Error formula : n-point rule (2n-1 degree)

interpolation error $= \dfrac{f^{(2n)}(\xi(x))}{(2n)!} \prod_{k=1}^{n} (x-x_k)^2$

quadrature error $= \dfrac{f^{(2n)}(\xi(x))}{(2n)!} (P_n(x), P_n(x))$ ⊛

Where does that square come from ?

We assume that the derivatives of f are continuous, therefore Hermite interpolation is the natural choice.

⊛ x_k are the roots of $P_n(x)$, so the product is $[P_n(x)]^2$

Quadrature error: $\int f(x)dx - Q(f) = \dfrac{f^{(2n)}(\xi(x))}{(2n)!} \int P_n(x) P_n(x) dx$

EXAMPLE Gauss rule : $[-1,1]$; $n=1$

Notice : Since we only want the roots, there is no need to normalise.

GS : $\tilde{q}_0 = 1$

$$\tilde{q}_1 = x \cdot 1 - \frac{\langle x, 1\rangle}{\langle 1, 1\rangle} \cdot 1$$

$$= x - \left[\left(\int_{-1}^{1} x\,dx\right) \Big/ \left(\int_{-1}^{1} 1\,dx\right)\right] \cdot 1$$

$$= x$$