Exhaustible resources

Lecture 9

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Definitions

- Re-producible goods
  - bicycles, software, services,
  - Total supply over time is unlimited)
- Renewable goods (resources)
  - Timber, fish, hydro electricity, agricultural commodities
  - Resource base is finite but the overall output is in principle unlimited)
- Non-renewable or exhaustible goods (resources)
  - Industrial metals: copper, nickel, aluminum
  - Fossil fuels: oil, coal, gas
  - Resource base and the output are both finite

Definitions

For exhaustible goods, overall consumption is finite

- Consumption today reduces what is available for future
- Two types of costs in extraction: (i) out-of-pocket marginal cost of production; (ii) opportunity cost of production (scarcity rent)
- Demand and supply meet at a different point in every period, depending the on the resource stock left

Equilibrium conditions

1. price p_t is equal to marginal cost of production MC plus the opportunity cost of production R_t (what would be achieved by leaving the last unit for tomorrow instead)

$$p_t = MC_t + R_t$$

2. the opportunity cost should grow at the rate of interest r

$$R_t = R_{t+1}/(1+r)$$

3. production over time is equal to the total resource endowment S

$$q_1 + ...q_T = S$$

Consumption, scarcity rent, and prices

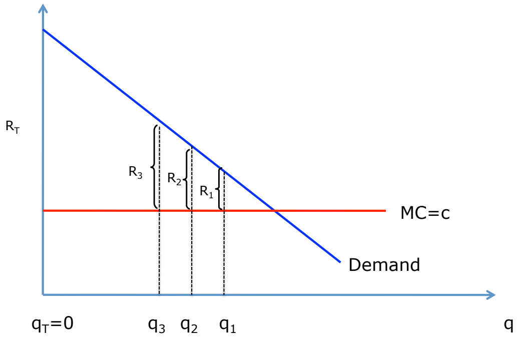

Figure: Observe: consumption declines, scarcity rent increases, producer receives a resource rent (despite constant MC), consumption ceases when the choke price is reached

A two-period resource-extraction model

A simple framework for developing the ”Hotelling rule”

- preferences

$$w = u_1(c_1) + \delta u_2(c_2)$$

  - u_t, c_t utility and consumption, resp.
  - discount factor captures the time preference, $0 < \delta \leqslant 1$
  - w is the welfare to be maximized

A two-period model

- budgets, technologies

$$c_1 + k_2 = f_1(k_1, z_1)$$
$$c_2 = f_2(k_2, z_2)$$
$$z_1 + z_2 = s_1$$

  - $P_1 = \frac{\partial f_1}{\partial z_1} > 0$, resource price $t = 1$
  - $P_2 = \frac{\partial f_2}{\partial z} > 0$, resource price $t = 2$
  - $R_{1,2} \equiv \frac{\partial f_2}{\partial k}$, capital return
  - $MRS_{1,2} \equiv \frac{u_1'}{\delta u_2'}$, marginal rate of substitution

A two-period model

- welfare:

$$w(k_1, s_1) = \max_{k_2, z_1} [u_1(f_1(k_1, z_1) - k_2) + \delta u_2(f_2(k_2, s_1 - z_1))]$$

Savings:

$$-u_1'(.) + \delta u_2'(f_2(k_2, z)) \frac{\partial f_2(.)}{\partial k} = 0 \quad (1)$$

$$\Rightarrow 1 = \frac{R_{1,2}}{MRS_{1,2}}$$

A two-period model

resource:

$$u_1'(.)\frac{\partial f_1(.)}{\partial z_1} + \delta u_2'(f_2(k_2, z))\frac{\partial f_2(.)}{\partial z_2}(-1) = 0 \qquad (2)$$

$$\Rightarrow MRS_{1,2} = \frac{\frac{\partial f_2(.)}{\partial z_2}}{\frac{\partial f_1(.)}{\partial z_1}}$$

A two-period model

Combining (1)-(2), and using definitions:

The general equilibrium Hotelling rule for resource prices:

$$P_1 = \frac{P_2}{R_{1,2}}$$

Recall $R_{1,2} = 1 + r_1$, where r_1 is the interest rate at period 1. Thus, resource prices grow at the rate of interest, and the interest rate is determined by the fundamentals of the economy:

- technologies: f_1 and f_2
- preferences: u_1, u_2, and δ

Note: Hotelling (1931) did not have general-equilibrium; for this, see Dasgupta and Heal (1979), and, e.g., Golosov et al. (2014). See references in the end.

Resource prices over long period

Generalizing the previous:

Current prices are linked to far-future prices:

$$P_1(1+r_1)(1+r_2)...(1+r_n) = P_{n+1}$$
$$\Rightarrow P_1 = \frac{P_{n+1}}{\prod_{i=1}^{n}(1+r_i)}$$

Note that the current price is an extremely forward-looking variable. Can we trust that the markets have such a long planning horizon?

- ”wrong” expectations (See Jovanovich, 2013)
- slow learning

Digression: continuous-time version of the Hotelling rule

There is a tradition of modeling the resource-use and price in continuous time (for a survey, see Gaudet (2007)). Suppose interest rate $r > 0$ is constant over time, and that there is a constant cost $c > 0$ per unit of resource produced. The Hotelling rule is an arbitrage condition for the sellers.
Sell a unit today:

$$P_t - c$$

Sell next period, after $\Delta > 0$ units of time:

$$[P_{t+\Delta} - c]e^{-\Delta r}$$

Continuous-time Hotelling

Indifference:

$$
\begin{aligned}
P_t - c &= [P_{t+\Delta} - c]e^{-\Delta r} \\
&\approx [P_{t+\Delta} - c](1 - \Delta r) \\
&\Rightarrow \\
\frac{P_{t+\Delta} - P_t}{\Delta} &= r(P_{t+\Delta} - c) \\
&\Rightarrow \\
\lim_{\Delta \to 0} = \frac{dP_t}{dt} &= r(P_t - c)
\end{aligned}
$$

This is a differential equation with solution

$$P_t = c + [P_0 - c]e^{rt}$$

We will discuss how to use this analyze the resource-extraction over time.

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