Chapters 7 and 8 Problems

One tool that can help students understand the Solow Growth model are my Excel "Helps." I would suggest working through these as you proceed with the problems below. The helps are located at: http://www.cbe.wwu.edu/krieg/Econ307/Excel%20Spreadsheets/Excel%20Spreadsheets.htm.

C Level Questions

1. Consider the Solow Growth Model which has technological progress and population growth. The economy is described by:

$$y = k^{.3}$$
$$s = .1$$
$$\delta = .03$$
$$n = .02$$
$$g = .02$$

a. Solve for the steady state level of capital per capita and output per capita. In the steady state, how fast does capital per capita grow? How fast does output per capita grow? How fast does total output grow?

b. 15 years ago (1991), American Real GDP was $6,720.9 billion and the labor force was 117,770 thousand people. 10 years later (2001), American Real GDP was $9348.6 billion and the labor force was 134,253 thousand people. Using these numbers and assuming the Solow Growth model is correct, determine the average annual technological growth rate for the United States over the last ten years.

2. The purpose of this problem is to simulate the Solow Growth model using Excel (or a similar spreadsheet. At the completion of this problem, you should be able to identify steady state levels of growth per capita, the speed of economic growth, and how the per capita variables translate into the total production, labor, and capital in an economy.

For this entire homework, you will use the following equations:

$$Y = K^{\frac{1}{3}}L^{\frac{2}{3}}$$
$$y = k^{\frac{1}{3}}$$
$$\text{savings} = s\,k^{\frac{1}{3}}$$
$$\text{depreciation} = \delta k$$

In the Solow Growth chapter, Mankiw estimates that for the United States, δ=.04 and the average savings rate in the U.S. over the last 30 years is s=.068. Using these values, create a spreadsheet that runs for 400 periods that contains the following information in period 1.

Period	y	k	savings	depreciation	c	Y	K	L
1		1.15						100
2								100

A few notes:
A. We begin by assuming that capital per person is 1.15. This will grow over time based upon the difference between savings and depreciation.
B. We also assume that there are 100 people in our economy for each year; this will not change until you are asked to change it.

C. You will need to fill in the rest of the blanks with formulas that compute the relevant numbers and copy down for 400 periods. The best way to do this is to compute the per capita variables according to the equations in Chapter 4 and then compute the variables Y and K by remembering Y=L*y and K=L*k. After you've succesfully done this for the first and second year, you should be able to use the "copy down" feature in Excel to paste your new equations in the remaining time periods.

Questions:
a. Given s=.068 and δ=.04, mathematically find the steady state level of k and y (this does not require Excel—as a matter of fact, you should attempt this before running any Excel program).
b. After 400 periods, have the values of y and k reached their steady state levels? Why or why not? Plot and print the values of y and Y to help answer this question.
c. What is the growth rate of total output between period 1 and period 400? Compare this to the growth rate between two periods (1 and 10) and (391 and 400). Which subperiod grows faster? Why? Is the growth rate of total output different than the growth rate of per capita output? Why or why not?
d. Now imagine that each period, the labor force grows by 2% (n = .02). Mathematically solve for the steady state level of capital per person and output per person.
e. Produce another computer model similar to the one above including the growth in labor force. How much do y and Y grow in the steady state? Plot and print both y and Y over time. Does this match what we observe in the United States?

3. [This question is meant to address the assumption that "per capita" refers to per worker variables rather than per population variables]

Consider the standard Solow Growth model where output is given by $Y = K^{1/2}L^{1/2}$. However, the population of this economy is given by P. Assume that a constant percentage, ψ, of the population chooses not to participate in the labor force (so $L = (1 - \psi)P$).

a. Solve for the per-population production function (I'll denote this *y* as opposed to y which will remain the per worker production function). Compare this to the per worker production function.

b. Given the evolution of capital through time is given by $K_{t+1} = (1 - \delta)K_t + sY_t$, solve for the per-population equation that describes the evolution of capital over time.

c. Use the equation found in parts a and b to solve for the steady state level of capital per population and output per population. How does this compare to the steady state level of capital per worker and output per worker?

d. What happens to the steady state level of capital per population as ψ falls to zero? Explain.

B Level Questions

4. Suppose the economy of Marineland can be described by the following equations:

$$y = k^{\alpha}$$
$$0<\alpha<1$$
$$\text{savings} = s\,k^{\alpha}$$
$$\text{depreciation} = (\delta + n)k$$

a. Solve for the steady state level of capital per capita, output per capita, and consumption per capita.

b. Solve for the golden rule level of savings. At this level of savings, what is the steady state level of output per capita, capital per capita, and consumption per capita?

c. Suppose Marineland was saving at the golden rule level of savings and in the steady state. At time t_0, a hurricane strikes Marineland destroying half of the country's capital but not killing any citizens. Draw the time sequences of the subsequent values of y, c, i=s, and total output. An example of time sequences is given in figure 4-10 on p. 198 and 199 of Mankiw.

d. Starting again at the steady state with the golden rule level of savings, suppose at time t_0 Marineland had a sudden influx of immigration that doubled the size of the labor force. After this influx, the amount of labor remains steady (at twice its original level). Draw the time sequences of the values of y, c, i=s, and total output after time t_0.

5. In the Solow Model with population and technological growth, consumption is maximized when the slope of the production function (the MPK) is equal to the slope of the depreciation curve $(n + \delta + g)$ where n measures the population growth rate and g measures the technological growth rate. A good first guess at American marginal productivity of capital is .12 (a 1 unit increase in capital leads to a .12 unit increase in production). Good guesses at the American depreication rate is 4%, and since real GDP has grown at about 3% for the last 30 years, a good guess for $n + g = .03$ (see section 8-2 of Mankiw for details). Given these figures, is America at the Golden Rule level of consumption? How can you tell? If not, explain what America needs to do to get closer to the Golden Rule.

Do problems #3 and 6 on p. 206 of Mankiw.
Do problem #1 in problems and applications on p. 227 of Mankiw.

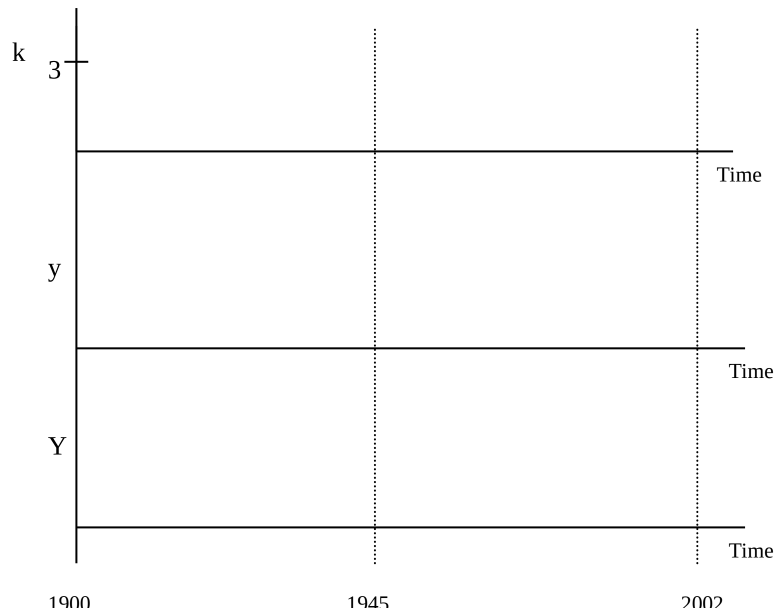
A Level Questions

6. Imagine the country of Japan could be described as a Solow model of the economy with the equations:

$$y = k^{1/4} \qquad s = .10 \qquad \delta = .04$$

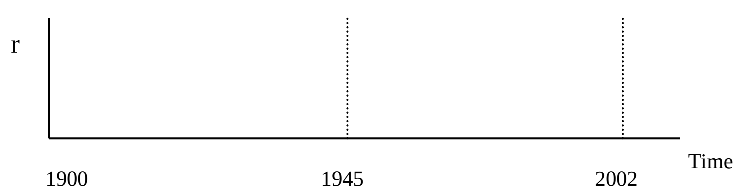
a. Solve for the steady state level of capital and output.

b. Imagine that in 1900, the country of Japan had 3 units of capital per person and followed the Solow growth model found in part a. On the plots below, graph the Solow model's prediction of y, k, and Y between 1900 and 1945.



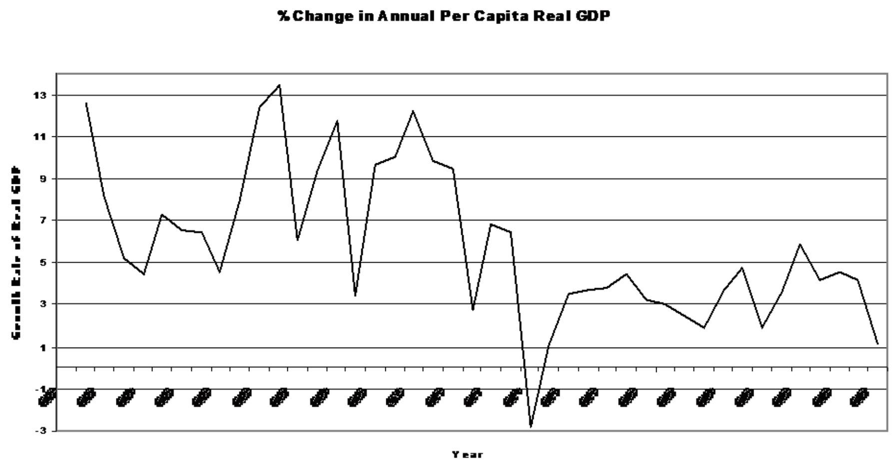
c. In 1945, Japan was the target of a number of large, heavy, explosive devices. Let's imagine that these devices destroyed half of the country's capital without damaging the Japanese population. On the above plot, show the effect of the capital destruction on y, k, and Y between 1945 and 2002. Be sure you show the Japanese economy achieving steady state by 2002.

d. In the plot below, show what happens to the Japanese real interest rate over the time period described in parts b and c. Hint: Imagine that investment demand grows at the same rate as real GDP.



e. The savings rate of 10% in the Japanese economy is not the golden rule savings rate. Solve for the golden rule savings rate for the Japanese economy. What is the significance of the golden rule?

f. Below is a graph of per capita annual growth rates in Japan. Do they follow what the Solow model predicts? Why or why not?



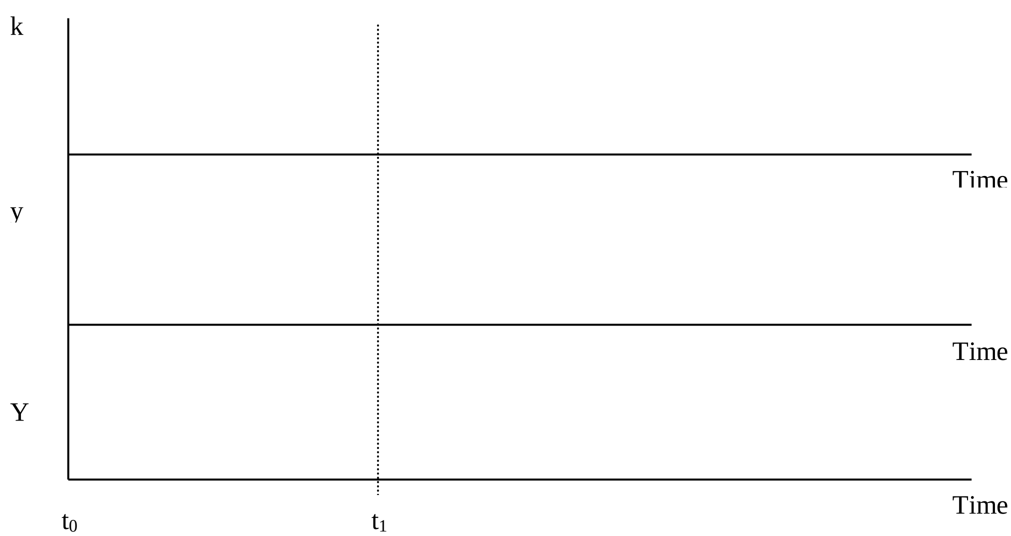
7. Consider the following Solow growth model:

$y = k^{.25}$ $s = .08$ $n = .02$ $\delta = .08$

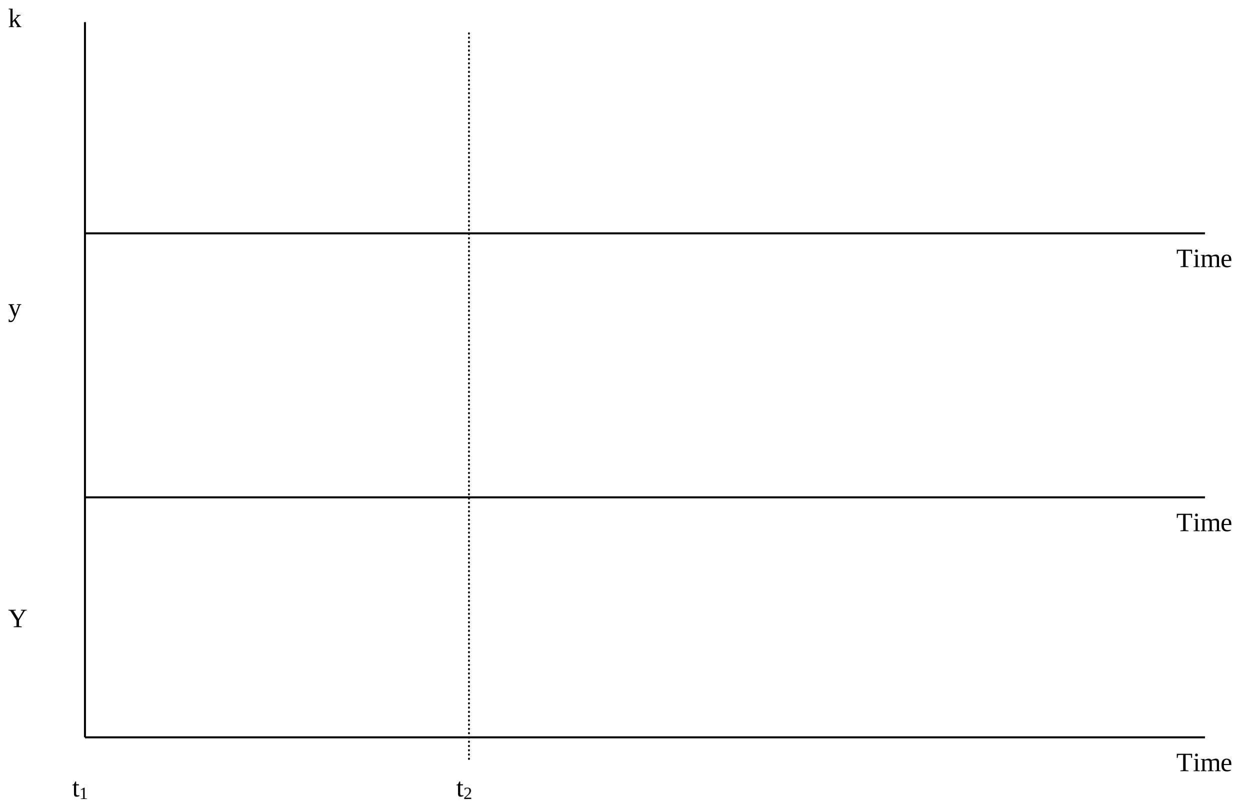
a. Solve for the steady state level of capital per capita and output per capita.

b. Is this economy at the golden rule? If not, give one governmental policy that will move this economy closer to the golden rule.

c. Imagine the economy described in part a has 1 unit of capital per person. On the graphs below, chart the progress over time of this economy. Assume that by time t_1, this economy achieves the steady state. Be sure to plot what happens to this economy over time after it reaches the steady state.



d. Imagine that at time t_2 the depreciation rate rose from 8% to 10%. On the charts below demonstrate the impact of this change on the Solow economy. Be sure to show what was happening in the economy before time t_2 and after the economy reaches its new steady state.



In general: Consider variations to the Solow Growth model that make the model fit reality better. For instance, what happens if the savings rate rises and income grows? What if the depreciation rate changes as more capital is accumulated?