Answers to Chapter 7 Problems

C Level Questions

1. The following question deals with the idea of a Natural Rate of Unemployment.

a. Data on a recent survey of WWU students is as follows: students can be classified as "dating" and "single." For each student, 10 percent of those that are single become dating and among those that are dating, 5 percent become single. If there are 1000 WWU students, what is the steady state number of students who are dating?

Three pieces of information are given in this problem: The number of students (P) are equal to 1000. We know that people can either be single or dating; thus P = S + D where S represents single and D dating. Thus, the "dating rate" is given by:

$$\frac{D}{P} = 1 - \frac{S}{P}.$$

Finally, we know that in equilibrium, the number of those entering a relationship (.10S) is equal to those who break-up (.05D); that is .10S = .05D or 2S = D. Substituting all of this information into the above equation gives:

$$\frac{2S}{1000} = 1 - \frac{S}{1000}.$$

Solving this gives S = 333.33. Thus, the steady state number of students dating is equal to 666.66.

b. Imagine that in order to stimulate campus harmony, the psychology department decided to subsidize tuition payments of students that were dating. How would this effect your answer to part a?

By subsidizing dating students, one would expect more people to want to date and fewer people to leave dating relationships. Thus, the percent of those entering relationships should rise above 10 percent and those leaving dating relationships should fall below 5 percent. Of course, this will lead to a higher number of students dating.

B Level Questions

2. Many jobs involve advancement after observation of employees. For instance, professors are either "pre-tenure" or tenured professors. This problem is intended to ask about the role of shirking and working hard in markets for laborers that advance by stages.

a. Consider the market for professors that is described by the equations:

$$Y = 800L^{1/2} \qquad\qquad L^s = \left(\frac{W}{P}\right)^2$$

where Y represents the production of professors (perhaps measured in added IQ points of their students). L^s represents the labor supply of professors.

Solve for the labor demand of professors and then solve for the equilibrium real wage and numbers of professors hired.

Labor demand is given by the equality of MPL and the real wage. Given the production function $Y = 800L^{1/2}$, the $MPL = \frac{400}{L^{1/2}}$. Setting this equal to the real wage gives the labor demand equation of $\frac{W}{P} = \frac{400}{L^{1/2}}$.

Setting labor demand equal to labor supply gives an equilibrium real wage of 20 and an equilibrium L = 400. If 400 individuals are hired, Y = 16,000.

b. One way to view the career of professors is to consider their employment consisting of two periods: before tenure and after tenure. To make this problem as easy as possible, imagine that in both periods professors earn a real wage of $\frac{W}{P}$. Further, imagine that professors make a one time decision upon being hired in the first period of working hard or shirking. Regardless of their decision, each professor takes home $\frac{W}{P}$ at the end of their first period. Professors who work hard will survive into the second period with probability p = $3/4$. Professors who shirk will survive into the second period with p = $2/3$. Professors who work hard receive payment of $\frac{W}{P}$ from their universities but also incur a cost of working hard that is valued at one unit of real wage. Assume that by working hard, the cost is incurred both in period 1 and period 2. Those who choose not to work hard incur no cost in either period. Finally, assume all professors, regardless of their work-level, choose to retire at the end of the second period.

How much should this firm pay in order to insure that all professors hired will work hard?

The expected value of the job for a shirker is $\frac{W}{P}+\frac{2}{3}\frac{W}{P}$. The first term represents the value to the worker of working the first period. Since the worker holds their job with 2/3rds probability in the second period, the second term represents the expected (average) amount expected to be received during that second period. The expected value of the job for a hard worker is given by $\left(\frac{W}{P}-1\right)+\frac{3}{4}\left(\frac{W}{P}-1\right)$. The first term is the value of working the first period net of the cost of working hard. The second term is the value of the second period. Since a firm wants to hire only hard workers, the firm must pay a wage such that $\left(\frac{W}{P}-1\right)+\frac{3}{4}\left(\frac{W}{P}-1\right)>\frac{W}{P}+\frac{2}{3}\frac{W}{P}$. Solving this for the real wage gives $\frac{W}{P}>21$.

c. If universities choose to pay their professors just enough to get them to work hard, what would the resulting unemployment rate equal?

A university paying a real wage of 21 will attract 21^2 = 441 applicants and demand $(400/21)^2$ = 362.8 workers leaving 78.2 workers unemployed for an unemployment rate of 17.7%.

3. Imagine that the heavy construction labor industry can be described by the following equations:

$$\text{Labor Supply: } \frac{W}{P}=5+2L$$

$$\text{Labor Demand: } \frac{W}{P}=20-L$$

Where L measures thousands of workers and W/P is the real wage.

a. What is the equilibrium real wage in this market? How much labor is hired? What is the unemployment rate in equilibrium?

To find the equilibrium, one remembers that supply must equal demand or 5+2L = 20 – L. It is straightforward to show L = 5 and W/P = 15.

b. As the construction industry is highly variable, workers in this industry have a 5% chance each month of being laid off. What is the expected value of this job to the worker?
Workers with a 5% chance of being laid off each month can expect to hold their job for 20 months (1/.05). Thus, at a wage of 15 per month, the value of this job is 300.

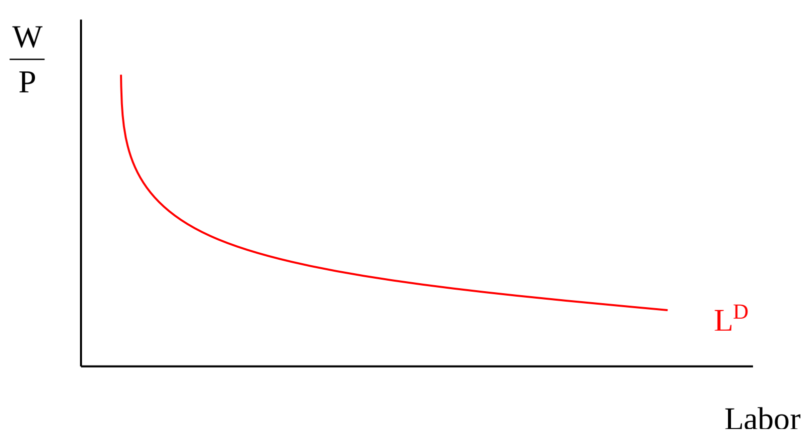
c. Imagine that when going to work, workers make a decision to work hard or to slack-off while working. If workers work hard they incur an extra burden in the form of sore muscles that costs the worker an equivalent of 6 units of real wage. Workers that slack incur no such additional costs but will be fired with 15% more frequency than workers that work hard (they lose their jobs with 20% probability as opposed to 5%). If firms want all of their workers to work hard, what wage will firms pay their workers? What is the unemployment rate?
As long as hard work is profitable to firms, firms will want to get their workers to work hard. To do this, firms will pay a wage that makes a job worked hard more valuable than a slacker's job. A firm will want to set the wage at:
$\frac{W-6}{.05} > \frac{W}{.2}$ where the value of a job worked hard is on the left and the value of a slacked job is on the right.
Solving for W gives W > 8. In order to get workers to work hard, firms must pay a real wage of at least 8. Since the equilibrium wage is already 15, the workers are working hard and firms would not choose to pay more. Hence the economy remains in equilibrium and no unemployment exists.

d. Does your answer to part c change if the cost of working hard is 12 instead of 6? If so, what is the new unemployment rate?
Yes. In this case workers will work hard if:
$\frac{W-12}{.05} > \frac{W}{.2}$ or W > 16. This firm will pay a wage of 16 units in order to get workers to work hard. At this wage, the firm demands 4 workers and households are willing to supply 5.5 units of labor leaving 1.5 units of labor unemployed. The unemployment rate is therefore 1.5/5.5 = 27.27%.

4. Imagine a firm that produces t-shirts with a production function of:

$$Y = 30L^{1/2}$$

where L is measured in thousands of people.
a. On the plot below, draw this firm's demand for labor. What is the equation for this firm's demand for labor?



$\frac{W}{P} = MPL = \frac{15}{L^{1/2}}$.

b. The labor supply in this market is given by $\frac{W}{P} = 5L$. What is the equilibrium level of labor hired? What is the equilibrium real wage? How many t-shirts does this firm produce?

Setting supply equal to demand gives $5L = \frac{15}{L^{1/2}} \Rightarrow L^{3/2} = 3 \Rightarrow L = 3^{2/3} \Rightarrow L = 9^{1/3} \Rightarrow L = 2.08008$. The equilibrium wage is therefore $5\times2.08008 = 10.4$ and the firm produces $30\times9^{1/6} = 43.267$.

c. A large number of t-shirt employees have unionized and demand a real wage of 12. If the labor demand curve remains the same and the t-shirt companies agree to this wage, how many workers will get hired? How many t-shirts will this firm produce?

The firm will hire labor such that $12 = \frac{15}{L^{1/2}} \Rightarrow L^{1/2} = \frac{15}{12} \Rightarrow L^{1/2} = 1.25 \Rightarrow L = 1.25^2 = 1.5625$. The firm will produce $30\times(1.25^2)^{.5} = 37.5$.

d. What is the unemployment rate when the unions negotiate a real wage of 12?
The firm hires 1.5625 workers of the L = 12/5 = 2.4 workers who want a job. The unemployment rate is thus (2.4 – 1.5625)/2.4 = 34.89%

e. Discuss the impact the union's higher wages have on workers. Address the impacts on total payments to workers as well as impact on individual groups of workers.
The 1.5625 workers who retain their job are made better off at the expense of the 2.08008-1.5625 = .51758 workers who lost a job they had in equilibrium. Total payments to workers fall from 2.08008×10.4 = 21.632 to 1.5625×12 = 18.75 so workers are made worse off in general. Of course the question didn't ask but it is also clear that the firm is worse off (it produces fewer t-shirts) and society is worse of (it has fewer t-shirts which raises the price of shirts).