section 5.5, we see how decision tree analysis can be employed to evaluate the consequences of alternative marketing and cost assumptions on project present values and thereby aid the decision-making process. Chapter 6 discusses how the capital asset pricing model can be used to determine the returns required for projects of varying risk.

5.1 Measuring Project Riskiness

Risk is normally measured as the variability of possible returns. The same factors that affect the variability of security returns also affect the variability of project returns. Macroeconomic risk factors, which affect all firms to a greater or lesser degree, include changes in the growth rate of the economy, the inflation rate, and the level of real interest rates. Risk factors that are company and project specific include competitor actions, shifting consumer tastes, technological uncertainty, uncertain exploration costs, and changing input costs and output prices. Macroeconomic risk factors are the primary source of systematic or beta risk, whereas firm-specific risk factors result in unsystematic risk.

Project risk is not an unambiguous concept. It can be measured relative to the project itself, the firm, or a well-diversified investor. Thus, in discussing project risk, one must distinguish among three separate types of risk: (1) total project risk, which is based on the variability of project returns, (2) company risk, which is measured by the contribution of project risk to the variability of total company returns, and (3) systematic risk, which is based on the project's beta, as measured by the correlation between project returns and returns on the market portfolio.

PROJECT RISK

The business risk of a project is primarily determined by the variability of sales and costs. The latter include both the initial investment cost and subsequent production costs. Natural resource projects are also affected by uncertainty over the location and magnitude of mineral deposits. Regardless of the source of the risk—whether from macroeconomic or project-specific factors—operating leverage magnifies its impact.

Operating Leverage

Any time a firm uses assets for which it must pay a fixed charge, regardless of the volume of production, it has **operating leverage**. These fixed costs include the opportunity cost of the funds tied up in the assets, economic depreciation of the plant and equipment, property taxes, insurance, management expenses, and a portion of the utility bills. By contrast, variable costs—including direct labor, raw materials, energy, sales commissions, and most utility bills—vary directly with the level of production and sales. Once the plant is built, the opportunity cost of funds is no longer a relevant fixed cost; it is now a sunk cost, unless the firm is considering selling the plant.

Firms pay careful attention to operating leverage in deciding the trade-off between more automated, capital-intensive production facilities with lower unit costs and more labor-intensive production facilities with higher unit costs. Suppose, for example, that a firm is trying to decide between two different microprocessor assembly facilities—facility A, which is highly automated, and facility B, which is less automated and more labor intensive. Facility A is expected to have fixed costs of \$8 million annually and unit variable costs of \$4. In contrast, B's fixed costs are expected to be only \$4 million annually but its variable costs are \$10 per unit. To summarize,

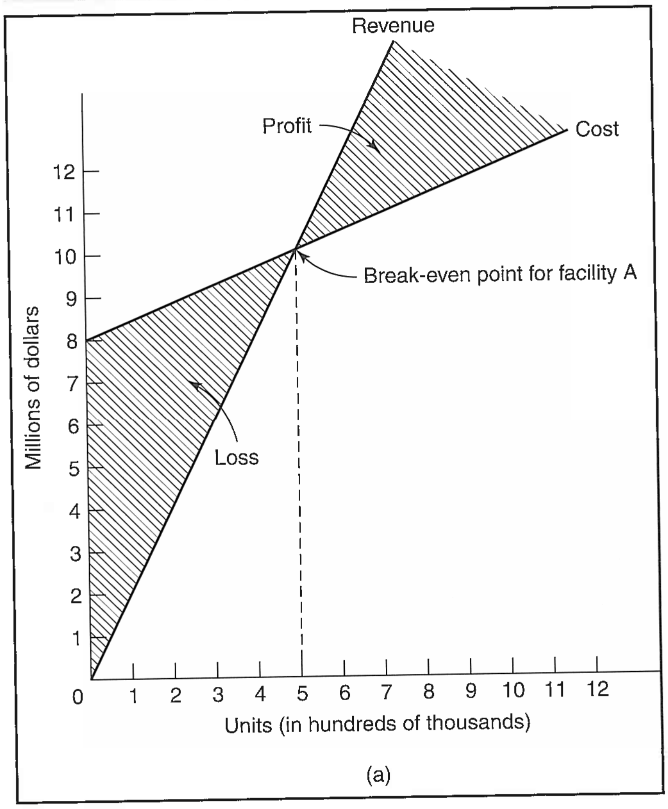
	Facility A	Facility B
Fixed costs (\$)	8 million	4 million
Variable costs (\$)	4/unit	10/unit

The microprocessor is expected to sell for \$20. We can study the relationship among profits, volume, and the different cost structures by means of the break-even charts in Exhibits 5.1(a), 5.1(b), and 5.1(c).

The intersection of the total revenue line with the total cost line is the point at which the project just breaks even, that is, the volume of sales at which project revenue just covers all project costs. This point is reached when

$$\text{Total revenue} = \text{Total cost}$$
$$P \times Q = F + V \times Q \tag{5.1}$$

EXHIBIT 5.1 (a) Break-Even Analysis for Facility A



where

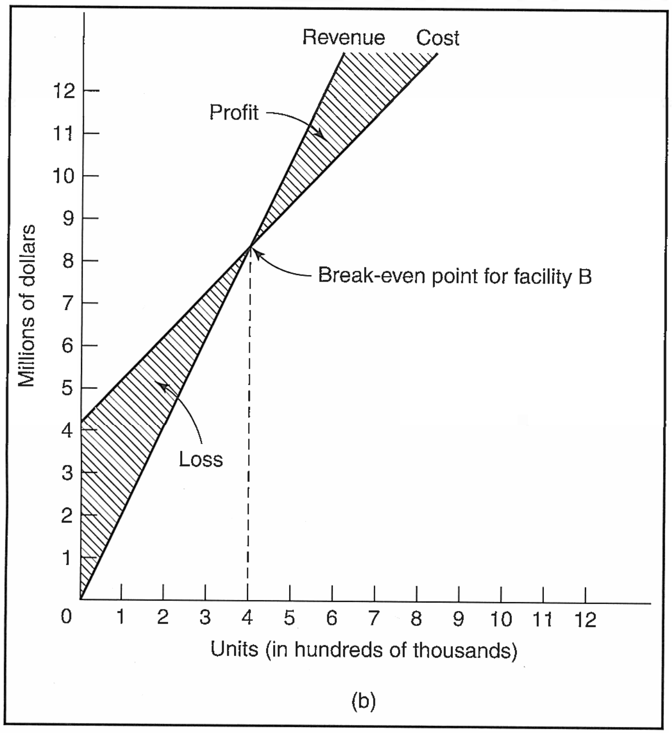
P = sales price per unit
Q = unit volume
F = fixed costs
V = unit variable cost

By rearranging terms, we can solve Equation 5.1 for the break-even quantity

$$Q^* = \frac{F}{(P - V)} \tag{5.2}$$

Applying Equation 5.2, we obtain the break-even point of 500,000 units for facility A (8,000,000/16) and 400,000 units for facility B (4,000,000/10). This is shown in Exhibits 5.1(a) and 5.1(b). Thus, if there is substantial risk that sales will be below 500,000 units annually, facility B will clearly be preferable. In fact, facility B is more profitable until annual sales reach 667,000. This can be seen by solving for the sales quantity at which A's profits are just equal to B's profits. The subscripts on the fixed

EXHIBIT 5.1 (b) Break-Even Analysis for Facility B



and variable costs refer to costs for A and B:

$$\text{Profit for A} = \text{Revenue} - \text{Cost} = \text{Profit for B} = \text{Revenue} - \text{Cost}$$

$$P \times Q - F_A - V_A \times Q = P \times Q - F_B - V_B \times Q$$

$$20Q - 8{,}000{,}000 - 4Q = 20Q - 4{,}000{,}000 - 10Q$$

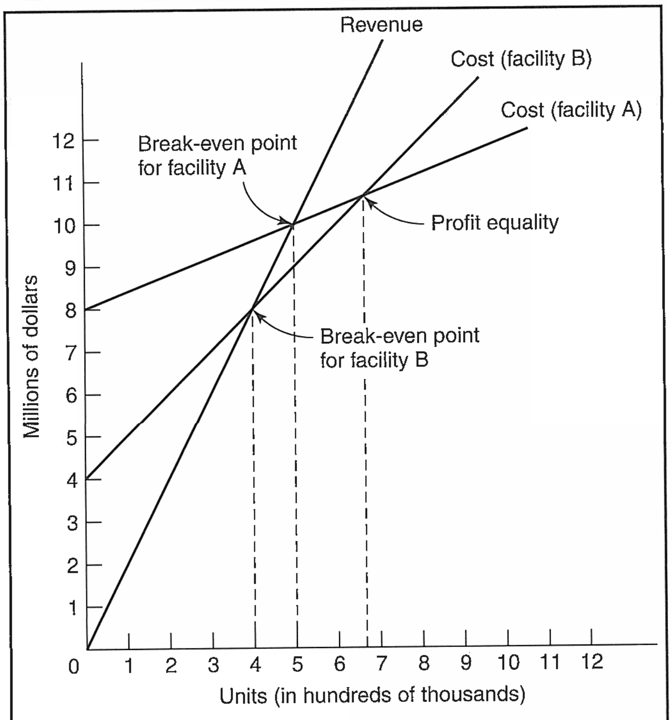
or

$$Q = 667{,}000$$

Beyond this level of sales, shown in Exhibit 5.1(c), facility A becomes increasingly profitable relative to facility B because the incremental profit on each additional unit sold by facility A is \$16 in comparison with B's incremental profit of \$10. The reason for facility A's higher incremental profit, of course, is that A's unit variable cost is \$6 lower than B's unit variable cost.

As sales expand beyond the break-even level, profit increases. This is represented by the shaded area to the right of the break-even point for each facility, as seen in Exhibits 5.1(a) and 5.1(b). On the other hand, the shaded areas to the left of the break-even points show the losses associated with each level of sales below these points. The higher is the ratio of fixed to variable costs, the more sensitive that profit will be to a

EXHIBIT 5.1 (c) Break-Even Analysis for Facilities A and B



(c)

change in sales. Similarly, an increase in the ratio of price to unit variable cost increases the sensitivity of profit to fluctuations in the quantity sold.

In general, the advantage of a labor-intensive process is that labor is typically a variable cost and can be reduced if demand falls off. Not so with capital equipment, for which the firm must continue to bear the opportunity cost of the funds tied up in it along with the cost of economic depreciation.

The logic of this analysis breaks down in the case of those Japanese firms adhering to a system of lifetime employment. For them, labor is treated as a fixed cost, giving these firms far greater operating leverage than firms that can easily adjust their work forces to changes in economic conditions. The same is true for firms located in those countries that restrict or prohibit layoffs—which these days include most countries aside from the United States. Viewing labor as a fixed cost helps explain why, once a plant is built and workers hired, Japanese firms and others that are unable to adjust their work force tend to keep producing at close to capacity even at low prices that do not come close to covering their costs.

Clearly, operating leverage does not cause variation in profit. But it can exacerbate any profit variation that does exist. The more fixed costs a project has, the more its profits will fluctuate with a given change in sales volume (i.e., all other things being equal, higher operating leverage leads to greater project risk).

The Relevance of Project Risk

Although most risk analysis techniques focus on the total risk of the project standing on its own, this emphasis is somewhat misplaced. Any single project is just one of the collection of projects that make up the firm, each with its own uncertain return. It is the overall riskiness of this project portfolio—which we will call *firm risk*—that matters to top management, not the riskiness of any individual project. Management pays attention to firm risk because this is what determines the possibility of financial distress and, ultimately, bankruptcy. Thus, project risk will be of concern to top management only to the extent that it affects the variability of total corporate returns.

From the even broader perspective of a well-diversified investor, the firm itself is just one of numerous firms, each with its own project portfolio. What matters to the well-diversified investor, therefore, is the project's contribution to *total* portfolio risk. Of course, the manager who is in charge of the project and who will be judged according to its results, will be quite concerned with total project risk. This may lead to risk-averse behavior that is suboptimal from the standpoint of the firm and its investors.

Companies do pay attention to project risk because it serves as a proxy for the project's contribution to firm risk. In the presence of uncertainty, it is often difficult to figure out just how much of a project's return variability will be diversified away. In general, though, the riskier the project is on its own, the more risk it is likely to contribute to firm risk. This contribution will not be one for one, however, as is evident from the following example.

FIRM RISK

A project that is highly risky in and of itself, like drilling for oil, may be of minimal risk from the firm's standpoint. For example, suppose the cost of drilling a well is $3 million and the chance of finding a commercial quantity of oil (an amount that makes the well economic to develop) is 10%. At the current price of oil, a successful well will yield an average net profit after development costs of $35 million. Although the firm has only a one-in-ten chance of striking oil, if it drills 100 wells, it is almost certain to be successful 10 times. As long as the price of oil is stable, the firm's expected profit will be $50 million with very little risk. The expected profit per well drilled will be $0.5 million. The more wells that are drilled, the lower the average risk will be per well. Thus, from the firm's standpoint, the incremental risk associated with drilling one more well is minimal.

No matter how many wells the firm drills, however, it still faces a major risk—the risk of fluctuations in the price of oil. The $0.5 million expected profit per well drilled is predicated on a specific price of oil. If the price of oil changes, the net profit per well will also change. Thus, drilling numerous wells can eliminate most of the *discovery risk*—uncertainty as to the quantity of oil that will be found—but it cannot eliminate the *price risk*—uncertainty about the price at which the oil will sell in the market.

Total Risk versus Systematic Risk

According to the capital asset pricing model (CAPM) and the more recent arbitrage pricing theory (APT), which are discussed in Chapter 6, even the firm risk of a project is irrelevant except that it affects the firm's systematic risk. This is true but in a limited sense. Both the CAPM and the APT demonstrate that under reasonable circumstances, diversifiable risks are not priced and hence do not affect the required rate of return on risky investments. Systematic risks are priced; but because the price of risk is the same for all market participants, or so the story goes, there is no gain to shareholders from "laying them off" to financial markets. And yet corporations are major purchasers of insurance, heavy use commodity, financial futures, and forward contracts; sign long-term purchase and sales contracts; and engage in a wide variety of other behavior, all designed to avoid or reduce total risk.

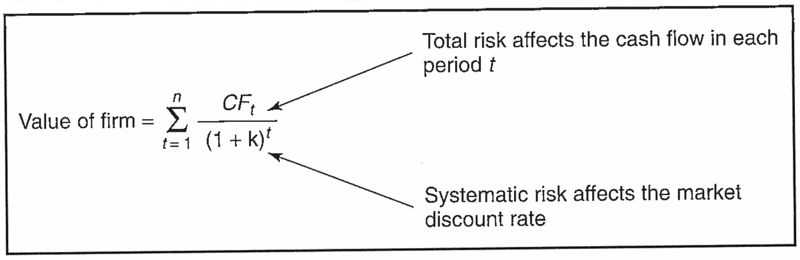
One explanation for this behavior, which is consistent with both shareholder wealth maximization and the focus of asset pricing models on systematic risk, is that reducing total risk can increase expected cash flows. This explanation does not conflict with either the CAPM or APT, both of which say that only the systematic component of total risk affects investors' required returns; the unsystematic or *avoidable* component is irrelevant. These models are concerned only with the effect of risk on market discount rates; for the most part, they ignore its effect on expected cash flow. To the extent that an increase in total risk can lower the firm's expected future cash flows, however, it makes sense to reduce total risk. The distinction between the effects of systematic risk and the effects of total risk on the value of the firm is shown in Exhibit 5.2.

How does a higher total risk lower the expectations of future cash flows? Firms with a higher total risk, all else being equal, are more likely to find themselves in financial distress. Thus, total risk, by increasing the threat of financial difficulties, can affect the level of future corporate cash flows by influencing the willingness of customers, suppliers, and employees to commit themselves to relationships with the firm, thereby affecting future sales, operating costs, and financing costs.[2] It can also affect the firm's ability to take full advantage of tax credits and write-offs. In addition, an increase in total risk may result in management actions detrimental to shareholder interests.

Impact of Total Risk on Sales

Reducing total risk can aid a firm's marketing efforts by providing greater assurance to potential customers that the company will be around in the future to service and upgrade its products. Purchasers of long-lived capital assets are especially concerned about the seller's longevity. They want to know if the manufacturer will be there to service the equipment and supply new parts as old ones wear out. If the original supplier goes

EXHIBIT 5.2 How Risk Affects the Value of the Firm



[2]The effects of total risk on sales and costs are explored in more detail in Shapiro, A.C., and Titman, S. "An Integrated Approach to Corporate Risk Management." *Midland Corporation Finance Journal*, Summer 1985, pp. 41–56.

out of business, parts and repairs may become a problem. Potential buyers of Chrysler cars in the late 1970s, for example, were understandably nervous about purchasing a product that they might have difficulty getting serviced if Chrysler went bankrupt.

Customers of products like computers that are undergoing rapid technological change are also very concerned about producers' staying power. For example, IBM exploits buyers' fears—of getting stuck with obsolete equipment and having to write off the associated nontransferable investments in software and learning—with the message, "What people want most from a computer company is a good night's sleep." Such fears hastened the downfall of Wang Laboratories, the pioneer of word-processing computers. As its financial outlook worsened, customers who feared becoming technological orphans took their business elsewhere, eventually forcing Wang into bankruptcy.

Fortunately for Beech, it was an affiliate of Raytheon—a profitable defense electronics company with "deep pockets"—that could afford to take some risks. This gave potential customers greater assurance that the company would be around in the future to supply parts and provide service for the Starship. Were this not the case, it is doubtful that Beech would have gone ahead and developed the plane on its own.

Impact of Total Risk on Operating Costs

A firm's cost of doing business is, in part, a function of its suppliers' view of the company's long-run prospects. A firm struggling to survive is unlikely to find suppliers bending over backwards to provide it with specially developed products or services, particularly if those products or services are unique—like special dies or castings—and suitable for use only by the firm in question.

In general, the value of investing in a long-term relationship with a customer will depend on whether the customer is expected to survive in the long run. The lower the likelihood is of future survival, the more of these relationship costs the customer will have to bear up front in the form of higher prices or less closely tailored services and products. Lower-risk firms also have an easier time attracting and retaining good personnel.

Impact of Total Risk on Taxes

As the variability of operating profits increases, so does the probability that a firm will be unable to fully utilize its tax credits and depreciation and interest expense tax write-offs. Because the resale market in tax credits and tax write-offs is imperfect, as indicated by the discount taken when tax benefits are sold, an increase in total risk will lead to a reduction in expected corporate cash flows. If the tax credit or tax loss is carried forward, the relevant cost will be the reduction in the present value of the tax benefit. By reducing its total risk, a firm can increase the expected value of its tax credits and tax write-offs and thereby increase its expected future cash flows.

Perhaps the costliest aspect of financial distress stems from the firm's loss of operating and investment flexibility owing to the constraints imposed by lenders. The riskier a firm is perceived to be, the more stringent will be the restrictions on the operating policies and the investment projects that it can pursue. For example, lenders may veto high-risk projects with positive net present values.

Decreasing total risk can reduce or eliminate some of the more onerous debt restrictions and covenants. Investment and operating policies with fewer restrictions on them should increase expected future cash flows and shareholder wealth.

SYSTEMATIC RISK

From the perspective of a well-diversified investor, the total firm risk associated with a project does not affect the expected return that the investor demands for undertaking the project. All that matters to such an investor is the project's contribution to the risk of the investor's portfolio. Clearly, the fraction of risk that is relevant from the standpoint of a well-diversified investor is less than that which is relevant to corporate management, which, in turn, is less than that of the project's management.

Consider, for example, the systematic risk of the general aviation business. This is the systematic risk that faces Beech Aircraft's new Starship venture, and it primarily depends on the state of the national economy. In a healthy economy, with expanding business activity, executives are more likely to travel, and so the value to their companies of investing in corporate planes will rise. Sales of executive aircraft, therefore, pick up during prosperous times and fall off during recessions.

Other risks faced by Beech's Starship project, including the uncertainty of its appeal to business executives and the possibility of competitors' developing their own canards, are unsystematic in nature. This is evident from the fact that a well-diversified investor, whose portfolio includes the securities of competitor general aviation manufacturers, can fully avoid these risks.

According to the CAPM, the systematic risk of a project—as measured by the correlation between the returns on the project and returns on the market portfolio—will affect the required return on the project. The important point here is that the required return on a project is a function of the riskiness of the project itself, not of the riskiness of the firm undertaking the project. This means that each project has its own cost of capital, which is independent of the firm investing in the project. Thus, when Coca-Cola entered the motion picture business through its acquisition of Columbia Pictures, its required return on this investment was determined by the risk of making films, not the risk of being in the soft drink business.

To summarize, only the project's systematic risk will determine its required return, but total risk could affect the project's actual cash flows through its impact on the expectations of customers, suppliers, employees, and creditors. Even the diversified investor, therefore, will be interested in the project's total risk to the extent that it is likely to affect the project's *actual* return.

5.2 Sensitivity Analysis

As we will see in the next section, most techniques for factoring risk into a capital-budgeting analysis reduce all uncertainties to just a single value for the risk-adjusted NPV. This is done by using the expected values of sales and costs and ignoring any variations in those numbers. The advantage of using a single number to represent the impact of risk is that it simplifies the decision process. If the calculated NPV is positive, the project should be undertaken; if it is negative, it should be rejected. Often, however, this single number hides a great deal of information about the riskiness of the proposed project: Because the future is unknowable, it is evident that today's estimates of future project prices, costs, and volumes are going to be wrong. It is natural, therefore, for decision makers to want to study, in advance, how potential estimation errors will affect the project NPV.

Moreover, before conducting a project risk analysis, it makes sense to see whether the risk has a significant impact on the project's net present value. There is no sense in doing a thorough risk analysis if the risk is not material. An important means of going beyond the information conveyed by a risk-adjusted NPV is sensitivity analysis.

Sensitivity analysis is a procedure to study systematically the effect of changes in the values of key parameters—including R&D costs, plant construction cost, market size, market share, price, and production costs—on the project NPV. It is best suited to address a series of "what if" questions, like the ones contained in the following quotation from the *Wall Street Journal*:

> The Chrysler Corp. of 10 years ago wouldn't have had any trouble making the decision that is occupying a good deal of top-executive time at the No. 3 auto maker: whether to invest some $350 to $400 million to expand production of the company's fast-selling minivans.
>
> Chrysler has the capacity at its Windsor, Ontario, plant to build 270,000 minivans a year, and it's currently producing them at that rate. But before adding the capacity to build about 150,000 more each year, Chrysler has asked itself some rough questions about the vehicle's market and potential.
>
> What if, as some predict, the economy should turn sour next year or the year after, just about the time the new capacity would be coming on stream? What if the minivans that General Motors Corp. and Ford Motor Co. are introducing later this year saturate the market or even eat into Chrysler's current share?
>
> Ultimately, Chrysler resolved those doubts to its satisfaction. In essence, top Chrysler executives have concluded that the minivan has a niche in a market that probably will continue to grow.[3]

The first step in a sensitivity analysis usually is to have the engineering, marketing, and production people specify pessimistic, most likely, and optimistic values for each of these variables. Then a series of project NPVs is calculated on the basis of setting each variable at its most pessimistic or optimistic value while holding all the other variables equal to their expected values. The purpose is to see how sensitive the project returns are to different cost and marketing assumptions.

[3] Lehner, U.C. "Chrysler Seen Spending $350 Million or More on Minivan, but Step Is Careful." *Wall Street Journal*, March 23, 1984, p. 6.

BOX 5.1

APPLICATION: CRYSTAL GLASS'S NEW PLATE GLASS PLANT

Crystal Glass Co. is thinking of building a plate glass plant in eastern Michigan. The plant, with an annual capacity of 100,000 tons, will cost $100 million to construct. The $100 million plant cost is regarded as pretty certain because it is guaranteed by fixed price contracts with the equipment manufacturer and the construction company. Profits on the project are assumed to be taxed at a combined federal and state rate of 50%.

Revenues. The glass plant is designed to capitalize on the rapidly growing market for high-quality plate glass within its sales territory. Total plate glass consumption in the plant's market area is currently running at 800,000 tons annually. This figure is expected to grow to 832,000 tons by next year, a 4% annual growth rate. Crystal Glass expects its new plant to produce about 90,000 tons of plate glass annually, an average of 90% of rated capacity. This would give it an approximate 11% market share in the first year of operation and annual revenues of $59.4 million. This revenue figure is based on an estimated $660 price per ton of glass sold by Crystal Glass, a price net of transportation charges.

Depending on market conditions, however, yearly production could rise to 100,000 tons or fall to 80,000 tons. Similarly, the price of plate glass could climb as high as $700 per ton or sink as low as $600 per ton.

Depreciation. For the sake of simplicity, the cost of the plant, $100 million, is assumed to be depreciated on a straight-line basis over 10 years, for an annual depreciation charge of $10 million.

Operating costs. Operating costs are divided into fixed costs and variable costs. Fixed costs include payroll costs, maintenance, repairs, supplies, and overhead expenses such as selling, general, and administrative costs. These costs are expected to average $12 million annually. Depending on current labor negotiations and other factors, however, these costs could be as low as $10 million or as high as $15 million. Payroll costs are treated as fixed costs here because the labor input does not vary with output.

Variable production costs—which include the costs of raw materials and power—are expected to average $140 per ton. Possible fluctuations in energy costs could reduce these costs to $110 a ton or raise them to as much as $170 a ton.

From these data, we can calculate the yearly cash flows for this project, along with its net present value and internal rate of return. The required return for this project is taken to be 15% because of the high degree of systematic risk associated with such a cyclical product as plate glass. As shown in the following table, the project NPV is $12.4 million. The term $PVIFA_{15,10}$ refers to the present value of an annuity of $1 for 10 years discounted at 15%. Its value is 5.0188.

Projected Cash Flows for Crystal Glass' New Plate Glass Plant

	Year 0	Years 1–10
Initial investment	–\$100,000,000	
Sales		
Tons sold		90,000
Price		× \$660
Revenue		\$59,400,000
Costs		
Variable costs (90,000 × \$140)		12,600,000
Fixed costs		
Depreciation		12,000,000
Total costs		10,000,000
Net income		34,600,000
Taxes (@ 50%)		\$24,800,000
After-tax income		12,400,000
Depreciation		\$12,400,000
Operating cash flow		10,000,000
Net cash flow		\$22,400,000
	–\$100,000,000	22,400,000

$$\text{Net present value (@ 15\%)} = -\$100{,}000{,}000 + PVIFA_{15,10} \times \$22{,}400{,}000$$
$$= \$12.4 \text{ million}$$

The following table shows what will happen to the NPV of Crystal Glass' proposed new plant when each variable is set, in turn, to its pessimistic and optimistic values, while simultaneously holding all other variables at their expected values. For example, if fixed costs rise to \$15 million annually (a jump of \$3 million from their expected value of \$12 million), then the plant's annual operating cash flow will decline by \$1.5 million (the after-tax increase in cost is half the before-tax increase because of the 50% tax rate) to \$20.9 million. The project net present value, using the same 15% discount rate as before, drops to

$$
\begin{aligned}
NPV &= -\$100{,}000{,}000 + PVIFA_{15,10} \times \$20{,}900{,}000 \\
&= -\$100{,}000{,}000 + 5.0188 \times \$20{,}900{,}000 \\
&= \$4{,}892{,}920
\end{aligned}
$$

The value of $5 million shown for this scenario in the following table is rounded to the nearest million.

Sensitivity Analysis of Crystal Glass' Proposal Plate Glass Plant

	Value for each variable under alternative scenarios			Project NPV under each scenario rounded to nearest million*	
Variable	Pessimistic	Expected	Optimistic	Pessimistic	Optimistic
Demand (tons)	80,000	90,000	100,000	0	25
Price per ton ($)	600	660	700	–1	21
Fixed cost ($)	15,000,000	12,000,000	10,000,000	5	17
Variable cost per ton ($)	170	140	110	6	19

*Discount rate is 15%.

The numbers from the sensitivity analysis in the table above indicate that the project is likely to be successful, even though its value is sensitive to changes in all the key variables. Only if the price drops to $600 a ton will the project have a negative NPV, and even then it will be just –$1 million.

But this analysis is somewhat misleading. It assumes that demand and price are independent of each other, a highly unlikely scenario. Typically, high demand and high prices go hand in hand, as do low demand and low prices. To see the effect of a low-demand, low-price scenario, rework the calculations to incorporate simultaneously, both a $600 per ton price and sales of 80,000 tons. This will produce a yearly operating cash flow of $17.4 million and a project NPV equal to –$12.7 million. Thus, the project turns out to be riskier than the simple sensitivity analysis indicates. We will explore further what happens when more than one variable is allowed to change at the same time in the section on simulation.

BREAK-EVEN ANALYSIS

In deciding whether to go ahead with a project, the real concern for investors is the possibility of losing money. Because production costs generally are fairly predictable, the key to making or losing money on most projects is the level of sales revenue. The major contributor to revenue uncertainty is uncertainty over the sales volume, Q. One approach to use in figuring this element of project risk is **break-even analysis**. This involves determining the quantity of sales, Q^*, at which the project NPV is just zero. If sales exceed Q^*, the project will have a positive NPV, whereas if sales are less than Q^*, the project NPV will be negative. The value of break-even analysis is that it is normally easier and requires less information to ascertain whether Q is less than or greater than Q^* than to decide on the absolute level of Q. For example, if $Q^* =$ 300,000, then it is unnecessary to spend time worrying whether actual sales will be, say, 375,000 or 400,000, because the result will not affect the decision (which is to accept the project).

Consider, for example, the Starship project described earlier. As you may recall, Beech's initial investment is expected to be $250 million and it plans on setting the price at $2.7 million apiece. Suppose the fixed costs of production are $15 million annually and the variable costs are $1.5 million per plane. At the time of the project, Beech was eligible to receive a 10% investment tax credit (ITC). The present value of the ITC and the accelerated depreciation write-offs taken together is assumed to be $120 million, based on an estimated 10-year project life and federal and state taxes of 50%. Hence, the initial investment net of the present value of the tax savings associated with the investment is $130 million. Exhibit 5.3 summarizes these data, and Exhibit 5.4 shows the cash flow estimates for the project and its net present values under various sales assumptions. The discount rate is set at 10%.

As expected, the NPV will be highly negative if Beech sells no Starships. With annual sales of 50 planes, the project NPV will be slightly positive and so will be acceptable; the NPV will be highly positive if annual sales are 75 planes. This suggests that the break-even sales point—the point at which the project NPV is just 0—is slightly fewer than 50 planes annually.

EXHIBIT 5.3 Summary of Data for the Starship Project

Initial investment: gross	$250,000,000
PV of investment tax benefits	120,000,000
Initial investment: net	$130,000,000
Price per plane	2,700,000
Variable cost per plane	1,500,000
Fixed costs	15,000,000

EXHIBIT 5.4 Break-even Analysis for the Starship ($ millions)

Annual Plane Sales	0	50	75
Revenue	0	135.0	202.5
Variable cost	0	75.0	112.5
Fixed cost	15.0	15.0	15.0
Net income*	−15.0	45.0	75.0
Taxes @ 50%	−7.5	22.5	37.5
After-tax income	−7.5	22.5	37.5
Present value @ 10%	−46.1	138.3	230.4
Initial investment	130.0	130.0	130.0
Project NPV @ 10%	−176.1	8.3	100.4

*Depreciation is already included in estimating the net investment required

The actual break-even sales quantity of 48 Starships annually is shown in Exhibit 5.5. It is the point at which the present value of future project cash flows just equals the \$130 million net initial investment. Although the cost figures presented here are just estimates, they illustrate the application of this powerful technique.

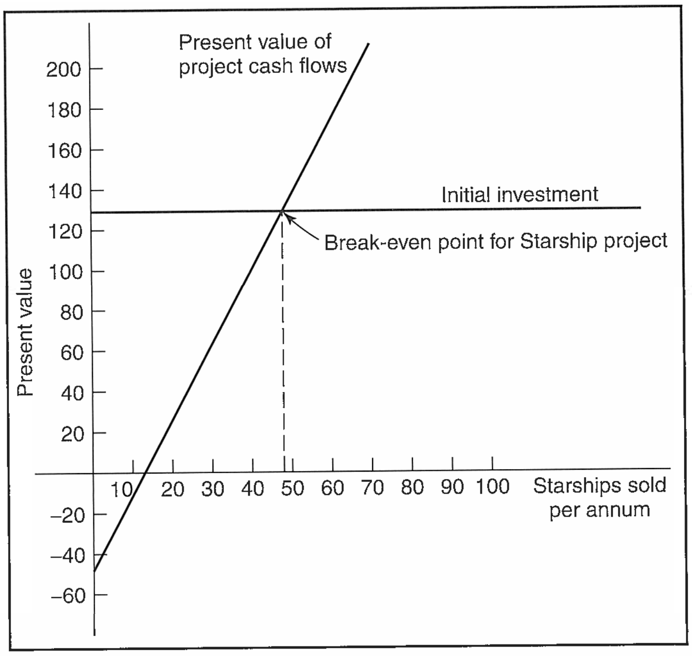
In general, the NPV of a project—assuming that annual fixed cash costs (which excludes depreciation), variable cash costs, prices, and quantities are constant—is:

$$NPV = -I_0 + D + [Q(P - V) - F](1 - t)\,PVIFA_{r,n} \quad (5.3)$$

where

I_0 = the initial investment
D = the present value of the depreciation write-off and any investment tax credits
Q = annual sales
P = unit sales price
V = unit variable cost
F = annual fixed cost
t = tax rate
n = project life
r = discount rate

EXHIBIT 5.5 Break-Even Analysis for Starship Project



We can find the break-even sales quantity by substituting Q^* for Q in Equation 5.3, setting NPV equal to 0, and solving for Q^*. This yields

$$Q^* = \frac{I_0 - D}{PVIFA_{r,n}(P - V)(1 - t)} + \frac{F}{P - V} \tag{5.4}$$

Applying Equation 5.4 to the various parameters for the Starship project, we see, as before, that the break-even level of annual Starship sales is

$$\begin{aligned} Q^* &= \frac{250 - 120}{6.1446 \times (2.7 - 1.5) \times 0.5} + \frac{15}{2.7 - 1.5} \\ &= 48 \end{aligned}$$

Of course, once the investment of $250 million has been made, and it becomes a sunk cost, the break-even point is lowered dramatically, to 13 (15/1.2 = 12.5) Starships annually.

Misuse of Break-Even Analysis

Some firms misuse break-even analysis by calculating the break-even point as that level of sales at which cumulative revenues just equal the sum of all development and production costs. This is known as the **accounting break-even point**. It can be converted to an annual basis by depreciating the initial investment on a straight-line basis and then finding the number of units that must be sold each year for the accounting profit to equal zero.

If we ignore the tax credit, this would mean an annual depreciation charge of $25 million for Beech's Starship project, given the initial project cost of $250 million and its 10-year life. With annual sales of Q planes, therefore, Beech's accounting profit (in millions) will be ($2.7 – $1.5)$Q$ – $15 – $25. Setting this profit figure equal to zero yields the accounting break-even point of 33 Starships per annum (40/1.2). At this level of sales, revenues will be just sufficient to cover all operating costs as well as recover the initial investment of $250 million. But unlike the NPV break-even analysis, the accounting break-even analysis makes no allowance for the *opportunity cost* of the funds tied up in the project. Hence, it is not surprising that the resulting break-even estimate of 33 is substantially below the NPV break-even estimate of 48 planes.

As a general rule, projects that break even on an accounting basis are sustaining an economic loss equal to the opportunity cost of the funds tied up in them. The case of Lockheed provides a dramatic illustration of the disastrous consequences that failure to heed this rule can have.[4] In 1971, Lockheed requested a federal guarantee for $250 million in additional bank debt to complete its L-1011 Tri Star program. A key issue during the resulting congressional debate was whether this program was economically viable. Lockheed's chief executive argued that sales would eventually reach or exceed the estimated break-even level of about 200 Tri Stars, making the program commercially

[4]This example is elaborated in Reinhardt, U.E. "Break-even Analysis for Lockheed's Tri Star: An Application of Financial Theory." *Journal of Finance*, September 1973, pp. 821–838.

sound. This break-even point, however, excluded the opportunity cost of the estimated $1 billion in project development and facilities costs. Had Lockheed allowed for this cost, its break-even sales level would have been closer to 500 aircraft, a number generally regarded as unattainable. This opportunity cost was irrelevant, of course, for that portion of the costs already incurred. This is in line with the proper focus on incremental cash flows pointed out in Chapter 3.

5.3 Simulation Analysis

Sensitivity analysis is a valuable technique for project risk analysis, and as such, it is widely used by business. Despite this, sensitivity analysis does have its limitations. Even if a project is very sensitive to changes in a particular variable, that variable may be very unlikely to change. Sensitivity analysis, however, ignores probabilities. In addition, it focuses on the effect of variations in one key parameter at a time instead of studying the effect on the value of the project of simultaneous changes in several key variables.

In contrast, computer simulation represents a project's NPV by a probability distribution rather than by a single number. In order to conduct a **simulation analysis**, you must first estimate probability distributions for each variable that will affect the project's cash inflows and outflows. For a new product introduction (e.g., Beech's Starship), these variables include the initial investment outlay (which determines not only the upfront cost but also the amount of depreciation recognized for tax purposes), market size, growth of the market, price, market share, variable costs, fixed costs, life of the project, and the project's terminal value. Some of these probability distributions, like the ones describing the initial investment cost and production costs, can be estimated with more confidence than can others, like the distributions for price and market share.

The next step is to program the computer to select at random one value apiece from each of these probability distributions. These values—for market size, market share, variable costs, and the like—are then used to calculate the net cash flow for each period. As each scenario is generated—a scenario being a particular set of values for the relevant project variables—the project NPV associated with that particular combination of parameter values is calculated and stored. This process is repeated, say, 600 times by the computer. The stored NPVs (all 600 of them) are then printed out by the computer in the form of a frequency distribution, along with the expected NPV and its standard deviation.

One caveat: *In calculating these NPVs, the cash flows should be discounted at the risk-free rate.* Using the opportunity cost of capital instead implicitly assumes that we already know what the project's risk is. And of course, if we knew that, we would not need to perform a simulation. Thus, assuming we are trying to evaluate project risk, we use the discount rate solely as a means to take into account the time value of money.

At each iteration, we could compute the project's IRR instead of its NPV. The result would be a frequency distribution for the project's rate of return. This frequency distribution could then be used to evaluate the expected return on the project and the standard deviation of that return. Many managers find this probability distribution more informative than the distribution of project NPVs.

To illustrate the simulation process, suppose the estimated probability distribution for the size of the executive aviation market is as follows:

Market size	200	250	300	350	400	450	500	550	600	650	700
Probability	0.02	0.05	0.08	0.10	0.15	0.20	0.15	0.10	0.08	0.05	0.02

The expected number of planes that will be sold is 450, but sales can range from 200 to 700 planes. To begin the simulation, the computer assigns numbers 1 to 100 to the different market sizes in proportion to their probabilities of occurrence. Specifically, numbers 1 and 2 are assigned to a market size of 200 planes, the next five digits to a market size of 250 planes, numbers 8 through 15 represent a market size of 300 planes, and so on. The computer's random number generator, which is like a roulette wheel with 100 slots, then selects a number at random from 1 to 100. Suppose it selects 37. This corresponds to a market size of 400 planes. Similarly, probability distributions and associated numbers are assigned to the other key variables.

The computer then picks values (listed in parentheses) for each of the other factors associated with the Starship, including market share (10%), price ($2.5 million), fixed costs ($12 million) and variable costs ($1.2 million), the initial investment outlay ($280 million), terminal value (0), and project life (10 years). On the basis of this particular scenario, annual sales are 40 Starships at a price of $2.5 million apiece. Thus, annual after-tax operating cash flows (excluding depreciation) are equal (in $ millions) to $40(2.5 - 1.2)0.5 - 12 = 14$. Discounting at the assumed risk-free rate of 6% yields a present value of $147 million over the 10-year life of the project. The initial investment net of tax credits and the present value of depreciation, discounted at 6%, is estimated at $140 million. For this scenario, therefore, the project NPV is $7 million.

PROBLEMS WITH SIMULATION ANALYSIS

Despite its appeal as a highly sophisticated, technologically advanced tool for evaluating project risk, simulation analysis is a controversial technique. Its difficulties are both theoretical and practical. Here are three potential problems.

Interdependencies

One major stumbling block to simulation analysis is that the more realistic the simulation is, the more complex it will turn out to be in practice. This is clearly shown in the case of interdependencies among the variables.

Our analysis so far has implicitly assumed that what happens in one period is independent of what happens in all the other periods. This is clearly unrealistic: A higher market share in one period is likely to mean better consumer acceptance and, therefore, a higher market share in subsequent periods. Similarly, lower-than-expected costs in one period will likely imply lower costs in the future.

This simulation process also assumes that within each period the variables are independent of one another. But as pointed out in the case of Crystal Glass' new glass plant, it is unrealistic to suppose that, for example, sales volume and sales price are unrelated to each other. In general, we would expect strong demand and high prices and weak demand and low prices to go together.

These interdependencies should be reflected in the simulation analysis. For example, if a high value for sales in the early years is selected, implying market acceptance, then higher sales volumes in subsequent years should also be selected. Similarly, within each period, higher prices should be used in conjunction with the higher sales volumes. Although specifying these interdependencies is difficult, ignoring them will render less valuable the resulting frequency distributions.

No Clear-Cut Decision Rule

The second problem is both practical and theoretical. After conducting a simulation analysis, you are left staring at a frequency distribution for either the project NPV or IRR. Should you accept or reject the project on the basis of this distribution? Simulation analysis gives no guidance in resolving what is ultimately the only important capital-budgeting issue—specifying an acceptable trade-off between project risk and return. By contrast, the NPV rule is quite specific: If the project NPV is positive when using the systematic risk-adjusted discount rate, accept the project; otherwise, reject it.

Disregards Diversification

The third basic problem is that the description of risk provided by a simulation analysis ignores the opportunities available to both the firm and its investors to diversify away a good portion of that risk. To the extent that the project returns are not highly correlated with returns on the rest of the firm's assets, the incremental risk of the project to the firm may be substantially lower than the project's total risk. Similarly, the less highly correlated the project's returns are with stock market returns, the less risky the project will be to highly diversified investors.

SURVEY OF RISK ASSESSMENT TECHNIQUES USED IN PRACTICE

The survey of corporate capital-budgeting practices by Graham and Harvey referenced in Chapter 2 also generated data on the risk assessment techniques used in practice.[5] These data show that about 53% of respondents used sensitivity analysis and 14% used simulation analysis to measure risk.

An earlier survey of risk assessment techniques used by companies was undertaken by Suk Kim, Trevor Crick, and Seung Kim.[6] These data are displayed in Exhibit 5.6. If the responses are taken at face value, 21% of the 367 firms surveyed ignore risk in evaluating projects. Another 52% assess risk subjectively, which could mean anything (see the application of Alaska Interstate for one such approach). These numbers indicate a major problem with such surveys: They are either unbelievable, or they raise more questions than they answer.

Consistent with the more recent survey by Graham and Harvey, the only risk assessment technique that is widely used is sensitivity analysis. Some companies use more than one technique, as is evidenced by the fact that the numbers in Exhibit 5.6 sum to more than 100%. Decision tree analysis is discussed in Section 5.5. The low incidence of quantitative technique usage could reflect skepticism over the value of these techniques. But more likely, they reflect a lack of faith in the data inputs that drive these methods.

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[5]Graham, J., and Harvey, C. "How Do CFOs Make Capital Budgeting and Capital Structure Decisions?" *Journal of Applied Corporate Finance*, Spring 2002, pp. 8–23.

[6]Suk, H.K, Crick, T., and Seung, H.K. "Do Executives Practice What Academics Preach?" *Management Accounting*, November 1986, pp. 49–52.

BOX 5.2

APPLICATION: ALASKA INTERSTATE

Diana Harrington described how one company, Alaska Interstate (AKI, its stock market symbol), a diversified company with holdings consisting of oil and gas ventures, a public utility, and an agricultural equipment manufacturer, tried to quantify risk and factor consideration of it into the strategic planning process.[7] AKI's 14-member top-management group first forecast the expected returns of each business unit by analyzing the market environment, competitive position, and available resources of each unit.

Next, AKI's top management turned to the question of expected risk. The group considered not only each unit's past history but also the various factors that could affect future returns, including the quality of management, the health of the industry, the unit's position in the industry, and the economic and political environments anticipated. Every one of the 14 officers estimated each unit's susceptibility to every factor identified, the importance of each factor, and—from these estimates—rated the risk of the unit on a scale of 1 to 10. The group aggregated these rankings to come up with an average projected risk for each unit.

AKI's risk assessment procedure, although subjective, included the best judgment of the most knowledgeable people. Moreover, the process of obtaining information in a systematic way forced managers to perform their evaluations with care. The end result was greater understanding by top management of the risks facing AKI and its business units. One outcome of this process was top management's decision to sell off all its nonoil and nongas businesses because the projected returns in those businesses did not compensate for their anticipated risks.

EXHIBIT 5.6 Risk Assessment Techniques Used by Respondents

Technique	Percentage
No method is used	21
Risk is determined subjectively	52
Standard deviation (variance)	4
Decision trees*	3
Computer simulations	7
Sensitivity analysis	27
Others	10

Source: Survey data from Suk H. Kim, Trevor Crick and Seung H. Kim. Do Executives Practice What Academies Preach? Management Accounting, November 1986, pp. 49–52.

*Decision trees are covered in Section 5.5.

[7]Harrington, D.R. "Stock Prices, Beta, and Strategic Planning." *Harvard Business Review*, May–June 1983, pp. 157–164.

5.4 Adjusting for Project Risk

So far we have seen how to assess the riskiness of a given project. This section discusses various methods of factoring risk into a capital-budgeting analysis. The principal means of incorporating project risk include adjusting the payback period, using a risk-adjusted discount rate, adjusting cash flows, and calculating certainty equivalents for the cash flows.

ADJUSTING THE PAYBACK PERIOD

Many firms treat risk in a subjective way instead of dealing with its impact on specific investments. This rather vague view of risk probably accounts for firms' frequent use of the somewhat haphazard requirement that riskier projects have shorter payback periods. For example, a project that is riskier than average may have a three-year payback requirement instead of the usual five-year requirement. But why three years? Why not four years or three and a half years? The arbitrariness of this procedure is evident.

Even in the absence of risk, we have already seen in Chapter 2 that payback is an inappropriate technique to use in investment analysis. Consequently, we shall not discuss this method further.

ADJUSTING THE DISCOUNT RATE

Another approach used to account for varying risks is to employ a risk-adjusted discount rate. This method is theoretically correct, but too often it is applied in an *ad hoc* manner. For example, a normal required return of 15% might be increased to 20% for a riskier project. Again, the question can be raised of why 20%? Why not 17% or 21.3%? In addition, when applying this method, decision makers often fail to distinguish between the project's total risk and the systematic component of that risk. Which is the relevant risk, and how should it be reflected in the discount rate? Chapter 6 shows how the CAPM can be used to supply the theoretically correct discount rate.

ADJUSTING CASH FLOWS

Most firms discount most-likely (modal) cash flows rather than expected (mean) cash flows. If a major new risk is anticipated, the mean value of the probability distribution of future cash flows will be significantly below its mode. Consider, for example, a proposed copper mining venture by Anaconda in Indonesia for which both the mean and mode of future cash flows are projected at $20 million annually. Suppose now that a change in government raises the possibility that the mine will be nationalized with no compensation paid to Anaconda. If the probability of nationalization is 25%, the mean value of future annual cash flows will decline to $15 million ($\$20 \times 0.75 + \$0 \times 0.25$); the modal value will remain at $20 million.

When faced with such a risk, many companies will apply one of the two methods discussed to the $20 million modal value, either shortening the payback period or raising the discount rate. Neither of these two approaches, however, lends itself to a careful evaluation of the actual impact of a particular risk on investment returns. Thorough risk analysis requires an assessment of the magnitude and timing of risks and their implications for the projected cash flows. In the case of the copper mine, the impact of the risk is to convert a $20 million expected cash flow into a $0 cash flow. Taking into account the likelihood of the risk's materializing reduces the expected cash flow to $15 million. In addition, both the payback and the risk-adjusted discount rate methods ignore the matter of timing. Nationalization five years hence is likely to be much less threatening than one expected next year, even though the probability of its occurring later may be higher.

In such a situation, using a uniformly higher discount rate just distorts the meaning of the present value of a project by penalizing future cash flows relatively more heavily than current ones; it does not avert the need for a careful risk evaluation. Instead, adjusting cash flows makes it possible to incorporate all available information about the impact of a specific risk on the future returns from an investment.

The method of cash flow adjustment requires that cash flows be adjusted to reflect the year-by-year expected effects of a given risk. Suppose, for instance, that nationalization of Anaconda's copper mine—should it occur—will not take place until year 4 at the earliest. If the venture is expected to have a seven-year life, then the expected year-by-year cash flows under this scenario will be

Year	1	2	3	4	5	6	7
Expected cash flow ($ millions)	20	20	20	15	15	15	15

If the additional risk being incorporated is systematic in nature, then the discount rate should also be adjusted to reflect the change in the project's systematic risk. This issue is discussed further in Chapter 6. Risks like nationalization, or most other project-specific risks—such as the possibility, when investing in new product development, that a competitor will come out with a similar product—are likely to be unsystematic in nature. Thus, when accounting for these risks, only the expected cash flows need be adjusted; there is no need to adjust the discount rate further.

USING CERTAINTY EQUIVALENTS

It is unlikely that management will be concerned solely with the systematic component of total risk—nor, as pointed out, should it be. An alternative approach is to use the certainty-equivalent method of Alexander Robichek and Stewart Myers, in which risk-adjusted cash flows are discounted at the risk-free rate r_f.[8]

The **certainty equivalent** of a risky cash flow is defined as that certain amount of money that the decision maker would just be willing to accept in lieu of the risky amount. For example, suppose a person would be willing to trade for $15,000, a lottery ticket having a 25% chance of winning $100,000 and a 75% chance of winning nothing (an expected value of $25,000). We would say that the certainty equivalent of this lottery ticket is $15,000.

This method is implemented by converting each expected cash flow into its certainty equivalent, by using a conversion factor that can range from 0 to 1.0. Specifically, the expected cash flow for period t, CF_t, is multiplied by the conversion factor for period t, a_t, and the resulting number, a_tCF_t, is the certainty-equivalent cash flow. The more certain the expected future cash flow is, the closer to 1.0 the value of a_t will be. Equivalently, a high a_t implies a more certain and, therefore, more valuable expected

[8]Robichek, A.A., and Myers, S.C. *Optimal Financing Decisions.* Prentice Hall: Englewood Cliffs, N.J., 1965.

cash flow. Less certain cash flows are valued less highly and, accordingly, have lower conversion factors.

The certainty-equivalent conversion factors are calculated as follows:

$$a_t = \frac{\text{Certain cash flow}}{\text{Expected cash flow}}$$

In the example of the lottery ticket, the conversion factor is 0.6 ($15,000/$25,000).

The certainty-equivalent cash flows, $a_t CF_t$, are then discounted at the risk-free rate of return to yield a certainty-equivalent net present value:

$$\text{Certainty-equivalent } NPV = -a_0 I_0 + \sum_{t=1}^{n} \frac{a_t CF_t}{(1 + r_f)^t}$$

where n is the expected life of the project and r_f is the risk-free rate.

To illustrate the application of the certainty-equivalent method, consider a project with a five-year life that has the expected cash flows and associated certainty-equivalent factors depicted in Exhibit 5.7. On the basis of a 9% risk-free rate of return, the calculations in Exhibit 5.7 show that the project has a net present value equal to $1,505.

The initial outlay is assumed to be known with certainty and so has a certainty-equivalent factor of 1.0. Subsequent cash flows, being risky, have certainty-equivalent factors of less than 1.0 but more than 0 because some return is expected; although they are risky, these returns have value. Note that the certainty-equivalent factors decrease over time. This is the typical pattern and reflects the greater risk associated with more distant cash flows. Despite this, the expected future cash flows are sufficiently large, relative to the initial investment of $15,000, so that the project is acceptable.

The certainty-equivalent method is considered by many financial theorists to be conceptually superior to the risk-adjusted discount rate method for several related reasons:

1. When valuing future cash flows, it is necessary to account for both the time value of money and risk. Adding a risk premium to the risk-free rate—which is what is done when using a risk-adjusted discount rate—may produce a present value factor that is not very meaningful. The certainty-equivalent method, on the other

EXHIBIT 5.7 Calculation of Certainty-Equivalent NPV

Year	Expected Cash Flow	Conversion Factor	Certainty-Equivalent Cash Flow	Present Value Factor @ 9%	Present Value
0	–15,000	1.0	–15,000	1.0	–5,000
1	8,000	0.8	6,400	0.9174	5,872
2	9,000	0.7	6,300	0.8417	5,303
3	6,000	0.6	3,600	0.7722	2,780
4	5,000	0.5	2,500	0.7084	1,771
5	3,000	0.4	1,200	0.6499	780
				Certainty-equivalent NPV =	1,505

hand, differentiates between the two distinct factors. It uses the discount rate to account for the time value of money and the certainty-equivalent factor to account for the riskiness of each individual cash flow.

2. The certainty-equivalent method allows each period's cash flow to be adjusted separately for its own degree of risk. It allows for different time patterns of risk, unlike the risk-adjusted discount rate method. Although the risk-adjusted discount rate method can also employ a time-varying risk premium, this procedure is considered to be more awkward.
3. With this method, decision makers can incorporate their own risk preferences directly in the analysis. This process increases the meaningfulness of the net present value number to the decision maker, even if this approach, by substituting managerial preferences for shareholder preferences, does not lead to shareholder wealth maximization.

Despite its conceptual superiority, however, the certainty-equivalent method is rarely used in practice. The main reason that it is not used is that no satisfactory procedure has yet been developed to generate the necessary certainty-equivalent factors. Moreover, it means losing some information about the valuation of future cash flows that is provided by shareholders in the form of their required yield on a typical firm investment.

SURVEY OF RISK ADJUSTMENT TECHNIQUES USED IN PRACTICE

The survey of corporate capital-budgeting practices by Kim, Crick, and Kim provide information on the risk adjustment techniques used in practice (unfortunately, the more recent survey by Graham and Harvey contains no information on this subject[9]). These data are summarized in Exhibit 5.8. The numbers in the table sum to more than 100%, indicating that some companies use more than one risk adjustment technique. Eighty-six percent of the 367 respondents claimed that their firms factored risk into the investment decision. But their methods for doing so vary.

EXHIBIT 5.8 Risk Adjustment Techniques Used by Respondents

Technique	Percentage
No adjustment is made	14
Adjustment is made subjectively	48
Certainty-equivalent method	7
Risk adjusted discount rate	29
Shortening payback period	7
Others	5

Source: Survey data from Suk. H. Kim, Trevor Cock, and Seung H. Kim, *Do* Executives Practice What Academics Preach? *Management Accounting,* November, 1986, pp. 49–52. Copyright IMA. Reprinted with permission from the Institute for Management Accountants, Montvale, N.J. www.imanet.org.

[9]Graham and Harvey point out that 51% of respondents said they would always or almost always use a risk-matched discount rate. However, it is not clear whether the risk adjustment is *ad hoc* or based on the CAPM.

It is unclear what "subjective adjustment" means, but this was the overwhelming favorite method. The risk-adjusted discount rate is also popular, but it is not clear whether the risk premium is subjectively assigned or is based on the CAPM. The odds are that the risk premium is decided on in an *ad hoc* manner. It is surprising that so few firms use payback to account for risk when so many use it as their primary or secondary capital-budgeting technique. Not surprisingly, given its difficulty in application, few respondents use the certainty-equivalent approach.

5.5 Decision Trees

Investment decisions are often not quite so cut-and-dried as the examples used so far would seem to indicate. Instead, as we saw in Chapter 4, many investment opportunities require a sequence of decisions through time, with each subsequent decision depending both on earlier decisions as well as the actual outcomes of those decisions. Consequently, what you plan to do today will often depend on what you plan to do in the future.

A useful aid in solving problems involving sequential decisions is to diagram the alternatives and their possible consequences. The resulting chart or graph is known as a **decision tree**, so called because it has the appearance of a tree with branches. A decision tree enables managers to visualize quickly the possible future events, their probabilities, and their financial consequences. It also helps in selecting the optimal sequence of decisions by facilitating the calculation of the NPVs associated with the alternative decision paths.

BOX 5.3

APPLICATION: PROCTER & GAMBLE'S NEW SOFT DRINK PLANT

To take a hypothetical example, suppose Procter & Gamble (P&G) has developed a new soft drink but is uncertain whether demand for the product will be high or low. P&G estimates that there is a 60% chance that demand will be high in the first year. High initial demand is a good omen; it raises the probability of high demand in the second and subsequent years to around 80%. On the other hand, if initial demand is low (with a 40% probability), there is a 70% chance that future demand will also be low.

Because of competitive pressures, P&G has decided to build a plant immediately to produce the drink rather than to spend additional time testing the product's market acceptance. It is uncertain, however, whether the plant should be large or small. If P&G constructs a large plant, at a cost of $15 million, it will keep the plant regardless of subsequent demand. Alternatively, P&G could build a smaller plant today (time 0) for $10 million, and then in the next year, if demand warrants, it could enlarge the plant at an additional cost of $7 million. (Thus, building a large plant at the outset is $2 million less expensive than building a small one initially and having to expand it later.)

The decision tree in Exhibit 5.9 on page 136 displays the choices available to P&G, along with their potential consequences. The squares represent decision points, and the circles represent chance events. After each decision has been made, fate selects the actual level of demand. This is represented by the branches of the decision tree. Each branch has its probability of occurrence written on it, along with the payoff associated with that combination of plant size and demand.

The sequence of events depicted by the tree is as follows: P&G builds a plant at time 0, after which fate selects either a high or a low demand. Each of the four possible combinations of plant size and demand level during that first year generates a different cash flow. At time 1 (the end of year 1), the small plant can be expanded (if it were the time 0 choice). Unfortunately, the permanent level of demand is still unknown at this point, although there exists somewhat less uncertainty about what it will be. Thus, in year 2, fate once more selects a high or low level of demand. The resulting payoff again depends on the particular demand-plant size combination involved. It is clear that the demand probabilities in year 2 depend on actual demand in year 1. For example, the probability of a high demand in year 2, given a high demand in year 1, is 0.8, but only 0.3 if year 1 demand was low. The chance of high demand in both years is $0.6 \times 0.8 = 0.48$.

Note that there are two points in time at which P&G can make decisions, now (time 0) and next year (time 1). The first decision is whether to build a large or a small plant. If the large plant is built, P&G's course is set—it can only hope that demand is high and remains high. If the time 0 decision is to build the small plant, P&G has another decision to make at time 1: Should it expand the small plant or should it sit tight? This decision depends on what the first-year demand turns out to be.

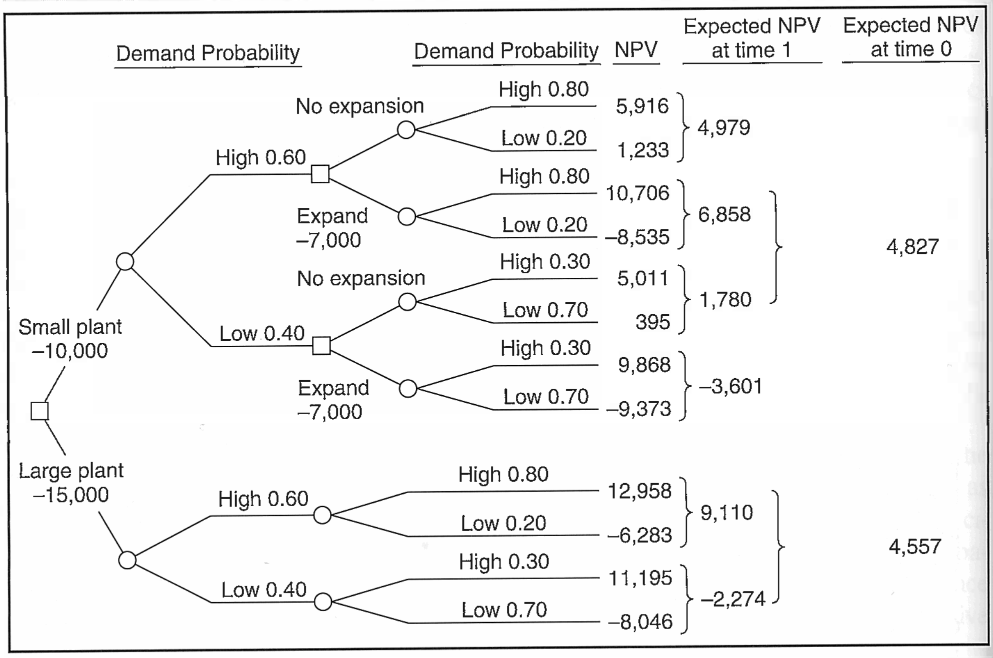
The problem confronting P&G today is whether to build a small or a large plant now. One solution procedure is to make the more distant decision first—the choice of whether P&G should expand its small plant next year—and then work backward to the present. This requires that we determine the expected net present value of expansion, given that the first-year demand turns out to be high or low. According to the numbers provided in Exhibit 5.9 on page 136, expansion when demand is low has an expected NPV of –$3,601,000. This is clearly dominated by the expected NPV of $1,780,000 for the no-expansion choice.

Expansion subsequent to high demand in year 1 has an expected NPV of $6,858,000. Alternatively, the no-expansion decision has an expected NPV of $4,979,000. Hence, the decision rule at time 1 is clear: If year 1 demand is high, expand; if it is low, do not expand. Now that we know the optimal decisions at time 1, conditional on year 1 demand, we can determine P&G's optimal decision at time 0.

The expected NPV at time 0 for a small plant, given that time 1 decisions are made optimally, is $4,827,000. If year 1 demand is low, we use the expected NPV associated with maintaining the small plant. If year 1 demand is high, we use the expected NPV associated with expansion at time 1. By contrast, the expected NPV associated with building the large plant initially is $4,557,000.

Thus, the expected NPV for building a small plant initially exceeds that for building a large plant by $270,000. But if there were no option to expand, the expected NPV of the small plant would be only $3,699,400 ($0.6 \times 4{,}979{,}000 + 0.4 \times 1{,}780{,}000$), and it would be optimal to build the large plant. This means that the option to expand, once the small plant is built, is worth $1,127,600 ($4,827,000 – $3,699,000). Before any investment, the option to expand has an expected NPV of $270,000, the difference between the expected NPVs of the optimal decisions with and without the expansion option ($4,827,000 – $4,557,000).

EXHIBIT 5.9 Decision Tree Analysis for P&G's Soft Drink Plant



5.6 Summary and Conclusions

Uncertainty of investment cash flows is an all-pervasive element in the capital-budgeting process, but it affects differently the riskiness of the project, the firm, and well-diversified investors. In this chapter, therefore, we identified three different types of risk associated with a project: (1) total project risk, which is based on the variability of project returns, (2) firm risk, which is measured by the contribution of project risk to the variability of total firm returns, and (3) systematic risk, which is based on the project's beta coefficient, as measured by the correlation between project returns and returns on the market portfolio. Although type 1 and type 2 risks should not affect the required return on a project, we have seen that these risks could affect project revenues and costs and, hence, alter actual project returns. Thus, it makes sense to incorporate total risk as well as systematic risk into capital-budgeting analyses.

This chapter described some of the methods that firms use to factor risk into their project analyses. These methods include adjusting the discount rate, adjusting project cash flows, and using certainty-equivalent cash flows. The end result of these analyses is the risk-adjusted project net present value. We have already seen that if calculated correctly, this number is sufficient to describe the relative desirability of each project.

Rather than settle for just one number, however, many firms also perform sensitivity analyses on project cash flows to appraise the effect of varying the values of certain key parameters on the project NPV, including construction costs, R&D costs, market size and share, price, and production costs. In addition, the advent of low-cost, high-powered computers and computer modeling techniques has encouraged a number of firms to simulate the effects of changing several of these parameters simultaneously, while taking into account the probability of these changes actually occurring. The output from a simulation analysis is a probability distribution of project NPVs. Finally, we considered the evaluation of projects in which a sequence of decisions must be made at different points in time, with each subsequent decision being affected by earlier decisions and the outcomes of those decisions. A useful aid in such situations, which is known as decision tree analysis, is to graph the alternatives and their possible consequences.

Although none of these techniques for assessing project risk and incorporating risk in a project analysis is perfect, some useful information may be gained from each. Employing several techniques in combination is likely to prove the most fruitful approach in practice.

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Sample Problems

1. Calculate the NPV of an investment project with the following characteristics:

   Units sold per year: 55,000

   Price per unit: $800

   Variable cost per unit: $720

   Fixed cost: 0

   Initial cost: $20 million

   Life of project: 10 years

   Discount rate: 10%

   Depreciation: straight-line

   Tax rate: 34%

   Answer: $NPV = -PV(\text{cost}) + PV(\text{depreciation tax shield}) + PV(\text{operating cash flows})$

   $PV(\text{cost}) = \$20{,}000{,}000$

   With a depreciable life of 10 years, annual depreciation is $2,000,000. This results in an annual depreciation tax shield of 2,000,000 × 0.34 = $680,000. Hence,

   $$PV(\text{depreciation tax shield}) = \$680{,}000 \times PVIFA_{10,10}$$
   $$= 680{,}000 \times 6.1446 = \$4{,}178{,}328$$

PV(operating cash flows) = (Price – Cost)(Units)(1 – tax rate)$PVIFA_{10,10}$
= (800 – 720)(55,000)(0.66)(6.1446) = $17,843,918
NPV = –$20,000,000 + 4,178,328 + 17,843,918 = $2,022,246

a. Suppose an additional investment of $5 million would reduce the variable cost per unit to $700. Calculate the NPV of this alternative.
Answer: PV(cost) = $25,000,000 (25% more than above)
PV(depreciation tax shield) = $5,222,882 (25% more)
PV(operating cash flows) = (800 – 700)(55,000)(0.66)(6.1446)
= $22,304,898
NPV = –25,000,000 + 5,222,882 + 22,304,898
= $2,527,780

b. What is the break-even (NPV) number of units for the two alternatives?
Answer: Break-even occurs when
PV(operating cash flows) = PV(cost) – PV(depreciation tax shield)
For the prior case,
(800 – 720)(X)(0.66)(6.1446) = 15,821,694.37
Break-even X = 48,767 units
For part (a),
(800 – 700)(X)(0.66)(6.1446) = 19,777,117.96
Break-even X = 48,767 units

The break-even quantities are identical. However, for any sales beyond 48,767 units, profits rise faster once the additional investment has been made. For sales less than break-even, losses mount more quickly if the investment is undertaken.

2. Multifoods, a retail grocery chain, is considering entering the fast-growing bulk food retail area. Although Multifoods is uncertain as to what its costs and revenues will be, it estimates that each of the relevant project parameters can take on only one of two possible values. Given below are the best estimates of the project's four parameters and their possible values over its five-year life:

Revenue/year	$100,000	$125,000
Fixed cost/year	20,000	15,000
Variable cost/year	10,000	5,000
Depreciation/year	10,000	10,000

Each parameter value has a 50% probability of being selected, and the selection of one parameter value is independent of the selection of any of the other parameter values.

a. Construct a probability distribution of the NPV for the project. Multifoods has a tax rate of 35% and a cost of capital of 12%. The initial investment is $150,000. (*Hint*: Since each of the four parameters can take on either of two values with probability 0.5, there are eight possible scenarios to examine.)

Answer: The eight (equally likely) scenarios are reported below:

Scenario	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Revenue	100.00	100.00	100.00	100.00	125.00	125.00	125.00	125.00
– Fixed cost	20.00	15.00	20.00	15.00	20.00	15.00	20.00	15.00
– Variable cost	10.00	10.00	5.00	5.00	10.00	10.00	5.00	5.00
– Depreciation	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
= Taxable income	60.00	65.00	65.00	70.00	85.00	90.00	90.00	95.00
– Tax	21.00	22.75	22.75	24.50	29.75	31.50	31.50	33.25
= After-tax income	39.00	42.25	42.25	45.50	55.25	58.50	58.50	61.75
+ Depreciation	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
= Cash flow	49.00	52.25	52.25	55.50	65.25	68.50	68.50	71.75
$\times$ $PVIFA_{12,5}$ =	176.63	188.34	188.34	200.06	235.21	246.92	246.92	258.64
– Investment	150.00	150.00	150.00	150.00	150.00	150.00	150.00	150.00
= *NPV*	26.63	38.35	38.35	50.07	85.21	96.93	96.93	108.64

The expected NPV of Multifoods' project, given these eight scenarios with equal probability of being realized, equals 67.64, with a standard deviation of 30.44.

b. If Multifoods prepared estimates of five alternative values for each parameter, how many possible outcomes would have to be considered?
Answer: The number of alternative scenarios would equal $5 \times 5 \times 5 = 125$.

QUESTIONS

1. Comment on the following statements:
   a. "Because our new expansion project has the same systematic risk as the firm as a whole, we need do no further risk analysis on the project."
   b. "Our company should accept the new potash mine project at Moosejaw. The cost of additional loans to fund the project is 12%, and our simulations lead us to expect a 14% return from the project."
   c. "It is difficult to decide whether to spend $10 million to reopen our mine because the price of gold is so uncertain. However, if we assume the price of gold grows at an average of 5% a year with a standard deviation of 20% a year, simulation indicates the mine has an average NPV of $500,000. Therefore, we should reopen."
2. Assess the impact of the following events on a firm's operating leverage:
   a. an increase in output price due to increased demand.
   b. a decrease in fixed cost.
   c. negotiation of a new contract with suppliers leading to higher commitments to purchase raw materials.
   d. lowered variable labor costs per unit of output.
   e. installation of new machine tools that lower variable production costs per unit of output.
3. Consider two firms, one American and the other Japanese, using identical production processes; that is, they use the same equipment and hire the same number of workers. However, the Japanese firm follows a no-layoff policy, whereas the American firm is willing to alter its work force in line with changing market conditions. Which company will have the larger amount of operating leverage? Why? How will the difference in amounts of operating leverage affect their marketing and production decisions and strategies?

4. The CAPM and the APT argue that only systematic risk matters; risk that is diversifiable is irrelevant to the well-diversified investor. Yet this chapter has argued that total risk matters, not just systematic risk. Is there an inconsistency here? Explain.
5. What is the advantage of using certainty-equivalent cash flows instead of risk-adjusted discount rates to calculate the NPV of an investment project?

PROBLEMS

1. Suppose that Bethlehem Steel has a current sales level of $2.5 billion, variable costs of $2 billion, and fixed costs of $400 million. If sales rise by 15%, how much will pre-tax profit increase in dollar terms? What will be the percentage increase in pre-tax profit? What explains the relationship between the percentage change in sales and the percentage change in pre-tax profit for Bethlehem?
2. In early 1990, Boeing Co. decided to gamble $4 billion to build a new long-distance, 350-seat wide-body airplane called the Boeing 777. The price tag for the 777, scheduled for delivery beginning in 1995, is about $120 million apiece. Assume that Boeing's $4 billion investment is made at the rate of $800 million a year for the years 1990 through 1994 and that the present value of the tax write-off associated with these costs is $750 million. On the basis of estimated annual fixed costs of $100 million, variable production costs of $90 million apiece, a marginal corporate tax rate of 34% and a discount rate of 14%, what is the break-even quantity of annual unit sales over the Boeing 777's projected 15-year life? Assume that all cash inflows and outflows occur at the end of the year.
3. The recently opened Grand Hyatt Wailea Resort and Spa on Maui cost $600 million, about $800,000 per room, to build. Daily operating expenses average $135 a room if occupied and $80 a room if unoccupied (much of the labor cost of running a hotel is fixed). At an average room rate of $500 a night, a marginal tax rate of 40%, and a cost of capital of 11%, what year-round occupancy rate do the Japanese investors who financed the Grand Hyatt Wailea require to break even in economic terms on their investment over its estimated 40-year life? What is the likelihood that this investment will have a positive NPV? Assume that the $450 million expense of building the hotel can be written off straight line over a 30-year period (the other $150 million is for the land which is not depreciable) and that the present value of the hotel's terminal value will be $200 million.
4. Conduct a sensitivity analysis for a project with the following characteristics. Each parameter can take on any of three different values but once a parameter value is selected, that value remains constant for the 10-year period. The discount rate is 10% and the project life is 10 years. Ignore taxes and depreciation.

	Low	Mean	High
(1) Sales (units)	160	500	960
(2) Price (per unit)	$3,000	$3,750	$4,000
(3) Variable cost (per unit)	3,000	3,000	3,000
(4) Fixed cost	100,000	200,000	4,000
(5) Initial investment	1,000,000	2,000,000	4,000,000

5. American Fruit Co. is considering constructing a new plant to process frozen fruit juices. One plant would be capital intensive, the other much more labor intensive. Although the final decision would hinge on the relative cost of capital versus labor in the northern

California area, management is curious about the behavior of the plants' return on assets during a typical business cycle.

a. Given the following information, calculate the break-even point in units of production for the two plants.
b. The economics department has prepared sales projections for three business scenarios: recession, normal, and recovery. Sales under each scenario are expected to be as follows: recession, 300,000 units; normal, 500,000 units; and recovery, 800,000 units. Calculate the return on assets for the two plants under these three scenarios.
c. If the three scenarios are all equally likely, what will be the variance of the return on assets for plant 1? For plant 2? What would you advise American Fruit?

	Plant 1	Plant 2
Fixed cost	$200,000	$600,000
Variable cost (per unit)	1.50	.50
Price (per unit)	2.00	2.00
Investment	1,000,000	1,000,000

6. For the following project, the chief financial officer has prepared a set of certainty-equivalent factors to adjust the cash flows for the estimated risk. The economics department has also prepared a set of risk-adjusted interest rates at which to discount the project's cash flow. The project's initial investment is $150,000 and the Treasury security rate is 8%.
   a. What is the NPV of the project from the finance department's estimates?
   b. What is the NPV from the economics department's estimates?
   c. What would you advise the company to do?

Year	1	2	3
Cash flows ($000)	$50	$75	$130
Certainty equivalents (finance department)	0.982	0.964	0.947
Risk-adjusted rates (economics department)	10%	12%	14%

7. A gold mine is considering replacement of some machinery. The new conveyor belt will cost $5 million and lower the cost of removing ore from the mine by $4 per ton. The old belt can be scrapped for $500,000. The following table shows that the life of the new machine is uncertain, as is the annual amount of ore that will be moved:

	Low	Mean	High
Tons per year	200,000	250,000	350,000
Life of new machine	6 years	9 years	13 years

Conduct a sensitivity analysis of the NPV of the replacement project assuming a discount rate of 10%. Ignore taxes.

8. Teletech Co. wants to use a decision tree in evaluating a venture capital investment in cable TV. The projected investment has a life of three years, and the associated after-tax cash flows ($000) and probabilities are as follows:

Year 1	Year 2	Year 3
Cash flow: $100 $P = 0.50$ $200 $P = 0.50$	If cash flow in year 1 = $100 Year 2 cash flow = $120 $P = 0.60$ = $9 $P = 0.40$ If cash flow in year 1 = $200 Year 2 cash flow = $250 $P = 0.50$ = $210 $P = 0.50$	If cash flow in year 2 = $120, can sell the investment for either $350 ($P = 0.70$) or $250. If cash flow in year 2 = $95, can sell the investment for either $125 ($P = 0.60$) or $75. If cash flow in year 2 = $250, can sell the investment for either $475 ($P = 0.80$) or $275. If cash flow in year 2 = $210, can sell the investment for either $140 ($P = 0.50$) or $110.

The initial investment for the firm is $500,000 after tax. The firm uses a cost of capital of 10%.

a. Construct a decision tree with the expected NPV of each alternative.
b. What is the expected NPV of the best possible outcome? What is its probability?
c. What is the expected NPV of the worst possible outcome? What is its probability?
d. Should Teletech make the investment? Why or why not?

9. Refer to the Starship project in Section 5.2.

a. What would the break-even quantity be initially if the cost of capital for the project were estimated at 14% rather than 10%?
b. What would the break-even quantity be if the Starship could be sold for only $2,000,000 each (assume a cost of capital of 10%)?
c. What would the break-even quantity be if cost overruns increased the initial investment from $130,000,000 to $230,000,000?

10. The following exhibit contains Beech's estimates of demand, price, and fixed and variable costs for the Starship under three alternative economic forecasts.

Variable (per year)	Pessimistic	Normal	Optimistic
Demand	50	75	125
Price*	2	2.7	3.2
Fixed cost*	22	15	7
Variable cost*	1.75	1.50	1.0

*Dollars in millions

a. If all other variables are assumed to be at their expected value (normal forecast), how sensitive is the project's NPV to changes in fixed cost? Use a cost of capital of 10%, tax rate of 35%, and project life of 10 years.
b. How sensitive is the project's NPV to changes in price?
c. How sensitive is the project's NPV to changes in variable cost?
d. Which factor seems most important to the success of the plane?
e. Is the Starship a risky project? Explain.