C Level Questions
1.   Imagine that the production of fishing lures is governed by the production function:
$$y = L^{.7}$$
where y represents the number of lures created per hour and L represents the number of workers employed per hour.
a. Does this production function exhibit constant, decreasing, or increasing returns to labor? How can you tell?
This production function exhibits decreasing returns to labor. The first worker hired produces 1 unit of output while the second worker hired produces .62 units of output. By definition, this is simply saying that the first derivative of the production function with respect to labor is positive (the more workers hired, the more output) and the second derivative is negative (as more workers are hired the additional output produced is less). In fact, this is the case:
$$\frac{dy}{dL} = .7L^{-.3} > 0$$
$$\frac{dy^2}{d^2L} = -.21L^{-1.3} < 0$$

b. Imagine that each fishing lure sells for $24 and that each worker is paid $6 per hour. How much labor is hired by this firm? What is the total output produced by this firm?
This firm maximizes its profits when its MPL is equal to the real wage. In this case the real wage is .25 and the MPL $= .7L^{-.3}$. Setting these equal and solving for L gives L $= 30.94$ and substituting this value for L back into the production function gives Y $= 11.05$.

c. Imagine now that the government charges a tax on wages of 10% (similar to the structure of the US social security taxes paid by employers). With this new tax in place, how much labor is hired by this firm? What is the total output produced by this firm? What is the effect on employment given an increase in wage taxes?
Profits $= 24 \times L^{.7}$ - $6 \times L \times (1+\tau)$ where $\tau = .10$. Maximizing this profit function gives:
$24 \times .7 \times L^{-.3}$ - $6(1+\tau) = 0$ or $L^{-.3} = 16.8/(6 \times (1+\tau))$. When $\tau = .10$, L $= 22.51$. A 10% labor tax decreases labor hired from 30.94 to 22.51 units (a 27.2% decrease from the original level of labor hired). Output will fall to $22.51^{.7} = 8.846$ from 11.05 (or a fall of 19.9%).

2.   Consider an economy described by the following equations:
        Y = 5,000   G = 1,000
        T = 1,000   C = 250 + .75(Y-T)
        I = 1,000 – 50r
a. In this economy, compute private savings, public savings, and national savings.
National Savings $= Y – C – G = 5000 – (250 + .75(5000-1000)) – 1000 = 750$
Private Savings $= Y – T – C = 5000 – 1000 – (250 + .75(5000-1000)) = 750$
Public Savings $= T – G = 1000 – 1000 = 0$.

b. Find the equilibrium interest rate (r).
$750 = 1000 – 50r$ thus $50r = 250$ and r $=5$.

c. Now suppose that G rises to 1,250. Compute private, public, and national savings. What is the new equilibrium interest rate?
National Savings $= Y – C – G = 5000 – (250 + .75(5000-1000)) – 1250 = 500$

Private Savings = Y – T – C = 5000 –1000 – (250 + .75(5000-1000)) = 750
Public Savings = T – G = 1000 – 1250 = -250.
500 = 1000 – 50r thus r = 10.

d. Now suppose that the government increases taxes and government purchases by equal amounts (both G and T equal 1,250). What happens to the interest rate and investment in response to this balanced budget change? Does your answer depend on the marginal propensity to consume? Why?
National Savings = Y – C – G = 5000 – (250 + .75(5000-1250)) – 1250 = 687.5
Private Savings = Y – T – C = 5000 –1000 – (250 + .75(5000-1250)) = 687.5
Public Savings = T – G = 1250 – 1250 = 0.
687.5 = 1000 – 50r thus r = 6.25
Yes, my answer does depend upon the MPC because an increase in taxes reduces my consumption by an amount dependent upon the MPC (which in turn increases savings).

B Level Questions
3.   Use the production function $Y = K^{.3}L^{.7}$ to answer the following questions:
a. Solve for the equilibrium real wage for an economy with this production function. (Your answer will be a function of K and L).
In class, we discussed the relationship between the real wage and the marginal productivity of labor; as a matter of fact, we claimed that the real wage must equal the marginal productivity of labor. The marginal productivity of labor is:

$$MPL = \frac{\partial Y}{\partial L} = .7K^{.3}L^{-.3} = \frac{W}{P}$$

b. Solve for the equilibrium real rental rate of capital for an economy with this production function.
The rental wage of capital occurs when the real payments to capital owners equals the marginal productivity of capital. Again:

$$MPK = \frac{\partial Y}{\partial K} = .3K^{-.7}L^{.7} = \frac{r}{P}.$$

c. What fraction of total income do workers receive in this economy? Capital owners?
Workers receive 70% of income and capital owners receive 30% of income. (See appendix to Chapter 3). This is fairly easy to see once you realize that each worker is paid the MPL and, given L workers, then total payments to workers must be MPL×L = $.7K^{.3}L^{-.3}$×L = $.7K^{.3}L^{.7}$ = .7Y. Total payments to workers amount to 70% of total output. The same technique works for payments to capital owners.

d. Suppose that immigration raises the labor force by 10%. What happens to total output (in percent)? What happens to the real rental rate of capital (in percent)? What happens to the real wage (in percent)? Carefully explain why the changes in the wage and rental rate of capital move in the direction you find.
Output prior to the increase was $Y = K^{.3}L^{.7}$. Output after was $Y = K^{.3}(1.1L)^{.7}$. The percentage change is therefore $K^{.3}(1.1L)^{.7}/ K^{.3}L^{.7}$ = 1.068 so output grew by 6.8%.

By the same process, the rental rate of capital also grows by 6.8%.

The real wage declines by $.7K^{.3}(1.1L)^{-.3}/.7K^{.3}L^{-.3}$ = .9718/1 so the real wage fell by 2.81%

4.   Consider an economy described by the following equations:

$$Y = 4000 \qquad C = 100 + .9(Y - T) \qquad G = 500 \qquad T = 500$$

In our society, a number of different types of investment occur. For instance, residential investment, inventory investment, and business investment are all different types of uses for real goods. Consider a society with two types of investment: residential and business. An equation for each are given below:

$$I_r = 50 - 10r \qquad\qquad\qquad I_B = 50 - 10r$$

where I_r represents residential investment and I_B represents business investment. Remember that total investment is equal to $I_r + I_B$.

a. On the plot below, graph national savings and total investment demand. (6)
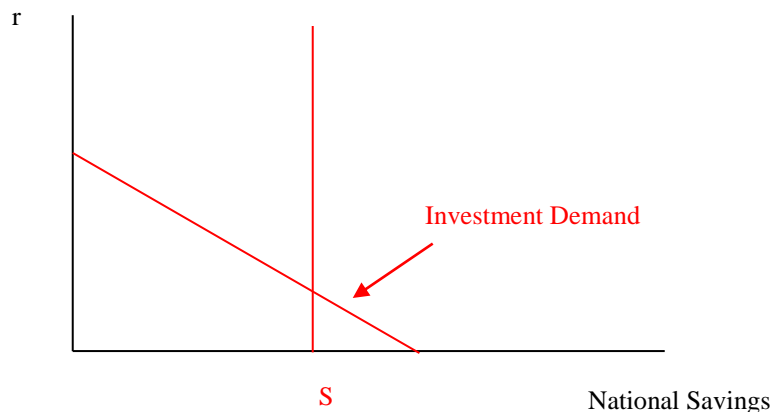
b. What is the equilibrium interest rate in this economy? What is the equilibrium level of investment? (10)
Savings equals investment when: $250 = 100 - 20r$. $r = -7.5$ and $S = I = 250$.

c. In March of 2001, President Bush signed into law a tax bill that gives tax credits to businesses which generates increased business investment demand. How will this policy impact the equilibrium interest rate? What will happen to the total quantity each of business investment and residential investment? (12)
Business investment demand increases while residential investment stays the same leading to an increase in total investment demand. If total investment demand increases then r rises. The amount of savings does not change (it remains 250) and given a greater amount of business investment demand, business investment grows at the expense of residential investment.

d. One flaw with the classical model introduced above is that it doesn't take into consideration the fact that interest rates are positively correlated with government revenues (e.g. during periods of high interest rates, late tax payers pay a greater penalty than during low interest rates). If interest rates positively influence tax revenues, plot the resulting savings and investment curves on the diagram below: (6)
If higher interest rates lead to more taxes and greater taxes reduce consumption less consumption increases savings, then higher interest rates lead to more savings. Graphically, this means that the savings curve is upward sloping.

r

Savings Supply when
interest rates impact taxes

Investment Demand

S   National Savings
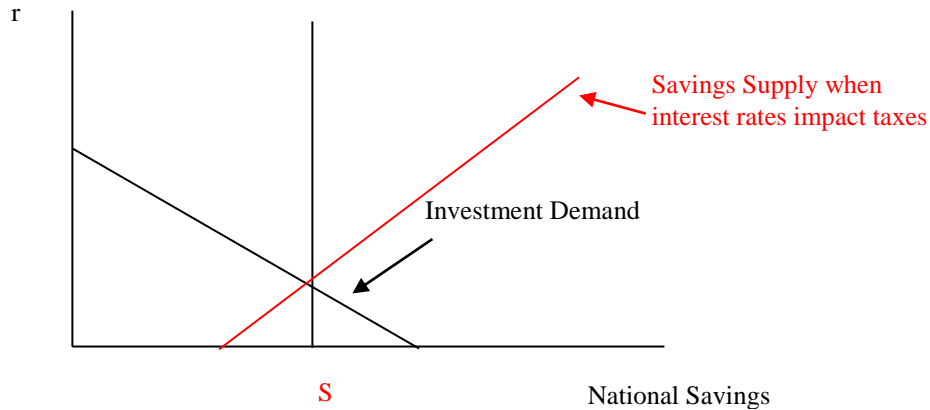
e. Contrast the effect of the Bush tax credit program on interest rates and total investment when interest rates effect tax revenues and when they don't. Under which scenario do interest rates change more? Why? (8)

r

Savings Supply when
interest rates impact taxes

Investment Demand

S   National Savings
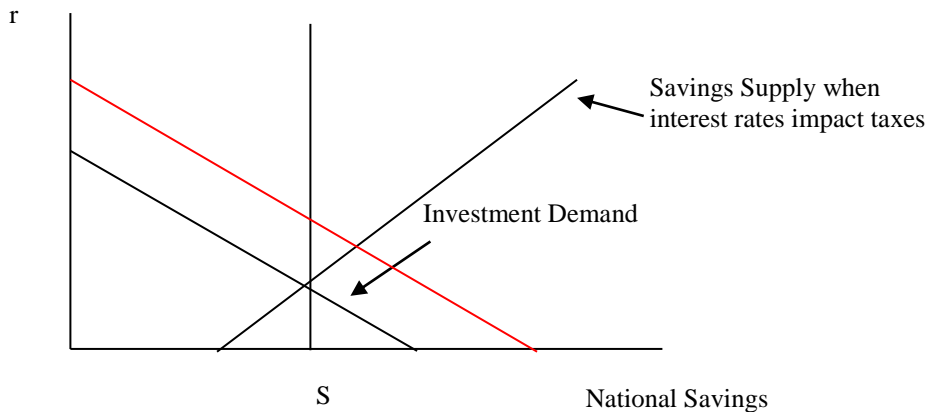
The Bush tax credit increases investment demand. If savings positively depends upon interest rates, then the increase in demand raises rates and encourages savings. Because more savings is flowing into the market, there is less opportunity for interest rates to rise. When savings does not depend upon interest rates the increase in demand just pushes rates up—no mitigating increase in savings. Thus rates rise more when savings does not depend upon interest rates.

A Level Questions
5.   Although the Cobb-Douglas Production function is widely used by economists to describe the production process, certainly some process are not described by this process. For instance, consider the Leontief production function which is defined as $Y = min(aK, bL)$ where a and b are coefficients and the function min indicates that Y is equal to the minimum of the two arguments in parenthesis. An example of this production process might be the number of pipe fittings produced in an hour. Given $a = 10$ and $b = 10$, hiring one unit of labor and no pipe wrenches (capital), one would produce $Y = min(10 \times 0, 10 \times 1) = min(0,10) = 0$ (because the minimum of zero or ten is zero). Likewise, buying a wrench and no labor also produces zero output: $Y = min(10 \times 1, 10 \times 0) = 0$. Hiring more wrenches than workers also produces only the amount that the workers can do (in other words hiring the extra wrench does not increase output).
a. Does the Leontief production function exhibit increasing, decreasing, or constant returns to scale?

The Leontief production function exhibits constant returns to scale. Given any set of inputs, doubling them will lead to an exact doubling of output.

b. What type of returns to labor does the Leontief production function exhibit?
Given a fixed amount of capital, an increase in labor by one unit increases output by a constant amount as long as bL<aK. If bL>aK then an increase in labor by one unit doesn't change output at all—in other words an increase in labor by one unit increases output by a constant amount (zero). Thus, in a sense, there is constant returns to labor.

c. Imagine that a = 10, b = 10, w = 5, r = 10, and P = 20. Can you find the profit maximizing level of labor and capital to hire?
The answer is an infinite amount of labor and capital will produce an infinite amount of profits. Consider the profit function:
Π = 20×min(10×K, 10×L) - 5×L - 10×K. Obviously, hiring either 1 labor and no capital or no labor and 1 capital will produce a negative profit so the minimum hiring of inputs a firm owner would embark with are K = 1 and L = 1. If capital and labor both equal 1, profits will be 200 – 5 -10 = 185. Hiring 2 units each of capital and labor will double profits to 370—thus increasing labor and capital to infinity will produce infinite profits. (because there is constant returns to scale and constant increasing costs).

6.   In most production activities, a large number of inputs are required. This problem asks you to consider a production function that includes two types of workers: skilled and unskilled. The production function is:

$$Y = K^{\frac{1}{4}}L_U^{\frac{1}{4}}L_S^{\frac{1}{2}}$$

where L_U is the number of unskilled workers and L_S is the number of skilled workers.

a. If unskilled workers get paid w_u, skilled workers get paid w_S, capital owners are paid r, and the firm receives price P for each of its goods it produces, write the firm's profit function.

$$\Pi = PK^{\frac{1}{4}}L_U^{\frac{1}{4}}L_S^{\frac{1}{2}} - w_uL_u - w_SL_S - rK$$

b. Solve for the equilibrium real wage for unskilled workers. What happens to wages unskilled workers earn when capital increases? Why?

$$\frac{d\Pi}{dL_U} = \frac{1}{4}PK^{\frac{1}{4}}L_U^{-\frac{3}{4}}L_S^{\frac{1}{2}} - w_u = 0 \Rightarrow \frac{w_u}{P} = \frac{K^{\frac{1}{4}}L_S^{\frac{1}{2}}}{4L_U^{\frac{3}{4}}}$$

Increase in capital raises unskilled wages because more capital increases the marginal productivity of the unskilled.

c. Solve for the equilibrium real wage for skilled workers. What happens to wages skilled workers earn when the number of unskilled workers increase? Why?

$$\frac{d\Pi}{dL_S} = \frac{1}{2}PK^{\frac{1}{4}}L_U^{\frac{1}{4}}L_S^{-\frac{1}{2}} - w_s = 0 \Rightarrow \frac{w_S}{P} = \frac{K^{\frac{1}{4}}L_U^{\frac{1}{4}}}{2L_S^{\frac{1}{2}}}$$

Skilled worker's wages rise as more unskilled workers are hired because more unskilled workers raise skilled MPL.

d. Imagine that the firm sells its products for $100 each, owns 1 unit of capital, and pays a w_U = $10 per hour and a w_S = $12.5 per hour. How many unskilled workers does this firm hire? How many skilled workers does this firm hire? How much total output is created?

Both of the above Marginal Products of Labor are used to determine how much skilled and unskilled labor are hired. After plugging in $w_S = \$12.5$, $w_U = \$10$, $K = 1$, and $P = 100$, the 2 MPL's are:

$$.1 = \frac{L_S^{1/2}}{4L_U^{3/4}}$$

$$.125 = \frac{L_U^{1/4}}{2L_S^{1/2}}$$

Using the first equation, I determine that:

$L_S^{1/2} = .4L_U^{3/4}$. I will then substitute this expression for L_S into the second equation and find:

$$.125 = \frac{L_U^{1/4}}{2\left(.4L_U^{3/4}\right)} = \frac{L_U^{1/4}}{.8L_U^{3/4}}.$$ From here we solve for L_U and discover that $L_U = 100$. Plugging $L_U = 100$ into

either of the above MPLs gives $L_S = 160$. 100 unskilled workers are hired and 160 skilled workers are hired.

e. Some activists argue for a more equal wage distribution in the United States at a "living wage." Imagine that the living wage in the above economy was $12.50 per hour and Congress passed a law forcing firms to pay each worker, whether unskilled or skilled, $12.50 per hour. Describe the effects of this legislation on the economy. What happens to total production? What happens to the number of unskilled and skilled workers hired? WHY? (10)

Higher W_U means fewer L_U will be hired. As fewer L_U are hired, the marginal product of L_S falls (remember the answer to part b). Since the MPL of L_S is lower, the firm does not want to hire as many L_S workers so it lays off both the unskilled (because the unskilled cost more) and skilled (because they are less productive). With fewer of both the skilled and unskilled, this firm reduces production.