Now, ρ_1 and $\bar{M}^{-1}\rho_2 M : \Pi \to GL_n(R_i)$ with same image under ϕ_0

$\Rightarrow (\rho_1, \bar{M}^{-1}\rho_2 M)\cdot\Gamma_n(R_3) \in E_3/\Gamma_n(R_3)$ whose image under b is the given $(\rho_1\Gamma_n(R_1), \rho_2\Gamma_n(R_2))$ #

Let $\rho_2 \in E_2$, $\rho_0 \in E_0$, $\rho_0 = \phi_2(\rho_2)$.

<u>Def</u>: $G_i(\rho_i) := \{ g \in \Gamma_n(R_i) \text{ s.t. } g\rho_i = \rho_i g\} \leq \Gamma_n(R_i)$

<u>Lemma 3.5.</u> If for all $\rho_2 \in E_2$ the map $G_2(\rho_2) \xrightarrow{\phi_2} G_0(\rho_0) \to 1$

$\Rightarrow b$ is injective.

<u>Proof</u> Supp $\vec{\rho} = (\rho_1, \rho_2) \in E_3$, $\vec{\psi} = (\psi_1, \psi_2) \in E_3$ inducing $\rho_1, \psi_1 \in E_1$, $\rho_2, \psi_2 \in E_2 \rightsquigarrow \rho_0 \in E_0$, ψ_0

$\vec{\rho} \mapsto \rho_0$, $\vec{\psi} \mapsto \psi_0$

Supp $\exists M_i \in \Gamma_n(R_i)$ s.t. $\rho_1 = M_1^{-1}\psi_1 M_1$, $\rho_2 = M_2^{-1}\psi_2 M_2 \Rightarrow \rho_0 = \bar{M}_1^{-1}\psi_0\bar{M}_1 = \bar{M}_2^{-1}\psi_0\bar{M}_2$

$\Rightarrow \bar{M}_2\bar{M}_1^{-1}\psi_0 = \psi_0\bar{M}_2\bar{M}_1^{-1} \Rightarrow \bar{M}_2\bar{M}_1^{-1} \in G_0(\psi_0) \Rightarrow \exists N \in G_2(\psi_2)$ mapping by ϕ_2 to $\bar{M}_2\bar{M}_1^{-1}$.

$N_2 := N^{-1}M_2 \Rightarrow N_2^{-1}\psi_2 N_2 = M_2^{-1}N\psi_2 N^{-1}M_2 = M_2^{-1}\psi_2 M_2 = \rho_2$.

Now, $\bar{N}_2 = (\bar{M}_2\bar{M}_1^{-1})^{-1}\bar{M}_2 = \bar{M}_1$. Since M_1, N_2 have same image in R_0, the pair $(M_1, N_2) = M \in \Gamma_n(R_3)$ s.t. $M^{-1}\underbrace{(\psi_1,\psi_2)}_{=\psi}M = \underbrace{(\rho_1,\rho_2)}_{=\rho}$ #

<u>Lemma 3.6</u> Property H3 is true

<u>Proof</u>: $R_0 = K$, $R_2 = K[\epsilon]$. By H1 ⊛ is surjective ($R_2 \to R_0$ small)

Now, $G_2(\rho_2) \to G_0(\rho_0) \to 1$ $\rho_2 : \Pi \to GL_n(K[\epsilon])$ $\rho_0 : \Pi \to GL_n(K)$, $\rho_0 = \bar{\rho}$

$G_0(\bar{\rho}) = G_0(\rho_0) = \{I\}$ since $\leq \Gamma_0(R_0) = \{I\} \Rightarrow$ Surjectivity is trivial

and by 3.5 ⊛ injective #.